

bulky and fire-hazardous equipment for storage of an onboard reserve of Hydrogen. The research of toxicity indexes of the fuel-filled for diesel gases with use of regular diesel fuel was conducted at the motor stand for three modes of its loading – 0.8; 1.0; 1.2 kW. At the following stage researches for the same load conditions with addition to fuel of the Hydrogen microdoses in a fraction – 0.49 ; 1.15; 1.83% (on mass) have been executed. Motor influence research results of hydrogen microdoses on diesel midget of toxicity indicators 1Ч8,5/11 have been brought to diesel fuel. The effect consists in the composition exception of the exhaust gases of the unburnt hydrocarbons and weakening of white damp in EG practically up to 0 and concentration of nitrogen oxide for 14–26%.

**Keywords:** hydrogen; diesel; toxicity of the exhaust gases; microdoses.

UDC 504.064.4 : 621.431.063 : 389.14 : 528.088

DOI: 10.20998/0419-8719.2019.1.09

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## DESCRIPTION OF MASS HOURLY EMISSIONS OF PARTICULATE MATTER OF DIESEL ENGINE BY BETA-DISTRIBUTION WITH TAKING INTO ACCOUNT THE PASSPORT ACCURACY OF GAS ANALYZER

*Relevance of the study is that qualitative and quantitative results of solving of its tasks suitable for developing of methodic of decreasing of methodical errors of determination of values of mass hourly emission of particulate matter in exhaust gas flow of reciprocating internal combustion engine with using of conversion formula and readings of opacimeter and gas analyzer. Purpose of the study is obtaining of parameters of beta-distribution that approximate the empirical law of distribution of values of mass hourly emission of particulate matter in exhaust gas flow of reciprocating internal combustion engine which achieved by indirect measuring with taking into account the accuracy of direct measuring of its components that has nonlinear impact. Task of the study is obtaining of rational values of number of multiple measuring of coefficient of attenuation of light flux and volume concentration of unburned hydrocarbons in exhaust gas on individual operational regime of diesel engine during bench motor tests for case of automation of measuring process. Methodic of the study is in consistently application of following methods: analysis of scientific and technical literature, analysis of data of bench motor tests, application of prof. Parsadanov conversion formula, mathematical apparatus of beta-distribution, numerical calculation studies. It was detected that empirical distribution of values of mass hourly emission of particulate matter in exhaust gas flow of reciprocating internal combustion engine which obtained with using of one of known conversion formula of prof. Parsadanov as the function of indicator of opacity and toxicity of exhaust gas, has a significant difference from the normal distribution law at number of measurements less than 50 even in case of confirmation of the hypothesis of normality of distribution law of readings of opacimeter into limits of its passport measuring errors. That results were described by the mathematical apparatus of beta-distribution by carrying out the numerical calculation studies with using of program product written of the language Borland Pascal 7.0. The parameters of beta-distribution were obtain as the function of number of multiple measuring.*

**Key words:** beta-distribution; opacimeter; multi-component gas analyzer; internal combustion engine; accuracy of measuring, engine bench tests; environment protection technologies; ecological safety.

### Introduction and relevance of the study

Vehicles with diesel reciprocating internal combustion engine (RICE) are powerful source of legislative normalized pollutants from the number of which particulate matter (PM) has the second place by the value of reduced toxicity [1 – 5]. PM are the disperse phase of aerosol of RICE exhaust gas (EG) and formed as the product of uncompleted combustion of motor fuel and consists of carcinogen substances and also forming the city smog [1 – 5].

Results of quantitative assessment of amount of emission of such pollutant in accordance with normative documents [6] usually expressed in units of mass hourly emission  $G_{PM}$  – kg/h, namely the assessment executes by the experimental way with using indirect measuring by gravimetric method during motor bench tests of PICE on stationary standardized testing cycle with using of complex of measuring equipment that na-

med as dilution tunnels. However, such complex of measuring equipment is difficult and high cost (from a hundred thousand to millions US dollars) and also should be certificated [1]. Nowadays on Ukraine territory there is only one certificate dilution tunnel of foreign manufacturing. That means what in most of departments of scientific institutions in our country that implements the scientific and researching works on topics which related with PICE and because of this fact they laboratories has motor test benches do not have the opportunity of obtaining the values of  $G_{PM}$ .

For execution of experimental researches the results of which can be compared with results that obtained with using of dilution tunnels, scientists of that departments develops and uses different conversion formulas which allows to convert the readings of more affordable, cheaper and common measuring equipment into values of  $G_{PM}$ . To the number of such measuring

equipment it can include opacimeters and multicomponent gas analyzers of different constructions because main components of PM are soot cores and adsorbed and condensed on its surfaces unburned hydrocarbons of motor fuel. To the number of such formulas it can include Parsadanov [3], Alkidas [7], Muntean [8] and MIRA [9].

Such conversion formula is function that related the value of  $G_{PM}$  with readings of opacimeter (smokiness – coefficients of weakening of the light flux  $N_D$  in % or coefficients of weakening or transmission of the light flux  $K$  in  $m^{-1}$ ) and also gas analyzer (volume concentration of unburned hydrocarbons in EG flow  $C_{CH}$  in ppm). That dependence has nonlinear character [1]. Such measuring equipment – opacimeter and multicomponent gas analyzer – are characterized certain values of accuracy of measurement which contains in its passports [10, 11].

As it known from the basics of metrology the character of distribution of random values of results of indirect measurements (because case of application of such conversion formulas can be considered as indirect measurement) in case of nonlinear dependences of its results from values of results of direct measurements (readings of opacimeter and multicomponent gas analyzer) has the character that substantially differs from the normal distribution law in values of asymmetry and excess. Wherein such effect present even in case of accordance of distribution laws of results of direct measurements to the normal law what caused by instrumental errors of measuring equipment. The effect is stronger than smaller is the number of repeated measurements at the same value of mathematical expectation (that means on the same regime of operational regimes area of PICE).

During the bench motor testing usually are limited to from 3 to 10 times measuring on one regime point. That means what for obvious reasons it is not expected availability of initial data set with 50 ... 1000 values (for different recommendations [12]) without automation of measuring process and joining the measuring equipment into single complex. Among these reasons are time of sampling and actual measurement, inertia of the device and also monetary and labor costs for the experiment.

As shown in studies [12, 13] in this case for more adequate description of observed phenomenon to physical reality the empirical distribution law of  $G_{PM}$  value in the form of histogram can be approximated with using of family of curves of probability densities of Pearson which not close to normal distribution law, namely beta-densities.

However, such studies have not yet been carried out therefore obtaining of parameters of beta-distribu-

tion of  $G_{PM}$  value which defined by known conversion formula with taking into account the passport accuracy of measuring equipment, readings of which has nonlinear influence on observed physical value, for several PICE special operational regimes, is relevant scientific and technical task which have scientific novelty and practical value. Qualitative and quantitative results of solving of the task are suitable for development of methodic of decreasing of value of methodic error of determination of  $G_{PM}$  value and also appropriate recommendation list.

*Purpose of the study* is obtaining of parameters of beta-distribution that approximate the empirical law of distribution of values of mass hourly emission of PM in EG flow of piston ICE which achieved by indirect measuring with taking into account the accuracy of direct measuring of its components that has nonlinear impact. *Object of the study* is distribution law of values of mass hourly PM emissions in PICE EG flow. *Subject of the study* is parameters of beta-density which approximate object of the study.

