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Sergiy ILLICHEVS'KYY
Postgraduate student,
Taras Shevchenko National University of Kyiv

ANALYSES OF UKRAINIAN INSURANCE MARKET BY THE BAYESIAN NETWORKS

This article is devoted to researching and development of new methods for calculating the time of occurrence of ruin probability for the insurance company with the help of Bayesian networks. The novelty of the article is that we use Bayesian Networks for measuring ruin probabilities.

Keywords: *Bayesian networks, ruin probability, insurance company.*

Contemporary insurance market is full of uncertainty. There is a great need in measuring, predicting and minimizing uncertainties Insurance services are one of the industries, which permanently experience risks of bankruptcy. That is why calculating the ruin probabilities for insurance companies are one of the problems that need well-developed mathematical models [1, p. 179].

Nowadays Ukrainian insurance companies are searching for new ways of profitability and competitiveness. Western European insurance companies have an option of investing their fund for additional profit. That is way there is a great necessity of creation and development of the bayesian networks for Ukrainian insurance to provide them the possibility of investing their fund for additional profit.

One of the first studies in this area was conducted in the beginning of the twentieth century. Since then, the mathematical methods of ruin probability calculation developed and accumulated a great variety of models and approaches. While the permanent growing of economic needs, insurance services increase steadily in the economies of all developed countries. Insurance services are one of the youngest industries any economy, which experience a stage of active development. In global practice of developed countries, well organized insurance services are involved in many economic sectors like investment activity of insurance companies. This article studies how the actuarial mathematical tools can positively affect the theoretical and practical development of insurance.

The development of theoretical, methodological, organizational and legal bases of insurance market have been contributed by many economists, such as: Alexandrova M., Alexandrova T., ArtyukhT., Bazylevych V., Baranovsky A. , Osadets S, Zaruba A., Kolomin E., Klapkiv M., Shah E., Reytman L., Slusarenko E, Yakovlev T., Facil M. and others.

One of the main problems at present for actuarial analysis of the Ukrainian insurance market is the lack of large statistical base, which is necessary for any econometric modeling. That is way there is a great necessity of actuarial models that involve fewer statistical information. We analyze methods of calculation of ruin probabilities for insurance company in presents of its investing activity. We consider an insurance company in the case when the premium rate is a bounded by some nonnegative random function and the capital of the insurance company is invested in a risky asset whose price follows a geometric Brownian.

The goal of the article is creation new types of actuarial models of the analysis of ruin probabilities that can be helpful for Ukrainian insurance companies under presence of their investment activities. There are different methods for approximating the distribution of aggregate claims and their corresponding stop-loss premium by means of a discrete compound