#### **Analysis of mathematical apparatus and features of application of prof. I.V. Parsadanov conversion formula**

Nowadays are known several conversion formulas for transformation of values of indicators of PICE EG smokiness into values of PM mass hourly emission, but only formula of prof. I.V. Parsadanov (NTU «KhPI») takes into account values of volume concentration of unburned hydrocarbons of motor fuel in EG flow [3, 7 – 9]. That dependence after certain transformations described by formulas (1) – (3).

$$G_{PM} = (a \cdot N_D + b \cdot N_D^2 + c \cdot C_{CH} + d \cdot C_{CH}^2) \cdot k, \text{ kg/h (1)}$$

$$a = 2,3 \cdot 10^{-3} \text{ kg/(h}\cdot\%), \quad b = 5,0 \cdot 10^{-5} \text{ kg/(h}\cdot\%^2),$$

$$c = 0,145 \cdot f, \text{ kg/(h}\cdot\text{ppm)}, \quad d = 0,33 \cdot f^2 \text{ kg/(h}\cdot\text{ppm}^2),$$

$$f = \frac{4,78 \cdot 10^{-3} \cdot (G_{air} + G_{fuel})}{0,7734 \cdot G_{air} + 0,7239 \cdot G_{fuel}}, \quad (2)$$

$$k = 0,001 \cdot (0,7734 \cdot G_{air} + 0,7239 \cdot G_{fuel}), \text{ kg/h. (3)}$$

#### **Family of Pearson family curves**

In Mathematical Statistics execute of approximation basing on typical distribution laws such as normal, logarithm-normal, exponential, Waybill, gamma-distribution and others. The advantage of application of typical distribution laws is they well-study and opportunity for gating of grounded, unassembled and relatively highly effective evaluations of parameters. However, the above-mentioned typical distribution laws do not have the necessary variety of forms, therefore their application does not provide the necessary generic representation of random variables.

Moments of the distribution of a random variable do not characterize it completely, but they define it

uniquely under certain conditions, which are executed for almost all distribution laws that are used in practice. When solving the problems of experimental data processing, knowing values of the moments is equivalent to knowing the distribution function and the coincidence of the values of first moments of the two distributions indicates that they are approximately uniform. In the case of not knowing the exact form of the distribution function, but with knowing values of the first moments, it is possible to select another distribution law with similar values of the first moments. In practice, such an approximation gives good results if values of the first four moments are coincide.

It is considered [12, 13] that an arbitrary nonnegative function  $f(y) \geq 0$  that satisfying the conditions of normalization  $\int_{-\infty}^{\infty} f(y) dy = 1$  can be considered as

the probability density  $P(y)$  of some random variable. The various nature of probability densities  $P(y)$  is given by the Pearson curves system, which is given by the differential equation (4), where coefficients  $a$  and  $b_i, i = 0,1,2$  completely specify the distribution system. Solving of the equation is written in the general form of equation (5). Conducting recurrent transformations, determine the senior moments through the younger ones, what allows determine the constants  $a$  and  $b_i$  through the selective estimates of the central moments of distribution and lead to a system of equations (6). In the general case, the Pearson distribution is determined by four moments  $\tilde{m}_1, \tilde{\mu}_2, \tilde{\mu}_3, \tilde{\mu}_4$ . The solution to this system is summarized in Table. 1. It is known that the nature of the curve may be different depending on the value of the roots of the equation  $b_0 + b_1y + b_2y^2 = 0$ . Marking the roots of the equation through  $y_1$  та  $y_2$ , their value is determined by the equation (7). For certainty signs are chosen in such a way that  $y_1 < y_2$ . The values of the roots depend on the magnitude of value  $K$ . If  $K < 0$ , than the roots are real and have different signs (Type I distribution by Pearson classification). If  $K > 1$ , than the roots are real and have different signs (Type VI distribution by Pearson classification). If  $0 < K < 1$ , than the roots are complex (Type IV of distribution). Type I of distribution by Pearson classification is beta-distribution which can be written in different forms, but the usual way is to write in the form of equation (8). The standard beta distribution is focused on a segment from 0 to 1. Through linear transformations, the beta-value can be converted so that it will take values at any interval. Type VI distribution is a beta distribution of the II kind and expressed in the form of equation (9). There is nothing defi-

nite to say about the type of IV distribution, there are only isolated cases of this distribution

$$\frac{dP(y)}{dy} = \frac{y-a}{b_0 + b_1y + b_2y^2} P(y), \quad (4)$$

$$P(y) = C \exp\left(\int \frac{y-a}{b_0 + b_1y + b_2y^2} dy\right). \quad (5)$$

$$\begin{cases} -a + b_1 = 0 \\ b_0 + 3b_2\tilde{\mu}_2 = -\tilde{\mu}_2 \\ -a\tilde{\mu}_2 + 3b_1\tilde{\mu}_2 + 4b_2\tilde{\mu}_3 = -\tilde{\mu}_3 \\ -a\tilde{\mu}_3 + 3b_0\tilde{\mu}_2 + 4b_1\tilde{\mu}_3 + 5b_2\tilde{\mu}_4 = -\tilde{\mu}_4 \end{cases}. \quad (6)$$

$$y_{1,2} = -\frac{b_1}{2b_2} \left(1 \pm \sqrt{1 - \frac{1}{K}}\right), \quad K = \frac{b_1^2}{4b_0b_2}. \quad (7)$$

$$f(y) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} y^{p-1}(1-y)^{q-1}, \quad 0 \leq y \leq 1. \quad (8)$$

$$f(y) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \frac{y^{p-1}}{(1+y)^{p+q}}, \quad 0 \leq y < \infty. \quad (9)$$

**Methodic of determination of beta-distribution parameters**

Generalized beta-distribution describes the distribution of random variable  $z = \alpha + (\beta - \alpha)y$ , which linear function of random variable  $y$ , that has beta-distribution of I type with parameters  $p, q$  and is distributed in interval  $\alpha \leq y \leq \beta$ . The opposite statement is also true – if the random variable  $z$  has a generalized beta distribution with the specified parameters, then the random variable  $y$  has a beta-distribution of I type ( $y = (z - \alpha)/(\beta - \alpha)$ ) with parameters  $p, q$  [19]. Going to the variable  $G_{PM}$ , which varies in range  $G_{PM \min} \leq G_{PM} \leq G_{PM \max}$  (it is easily determined from the analysis of empirical data), we can record the density of probabilities by the formula (10), the initial moments – by the formula (11), and the central ones – by the formula (12).

Table 1 – Coefficients for definition of type of distribution by Pearson classification

$d = 10\tilde{\mu}_2\tilde{\mu}_4 - 18\tilde{\mu}_2^3 - 12\tilde{\mu}_3^2$	
$c_0 = -\tilde{\mu}_2(4\tilde{\mu}_2\tilde{\mu}_4 - 3\tilde{\mu}_3^2)$	$b_0 = c_0 / d$
$c_1 = -\tilde{\mu}_3(\tilde{\mu}_4 + 3\tilde{\mu}_2^2)$	$b_1 = c_1 / d, a = b_1$
$c_2 = -2\tilde{\mu}_2\tilde{\mu}_4 + 6\tilde{\mu}_2^3 + 3\tilde{\mu}_3^2$	$b_2 = c_2 / d$

In this formulas  $C_n^k = \binom{n}{k} = \frac{n!}{k!(n-k)!}, k \leq n$  – binomial coefficients,  $\Gamma(\bullet)$  – gamma-function, which by definition  $\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt, (\text{Re } z > 0)$  [19].

For an integer argument, the calculation of the gamma-function is related to the calculation of the factorial, but for the general case, it is necessary to use the approximation, for example, the Sterling formula (12), or other approximations through a continuous fraction. This provides quite accurate research accuracy.

For the continuous distributions, the relations between the initial and the central moments described by formula (14) are also valid [13], which can also be used in the calculations, which for the moments up to the 4<sup>th</sup> rank inclusive gives the expressions, summarized in Table. 2. All calculations of the Pearson curves require high accuracy of calculations (it is necessary to hold up to 8...10 characters after a comma), which is explained by the multiplicative scheme of accumulation of errors in power terms. For generalized beta-distribution the mathematical expectation  $m_1(p, q)$ , dispersion  $D(p, q)$ , coefficients of asymmetry  $Sk$  and excess  $Ex$  describes by formulas (15) and (16).

Replacing  $m_1(p, q)$  and  $D(p, q)$  in the corresponding sample estimates  $\tilde{m}_1$  and  $\tilde{S}^2$  (that defined by the formulas from [12, 19]) we can use the relationships to determine the distribution parameters  $(p, q)$  and get the formulas (17) and (18).

These equations takes into account moments of 1<sup>st</sup> and 2<sup>nd</sup> rank, which is quite natural, but we can definite the parameters  $(p, q)$  with the coincidence of 1<sup>st</sup>, 3<sup>rd</sup> or 4<sup>th</sup> rank initial moments. Equations for finding of  $(p, q)$  are nonlinear and so solutions of system of nonlinear equations we got approximated with order of accuracy close to  $10^{-8}$  [17]. These new parameters we marks as  $p^*$ ,  $q^*$ .

To determine the distribution form, to use the consent criteria, etc., to put forward hypotheses about the distribution form, the sample should be presented as a histogram, which consists of  $m_{col}$  columns of a

certain length  $\Delta G_{PM}$ . That histograms allow us to see how values of the variables are distributed at the grouping intervals, that is, how often the variables take values from different intervals. It is generally accepted that intervals of the same length should be used. In the literature on the statistical processing of experimental data [18] specific recommendations are given regarding the choice of the number of intervals of grouping  $m_{col}$ , which differ significantly from each other.

Table 2. Moments of the distribution

Moment rank	Initial moments	Central moments
0	$m_0 = 1$ (by definition)	$\mu_0 = 1$ (by definition)
1	$m_1$ (determined)	$\mu_1 = 0$
2	$m_2 = m_1^2 + \mu_2$	$\mu_2 = m_2 - m_1^2$
3	$m_3 = m_1^3 +$ $+ 3\mu_2 m_1 + \mu_3$	$\mu_3 = m_3 -$ $- 3m_2 m_1 + 2m_1^3$
4 (initial)	$m_4 = m_1^4 + 6\mu_2 m_1^2 + 4\mu_3 m_1 + \mu_4$	
4 (central)	$\mu_4 = m_4 - 4m_3 m_1 + 6m_2 m_1^2 - 3m_1^4$	

According to our recommendations, we use the Stargess formula (19).

Derivative from decomposition of Gamma-function in series by the Stirling formula (13).

Derivative from Gamma-function with using of Psi-function that obtained by asymptotical formula (21).

$$f(G_{PM}) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \frac{(G_{PM} - G_{PM \min})^{p-1} (G_{PM \max} - G_{PM})^{q-1}}{(G_{PM \max} - G_{PM \min})^{p+q-1}}, \tag{10}$$

$$m_n(p, q) = \frac{\Gamma(p+q)}{\Gamma(p)} \sum_{k=0}^n C_n^k (G_{PM \max} - G_{PM \min})^k G_{PM \min}^{n-k} \frac{\Gamma(p+k)}{\Gamma(p+q+k)}, \tag{11}$$

$$\mu_n(p, q) = (G_{PM \max} - G_{PM \min})^n \frac{\Gamma(p+q)}{\Gamma(p)} \sum_{k=0}^n (-1)^k C_n^k \left(\frac{p}{p+q}\right)^k \frac{\Gamma(p+n-k)}{\Gamma(p+q+n-k)}. \tag{12}$$

$$\Gamma(z) \approx e^{-z} z^{z-\frac{1}{2}} (2\pi)^{\frac{1}{2}} \times \left[ 1 + \frac{1}{12z} + \frac{1}{288z^2} - \frac{139}{51840z^3} - \frac{571}{2488320z^4} + \frac{163879}{209018880z^5} + \frac{5246819}{75246796800z^6} \right]. \tag{13}$$

$$m_n = \sum_{k=0}^n C_n^k \mu_k m_1^{n-k}, \quad \mu_n = \sum_{k=0}^n (-1)^{n-k} C_n^k m_k m_1^{n-k}, \tag{14}$$

$$m_1(p, q) = \frac{G_{PM \min} \cdot q + G_{PM \max} \cdot p}{p + q}, D(p, q) = \frac{(G_{PM \max} - G_{PM \min})^2 \cdot p \cdot q}{(p + q)^2 \cdot (p + q + 1)}, \quad (15)$$

$$Sk = \frac{2 \cdot (q - p)}{p + q + 2} \sqrt{\frac{p + q + 1}{p \cdot q}}, Ex = \frac{6 \cdot ((p - q)^2 (p + q + 1) - p \cdot q \cdot (p + q + 2))}{p \cdot q \cdot (p + q + 2) \cdot (p + q + 3)}. \quad (16)$$

$$p = \frac{\tilde{m}_1 - G_{PM \min}}{G_{PM \max} - G_{PM \min}} \times \left[ \frac{(\tilde{m}_1 - G_{PM \min})(G_{PM \max} - \tilde{m}_1)}{\tilde{S}^2} - 1 \right], \quad (17)$$

$$q = \frac{G_{PM \max} - \tilde{m}_1}{G_{PM \max} - J_{P \min}} \times \left[ \frac{(\tilde{m}_1 - G_{PM \min})(G_{PM \max} - \tilde{m}_1)}{\tilde{S}^2} - 1 \right]. \quad (18)$$

$$m_{col} = \log_2 N + 1 = 3,322 \cdot \lg N + 1. \quad (19)$$

$$\Gamma_1'(z) \approx e^{-z} z^{z-\frac{1}{2}} (2\pi)^{\frac{1}{2}} \times \left[ \left( 1 + \frac{1}{12z} + \frac{1}{288z^2} - \frac{139}{51840z^3} - \frac{571}{2488320z^4} + \dots \right) \times \left( \ln z - \frac{1}{2z} \right) - \left( \frac{1}{12z^2} + \frac{1}{144z^3} - \frac{139}{17280z^4} - \frac{571}{622080z^5} + \dots \right) \right] \quad (20)$$

$$\Gamma_2'(z) = \Psi(z)\Gamma(z), \Psi(z) = d[\ln(z)]/dz, \Psi(z) \approx \ln z - (2z)^{-1} - (12z^2)^{-1} + (120z^4)^{-1} - (252z^6)^{-1} \quad (21)$$

Table 3. Comparison of results of application of different manner of obtaining of derivate of Gamma-function

Variant	$\Psi(1)$	$\Gamma_1'(1)$	$\Gamma_2' = \Psi(1)$
Note	-C (Euler's constant)	-	$\Gamma'(1) = 1$
Magnitude	-0,5772157	-0,5791283	-0,57896825

**Obtaining of initial data**

In order to carry out the calculation study, data is required regarding the magnitude of the mathematical expectation of PM mass-hourly emissions with EG flow of the autotractor diesel engine 2Ch10.5/12 and instrumental errors. The first one obtained with using of formula (1) herewith we were used data from motor

bench tests obtained in researches [14, 15] and summarized in Table 4. The second one obtained from technical documentation of opacimeter [10] and multi-component gas analyzer [11] and also summarized in Table 4.

This study carried out for two points of operational regimes area of diesel engine 2Ch10.5/12: a) regime with maximal smokiness of EG – this is regime of maximal torque with following coordinates of operational regimes area:  $n_{ks} = 1200$  rpm and  $M_T = 110$  N·m; b) regime with maximal volume concentration of unburned hydrocarbons of motor fuel in EG – this is regime of nominal power with following coordinates of operational regimes area:  $n_{ks} = 1800$  rpm and  $M_T = 95$  N·m.

Table 4. Initial data for calculation study [10, 11, 14, 15]

Point	$n_{ks}$ , rpm	$M_T$ , N·m	$G_{air}$ , kg/h	$G_{fuel}$ , kg/h	$N_D$ , %	$C_{CH}$ , ppm	Passport accuracy, %	
							Opacimeter $N_D$ [10]	Gas analyzer $C_{CH}$ [11]
$M_{T \max}$	1200	110	72.315	3,657	71.6	27	± 2.5	± 5.0
$N_{enom}$	1800	95	109.218	4,312	38.9	72	± 2.5	± 5.0

**Description of empirical law of distribution of values of PM mass-hourly emission of diesel engine 2Ch10.5/12 by mathematical apparatus of Pearson family curves as the function of passport accuracy of opacimeter and gas analyzer**

In order of implementation of researches which relates with empirical data approximation it necessary to gat the random numbers with a given distribution law. In terms of mathematical statistics its numbers are

random sample of volume  $N$  from general totality that is distribute with the law.

Mechanism of generating of random numbers that was used in the study was described in [18]. With using of random numbers generator **RANDOM** it generates the realization of random variable  $r_i$  ( $i=1 \dots N$ ), which distributed evenly in the range [0, 1] and has the name of a standard uniform sequence.

For the normal distribution with zero mathematical expectation ( $m_u = 0,0$ ) and unit dispersion

( $\sigma_u=1,0$ ) basing on two consecutively taken numbers ( $r_1, r_2$ ), ( $r_3, r_4$ ), ..., ( $r_{N-1}, r_N$ ) (for  $N$  values we has  $N/2$  couples of numbers) we gat respectively two numbers ( $u_1, u_2$ ), ( $u_3, u_4$ ), ..., ( $u_{N-1}, u_N$ ) of normal distributed random variable  $r$ , described by formula (20). When we get realization of researching parameter in form of normal random variable with non-zero mathematical expectation  $m_{PAR}$  and standard deviation

$\sigma_{PAR}$  that is different from the unit, we use the equati-on (23) from [18] for linear transformation of random variable with no violation of normality.

$$u_i = \sqrt{-2 \cdot \ln r_i} \cdot \cos(2 \cdot \pi \cdot r_{i+1}),$$

$$u_{i+1} = \sqrt{-2 \cdot \ln r_i} \cdot \sin(2 \cdot \pi \cdot r_{i+1}), (i=1,3,\dots,N-1). (22)$$

$$x_i = m_{PAR} + u_i \cdot \sigma_{PAR}, (i=1\dots N). (23)$$

Table 5. Dependences of quantity of grouping intervals from volume of sample

Volume of sample of random variable	N						
	50	100	200	300	400	500	1000
Exact value, g/h	6.643	7.644	8.644	9.230	9.644	9.966	10.966
Quantity of grouping intervals $m_{col}$	7	9	9	11	11	11	11

For definiteness in work, the sample size  $N$  we was set which in any case will be limited by quantity of observing or experiments (see Table 5). In accordance with formula (19) we obtained the exact values which with taking into account of unimodality of distribution due to recommendations round to the nearest larger odd number.

In the study we consider the case when magnitude of value  $C_{CH}$  is considered constant and derivation of determination of magnitude of value  $N_D$  is considered to be normally distributed within the limits indicated in Table 4 passport accuracy of measuring equipment. For value  $N_D$  carried out numerical modeling with following parameters: 1)  $m_{PAR} = 71.6 \%$ ;  $\sigma_{PAR} = 0.01$  with appropriate parameters for point with  $M_{Tmax}$ ; 2)  $m_{PAR} = 38.9 \%$ ;  $\sigma_{PAR} = 0.01$  with appropriate parameters for point with  $N_{enom}$ . Calculation results with using formulas (1) – (3) for these variants are following: 1)  $a = 2.3 \cdot 10^{-3}$  kg/(h·%),  $b = 5.0 \cdot 10^{-5}$  kg/(h·%<sup>2</sup>),  $c = 0.00279$  kg/(h·ppm),  $d = 1.268 \cdot 10^{-5}$  kg/(h·ppm<sup>2</sup>),  $f = 0.006199$ ,  $k = 0.05858$  kg/h,  $G_{PM} = 0.029617$  kg/h; 2)  $a = 2.3 \cdot 10^{-3}$  kg/(h·%),  $b = 5.0 \cdot 10^{-5}$  kg/(h·%<sup>2</sup>),  $c = 0.002788$  kg/(h·ppm),  $d = 1.264 \cdot 10^{-5}$  kg/(h·ppm<sup>2</sup>),  $f = 0.006196$ ,  $k = 0.08758$  kg/h,  $G_{Tq} = 0.037781$  kg/h.

Given the nonlinearity of the parameter  $G_{PM}$  dependence from value  $N_D$ , formally, under the condition of a normal distribution of the error of measurement of the second value, distribution of  $G_{PM}$  must differ from the normal. But the acceptance or rejection of the hypothesis of normality should be confirmed by calculations of the coefficients of asymmetry and excess. Selective (depending on the sample size) coefficients of asymmetry  $\tilde{S}k$  and excess  $\tilde{E}x$  are determined by

the formulas (24).

$$\tilde{S}k = \tilde{\mu}_3 / \tilde{S}^3, \tilde{E}x = \tilde{\mu}_4 / \tilde{S}^4 - 3. (24)$$

For normal distribution  $Sk = 0$ , that is the distribution is symmetrical relative to the mathematical expectation. If the right tile of histogram are longer than the left one, than  $Sk > 0$  and  $Sk < 0$  in the opposite case. For normal distribution  $Ex = 0$ . If the peak of histogram are conditionally sharp, than  $Ex > 0$  and  $Ex < 0$  if the peak are conditionally smooth (rounded). For previous conclusion about ability of approximation of empirical data by the normal distribution law in conditions of availability of data set of pretty large volume (about  $10^3$ ) we provide calculation of selective mean square deviations of coefficients of asymmetry ( $S_1$ ) and excess ( $S_2$ ) [13] with using formula (25). In some cases, such coefficients calculate by formula (26). If the selective distribution are normal or close to it, than coefficients  $\tilde{S}k$  and  $\tilde{E}x$  calculate by formula (26) has asymptotic to normal distribution laws with zero values of mathematical expectations and mean square deviations, what describes by formula (27).

It is believed that if the condition  $|\tilde{S}k| \leq 3S_1$  is fulfilled, then the distribution is symmetric. If at the same time, inequality  $|\tilde{E}x| \leq 5S_2$  is fulfilled for the coefficient of excess, the distribution can be considered normal. It should be noted that in literature there are other options ( $|\tilde{S}k| \leq 2S_1$ ,  $|\tilde{E}x| \leq 2S_2$  etc.), which indicates the ambiguity of the application of this criterion of normality distribution.

$$S_1 = \sqrt{\frac{6 \cdot (N - 2)}{(N + 1) \cdot (N + 3)}},$$

$$S_2 = \sqrt{\frac{24 \cdot N \cdot (N-2) \cdot (N-3)}{(N+1)^2 \cdot (N+3) \cdot (N+5)}} \quad (25)$$

$$\tilde{S}k = \frac{k_3}{\sqrt{k_2^3}}, \quad \tilde{E}x = \frac{k_4}{k_2^2} - 3,$$

where  $k_2 = \tilde{\mu}_2 / \left(1 - \frac{1}{N}\right)$ ,  $k_3 = \tilde{\mu}_3 / \left(\left(1 - \frac{1}{N}\right) \cdot \left(1 - \frac{2}{N}\right)\right)$ ,

$$k_4 = \tilde{\mu}_4 / \left(\left(1 - \frac{2}{N+1}\right) \cdot \left(1 - \frac{2}{N}\right) \cdot \left(1 - \frac{3}{N}\right)\right) - 3 \cdot \tilde{\mu}_2^2 / \left(\left(1 - \frac{2}{N}\right) \cdot \left(1 - \frac{3}{N}\right)\right) \quad (26)$$

$$S_1 = \sqrt{\frac{6 \cdot N \cdot (N-1)}{(N-2) \cdot (N+1) \cdot (N+3)}}$$

$$S_2 = \sqrt{\frac{24 \cdot N \cdot (N-1)^2}{(N-3) \cdot (N-2) \cdot (N+3) \cdot (N+5)}} \quad (27)$$

The developed in previous studies methodic of application of beta distribution was improved.

Firstly, it was changed the manner of obtaining of value of derivative of Gamma-function from the directly taking the derivative from Stirling formula to the using of Psi-function that obtained with using of asymptotical formula. The results of comparison of results of the both applied approaches between which other and with the theoretical value is showed in this table.

Secondly, it was changed the manner of determination of normality of distribution law from using of Student criterion of consent to using of entropy coefficient that allows assessing the degree of approximation of empirical distribution law to the normal law not only qualitatively but also quantitatively.

Along with the analysis of the «proximity» of the empirical distribution to the normal using the coefficients of asymmetry  $Sk$  and excess  $Ex$ , the criterion in the form of an entropy coefficient  $K_{entr}$  is also used. By histogram this estimate is determined by formula (28) [18].

$$K_{entr} = \frac{d_H \cdot N}{2 \cdot \tilde{S}} \cdot 10^X, \quad X = -\frac{1}{N} \cdot \sum_{j=1}^{m_{col}} r_j \cdot \lg(r_j), \quad (28)$$

where  $d_H$  (or  $\Delta G_{PM}$ ) – width of bar graph column;  $N$  – volume of the data set (amount of measurements);  $\tilde{S}$  – mean square deviation;  $m_{col}$  – number of bar graph columns;  $r_j$  – number of observations (number of «hits») in  $j$ -th column.

For normal distribution  $K_{entr} = 2.066 \approx 2.07$ , his is generally the maximum value for all of distributions, the minimal magnitude of value  $K_{entr}$  equal 1.11 for arcsine distribution law and other is in the specified boundaries. Under this criterion, any distribution other

than normal will have a magnitude of entropy coefficient less than 2.066. This applies to continuous distributions, for a histogram one can expect some exaggeration in the magnitude of the entropy coefficient.

Student`s criterion of consent:

$$\chi^2 = \sum_{j=1}^{m_{em}} \left( (r_j - N \cdot e_j)^2 / (N \cdot e_j) \right) \quad (29)$$

In order of carrying out of calculation study it was improved in accordance with worded above principals the software product “Applied Beta-Distribution” that was developed in the program space Borland Pascal 7.0.

Improved software product “Applied Beta-Distribution” that was used for calculations presented on Fig. 4. as well as the results of calculated comparative study in this software product.

It was carried out such modeling for samplings of initial data with volume 50, 100, 200, 300, 400, 500, 700 and 1000 items of influencing factors of both types. For each magnitude of sampling volume such modeling was carried out 15 times.

#### Determination of dependence of beta-distribution parameters of mass-hourly PM emissions of diesel engine 2Ch10.5/12 from quantity of initial data in sample

Carried out the calculated study for  $C_{CH} = m_{PAR} = 27$  ppm and  $\sigma_{PAR} = 0.02$  with appropriate parameters for the point  $M_{Tmax}$ .

For  $N = 50$  was provide 4 independent modeling studies, for  $N = 100, 200$  and  $1000 - 2$  studies for each.

When analyses data from this calculations and also results of modeling for sample with  $N = 300, 400$  and  $500$ , which due to lack of space are not presented in the study but fundamentally not different from those given in the study, it possible to conclude what there is the principal opportunity for approximation of empirical distributions of values of such nature with using the beta-density.

In all variants without any exceptions values of coefficient  $K$  are negative, roots of equation (27) are real and with different signs. Entropy coefficient  $K_{entr}$  are increasing with rising of volume of sample data set  $N$  gradually approaching the its limit value 2.07.

Carried out the calculated study for  $C_{CH} = m_{PAR} = 72$  ppm and  $\sigma_{PAR} = 0.01$  with appropriate parameters for the point  $N_{enom}$ . The character of empirical distributions for all researched volume of sample data set (from  $N = 50$  to  $N = 1000$ ) allows to approximate them with using the beta-density. On Fig. 1 – 15 presented some results of the study. It can be seen

that for relatively small volume of sample data set with  $N = 50$  the distributions has significant differences for different single modeling calculations (see Fig. 1 – 3). On all of Figures: histogram – empirical distribution; solid curve – normal distribution; dotted curve – beta-distribution.

Operating window of developed and improved program product “Applied Beta-Distribution” as well as results window is showed on Fig. 4.

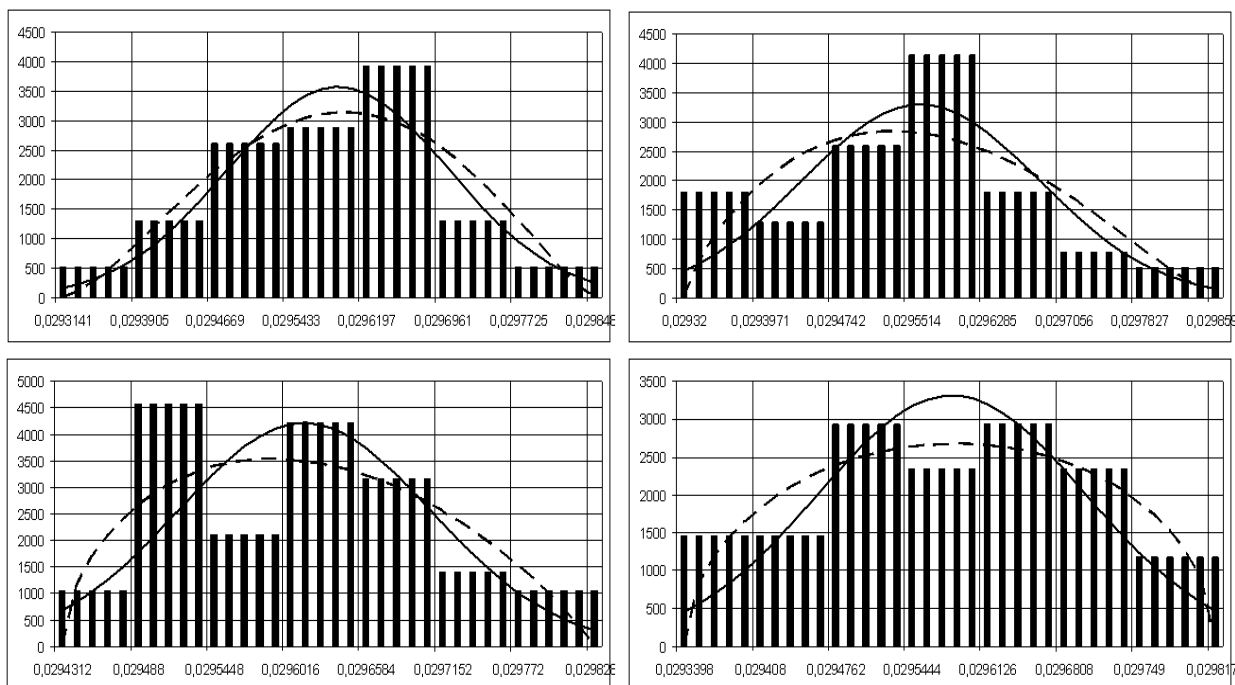


Fig. 1. Empirical, normal and beta-distribution for volume of sample data set  $N = 50$  and PICE operational point with  $M_{Tmax}$

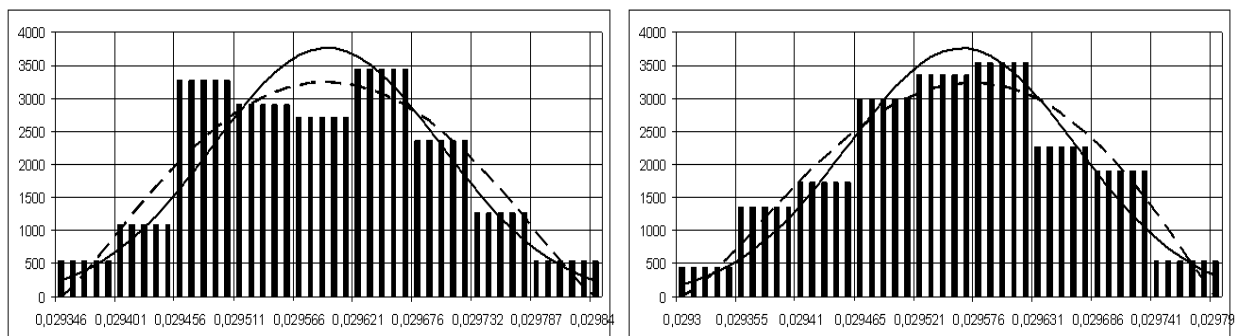


Fig. 2. Empirical, normal and beta-distribution for volume of sample data set  $N = 100$  and  $200$  and PICE operational point with  $M_{Tmax}$

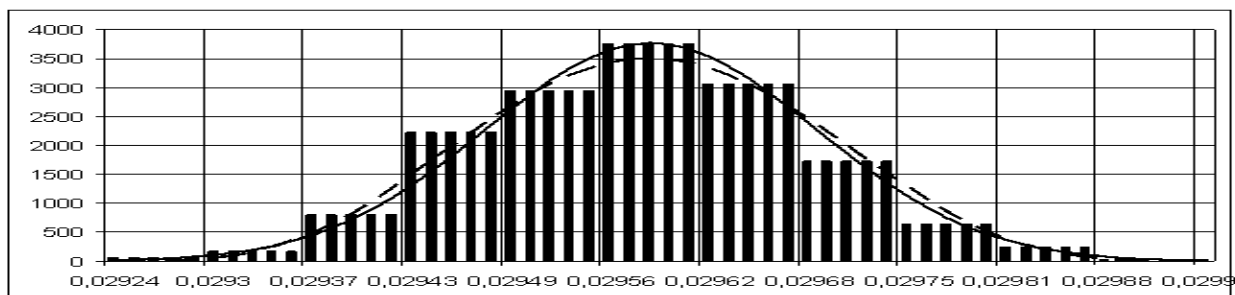


Fig. 3. Empirical, normal and beta-distribution for volume of sample data set  $N = 1000$  and PICE operational point with



Entropy coefficients equal respectively: 1) for  $N = 50$  respectively for modeling № 1 – 4  $K_{entr} = 1.882; 1.875; 1.763; 1.640$ ; 2) for  $N = 100$  respectively for modeling № 1, 2  $K_{entr} = 1.989; 2.055$ ; 3) for  $N = 200$  respectively for modeling № 1, 2  $K_{entr} = 2.017; 2.020$ ; 4) for  $N = 1000$  respectively for modeling № 1, 2  $K_{entr} = 2.059; 2.089$ .

```

F:\BP\BIN\TURBO.EXE
File Edit Search Run Compile Debug Tools Options Window Help
[ ]
PROGRAM KONKURS; (<N+>) KONKURS.FAS
uss: Fctns;
const
a_coef = 2.3e-03;
k_coef = 5.0e-05;
n_ran = 1000;
n_lin = n_ran;
coef = pi/180;
par_end = 2;
n_non = 4;
n_col_max = 15;
nend = 25;
neps = 10;
nonon = 4;
type
vecn_ran = array[1..n_ran] of real;
vecn_lin = array[1..n_lin] of real;
vecn_non = array[0..n_non] of real;
vecn_col = array[1..n_col_max] of real;
vecn_den = array[0..10*n_col_max] of real;
1:1
F1 Help F2 Save F3 Open Alt+F9 Compile F9 Make Alt+F10 Local menu
    
```

a)

```

F:\BP\BIN\TURBO.EXE
File Edit Search Run Compile Debug Tools Options Window Help
[ ]
E0100_13_1X1 2=17
n_ran=100 n_col=9
ND_exp= 71.4983915 ND_var= 0.58198193 ND_sko= 0.76287740
as_ND= -0.11820809 ex_ND= -0.60528758
GTCH<ND>_exp= 0.02952265 GTCH<ND>_var= 0.00000018 GTCH<ND>_sko= 0
GTCH<ND>_min= 0.02852932 GTCH<ND>_max= 0.03053092
as_en= -0.10225627 ex_en= -0.55479430
sko_as_en1= 0.23774389 sko_ex_en1= 0.45474705
sko_as_en2= 0.24137728 sko_ex_en2= 0.47833113
c1= -0.00000000 c1= 0.00000000 c2= 0.00000000 d= 0.00000000
b0= -0.00000027 h1= 0.00003710 h2= 0.18842717 a= 0.00003710
kden= -0.00703448 xden1= 0.00112746 xden2= -0.00133311
qdenbase= 2.3003813 qdenbase= 2.3350013
as_pq= 0.01068879 ex_pq= -0.78573009
pden= 2.3316456 qden= 2.3667362
as_pq= 0.01064570 ex_pq= -0.77909151
KENTR= 1.9833159
1:1
F1 Help F2 Save F3 Open Alt+F9 Compile F9 Make Alt+F10 Local menu
    
```

b)

Fig. 4. Improved software product “Applied Beta-Distribution” (a) and results of its application (b)

Fig. 5 and 6 contains the histogram of dependence of values of number of break intervals  $m_{col}$  and graphs of dependence of values of coefficients of asymmetry  $As$  and excess  $Ex$  of beta-distribution from the magnitude of volume of sampling of initial data  $N$  for both influencing factors.

Fig. 7 contains the graphs of dependence of values of parameters  $p$  and  $q$  of beta-distribution and also entropy coefficient  $K_{entr}$  from the magnitude of volume of sampling of initial data  $N$  for both influencing factors.

On graphs on Fig. 5 and 6 can be seen that all curves of density of beta-distribution which approximate the empirical data in case of variation of both influencing factors has the same pronounced characteristic shape of peak with low sharpness (smooth with  $Ex < 0$ ). The sharpness of peaks has nonlinear dependence from volume of sampling of initial data  $N$  and increases with its increasing approaching to sharpness of peak of the normal distribution law ( $Ex = 0$ ).

It also can be seen that there is insignificant left-sided asymmetry ( $As > 0$ ) of all worded above curves with magnitude that are invariant from volume of sam-

pling of initial data  $N$ . For the normal distribution law  $As = 0$ .

On graphs on Fig. 7 can be seen that the magnitude of volume of sampling  $N$  has nonlinear influence on the magnitudes of all physical values on these graphs. Values of parameters  $p$  and  $q$  are almost equal to each other and increases with its increasing of sampling of initial data  $N$ .

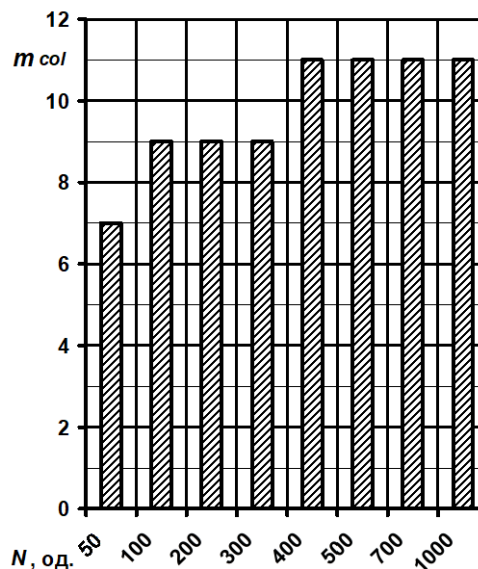


Fig. 5. Histograms of dependences  $m_{col} = f(N)$

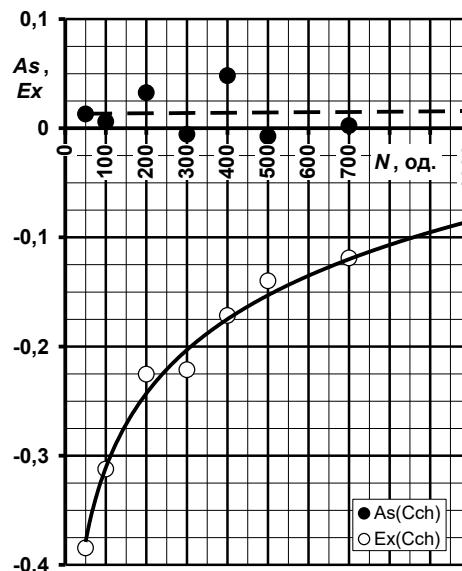


Fig. 6. Graphs of dependences  $As = f(N)$  and  $Ex = f(N)$

Values of entropy coefficient  $K_{entr}$  intensive increases with its increasing of sampling of initial data  $N$  in diapason 50 – 300 measurements from 1.84 to 2.05 and then “goes on shelf” asymptotically approximates to value of 2.07 that are characteristic of the normal distribution law.

Thus, graphics of the Fig. 6 and 7 was described as formulas (28) – (31) by method of less squares.

$$As = 2.412 \cdot 10^{-6} \cdot N + 1.322 \cdot 10^{-2}; R^2 = 0.975, (30)$$

$$Ex = 9.815 \cdot 10^{-2} \cdot \ln(N) - 0.763; R^2 = 0.986, (31)$$

$$p, q = 1.674 \cdot N^{0.033}; R^2 = 0.912, (32)$$

$$K_{entr} = 0.863 \cdot \ln(N) - 1.548; R^2 = 0.978. (33)$$

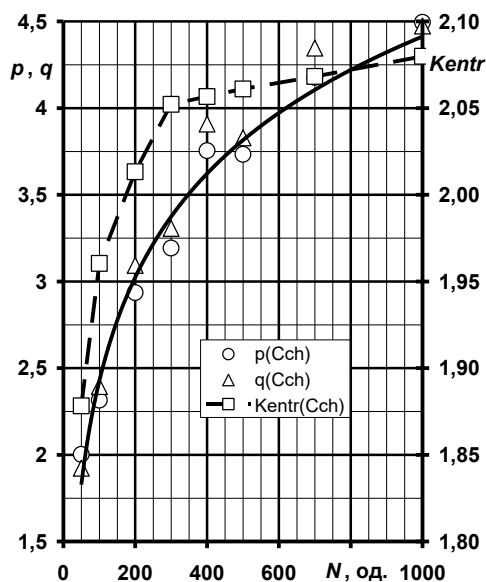


Fig. 7. Graphs of dependences  $p = f(N)$ ,  $q = f(N)$  and  $K_{entr} = f(N)$  for various values of  $C_{ch}$  and constant value of  $N_D$

### Conclusions

Thus, in accordance to results of the study it can make following general conclusions.

1. Results of analysis of scientific and technical literature on topic of the study allows to conclude that obtaining of values of mass-hourly PM emission with PICE EG flow as the legislative normalized pollutant with using of dilution tunnels at the time is not available for departments of scientific institutions in Ukraine. That's why in practice it been used different conversion formulas components of which determines by direct measurements with using of more available measuring equipment, namely opacimeters and gas analyzers. This is the relevance of the study.

2. From the results of analysis of mathematical apparatus of known conversion formulas – Parsadanov, Alkidas, Muntean and etc. – follows that obtained by indirect measurements with using of opacimeter and gas analyzer values of components of conversion formulas has nonlinear influence. Wherein, appears the problem of selection of law that is adequate to physical reality and describes the empirical distribution of researched value because its denormalization is appears even in case of normality of distribution law of values of

influencing factors.

From the results of analysis of special literature and passports of opacimeter and multi component gas analyzer was determined that for the first magnitude of coefficient of weakening of the light flux determines with accuracy  $\pm 2.5\%$  and for volume concentration of unburned hydrocarbons in PICE EG flow – with accuracy  $\pm 5.0\%$ . Results of measurements in limits of these errors are distributed by normal law.

2. From the results of analysis of mathematical apparatuses of family of Pearson family curves its became obviously that this is universal calculation instrument for approximation that allows to describe by formulas the empirical distributions which presents by histograms of almost any forms. For researched case solution of characteristic equation of general distribution using differential equation of Pearson family curves gives such magnitudes of coefficients which attests about accordance of empirical distribution of studied physical value to I type of distributions of Pearson classification – beta-density.

3. For obtaining of initial data set for description of empirical distribution of values of mass-hourly PM emissions with PICE EG flow, namely magnitudes of mathematical expectation of studied value, was analyzed data of motor bench testing of autotractor diesel engine 2Ch10.5/12 (values of mass-hourly PICE fuel and air consumption) and according to them initial data set: in the first case – sample of random variable of values of coefficient of weakening of the light flux and constant value of volume concentration of unburned hydrocarbons in PICE EG flow, and in the second case these variables swaps their places.

4. Carried out the calculation assessment of values of parameters of beta-density for approximation of magnitudes of values of mass-hourly PM emission with PICE EG flow as the function of values of coefficient of weakening of the light flux in EG flow in case of distribution of it by normal law in limits of passport measurement accuracy of opacimeter and constant value of volume concentration of unburned hydrocarbons in PICE EG flow in the first case and in the second case these variables swaps their places.

5. According to the results of the settlement research, it was established that empirical distribution of values of mass-hourly PM emission with PICE EG flow which obtained using one of known conversion formula as the function of values of coefficient of weakening of the light flux in EG flow has substantial difference from normal law at amount of measurements less than 50 even in case of hypothesis confirmation about normality of readings of opacimeter and gas analyzer in limits of passport measurement accuracy. Thus, in the study carried out the selection of ratio-

nal number of multiple measurements of values of coefficient of weakening of the light flux in EG flow in the first case and volume concentration of unburned hydrocarbons in EG flow in the second case on PICE individual operational regime during motor bench testing for the case of automation of measuring process.

6. It was obtained parameters of beta-density for description of studied indicator of ecological safety as well as its dependencies from number of measurements.

**Scientific novelty** of obtained results is in the following paragraphs.

1. For the first time it was proposed for description of empirical distribution law of values of mass hourly PM emissions with PICE EG flow which obtained with using of conversion formulas with taking into account nonlinear influence of passport measuring accuracy of opacimeter and gas analyzer what distributed by normal law, with using of Pearson family of curves of probability densities.

2. For the first time it was obtained dependencies of parameters of beta-distribution of values of mass hourly PM emissions with PICE EG flow from quantity of initial data in set for main points of its operational regimes area.

**Practical value** of obtained results is in the following paragraphs.

1. Obtained parameters of beta-distribution of values of mass hourly PM emissions with PICE EG flow as the function of readings of opacimeter and gas analyzer with taking into account its passport measuring accuracy are available for developing of methodic for assessment and decreasing of value of methodic error of  $G_{PM}$  value determination in case of small amount of initial data in set at automation of measuring process and also for ranking of known conversion formulas.

2. Obtained dependencies of parameters of beta-distribution of values of mass hourly PM emissions with PICE EG flow from quantity of initial data in set for main points of its operational regimes area allows justification of selection of minimal number of measurements of EG smokiness and toxicity of EG flow on single operational regime in case of automation of measuring process.

3. Developed software in Borland Pascal environment is available to following calculation researches.

Results of the study in form of appropriate methodica and software product was implemented into educational process of Applied Mechanics and Environment Protection Technologies Department of National University of Civil Defence of Ukraine when teaching discipline "Metrology, Standardization, Certification".

The research was carried out in the science and research work of Applied Mechanics and Environment

Protection Technologies Department of National University of Civil Defence of Ukraine «Using of fuzzy logic and psychophysical scales in a critical assessment of the level of ecological safety» (State Reg. № 0119 U001001, 2019 – 2021) and also Scientific work of young scientists that carried out at the expense of state budget of Ukraine of Berdyansk State Pedagogical University «Development of technology for assessing the quality and safety of nanotechnology products throughout the life cycle» (State Reg. № 0117U003860, 2017 – 2020).

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Received to the editorial office 02.07.2019

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## ОПИСАННЯ БЕТА-РОЗПОДІЛОМ МАСОВИХ ГОДИННИХ ВИКИДІВ ТВЕРДИХ ЧАСТИНОК ДИЗЕЛЯ З УРАХУВАННЯМ ПАСПОРТНОЇ ТОЧНОСТІ ГАЗОАНАЛІЗАТОРА

*О.П. Строчков, О.М. Кондратенко, В.Ю. Колосков, І.В. Міщенко*

Актуальність дослідження полягає в тому, що кількісні та якісні результати вирішення задач дослідження придатні для створення методики зниження методологічної похибки визначення величини масового годинного викиду твердих частинок з потоком відпрацьованих газів поршневого двигуна внутрішнього згорання з використанням формул перерахунку та показів димоміра й газоаналізатора. Метою дослідження є отримання параметрів бета-щільності, що апроксимують емпіричний закон розподілу значень годинних масових викидів ТЧ з потоком ВГ поршневого ДВЗ, отриманих непрямыми вимірюваннями з урахуванням точності прямих вимірювань їх складових, що чинять нелінійний вплив. Завданням дослідження є отримання раціональних значень кількості багаторазових вимірювань коефіцієнта послаблення світлового потоку та об’ємної концентрації незгорілих вуглеводнів у ВГ на одному режимі роботи дизеля у стендових моторних дослідженнях для випадку автоматизації процесу вимірювань. Методика виконання даної наукової роботи полягала у послідовному використанні наступних методів: аналіз науково-технічної літератури, методика аналізу даних моторних стендових випробувань, методика застосування формули перерахунку проф. І.В. Парсаданова, математичний апарат бета-розподілу, метод чисельних розрахункових досліджень. Встановлено, що емпіричний розподіл значень масового годинного викиду ТЧ з ВГ поршневого ДВЗ, отриманих при застосуванні однієї з відомих формул перерахунку проф. І.В. Парсаданова, як функція показників щільності й токсичності ВГ, суттєво відрізняється від нормального закону за кількості вимірювань, меншій за 50, навіть при підтвердженні гіпотези про нормальність розподілу показів димоміра у межах паспортної похибки вимірювань. Результати описано математичним апаратом бета-розподілу шляхом здійснення розрахункового дослідження з використанням програмного продукту, розробленого у середовищі Borland Pascal 7.0. Отримано параметри бета-щільності для описання досліджуваного показника екологічної безпеки та їх залежності від кі-

лькості вимірювань.

**Ключові слова:** бета-розподіл; димомір; багатоконпонентний газоаналізатор; двигун внутрішнього згорання; похибка вимірювань; стендові моторні дослідження; технології захисту навколишнього середовища; екологічна безпека.

## ОПИСАНИЕ БЕТА-РАСПРЕДЕЛЕНИЕМ МАССОВЫХ ЧАСОВЫХ ВЫБРОСОВ ТВЕРДЫХ ЧАСТИЦ ДИЗЕЛЯ С УЧЕТОМ ПАСПОРТНОЙ ТОЧНОСТИ ГАЗОАНАЛИЗАТОРА

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Актуальность исследования заключается в том, что количественные и качественные результаты решения задач исследования пригодны для создания методики снижения методической погрешности определения величины массового часового выброса твердых частиц с потоком отработанных газов поршневого двигателя внутреннего сгорания с использованием формул пересчета и показаний дымомера и газоанализатора. Целью исследования является получение параметров бета-плотности, аппроксимирующей эмпирический закон распределения значений часовых массовых выбросов ТЧ с потоком ОГ поршневого ДВС, полученных косвенными измерениями с учетом точности прямых измерений их составляющих, оказывающих нелинейное влияние. Задачей исследования является получение рациональных значений количества многократных измерений коэффициента ослабления светового потока и объемной концентрации несгоревших углеводородов в ОГ на одном режиме работы дизеля в стендовых моторных исследованиях для случая автоматизации процесса измерений. Методика выполнения данной научной работы состояла в последовательном использовании следующих методов: анализ научно-технической литературы, методика анализа данных моторных стендовых испытаний, методика применения формулы пересчета Парсаданова, математический аппарат бета-распределения, метод численных расчетных исследований. Установлено, что эмпирическое распределение значений массового часового выброса ТЧ с ОГ поршневого ДВС, полученных при применении одной из известных формул пересчета проф. И.В. Парсаданова как функция показателей дымности и токсичности ОГ существенно отличается от нормального закона при количестве измерений, меньшем 50, даже при подтверждении гипотезы о нормальности распределения показаний дымомера в пределах паспортной погрешности измерений. Результаты описаны математическим аппаратом бета-распределения путем расчетного исследования с использованием программного продукта, разработанного в среде Borland Pascal 7.0. Получены параметры бета-плотности для описания изучаемого показателя экологической безопасности и их зависимости от количества измерений.

**Ключевые слова:** бета-распределение; дымомер; многокомпонентный газоанализатор; двигатель внутреннего сгорания, погрешность измерений; стендовые моторные исследования; технологии защиты окружающей среды; экологическая безопасность.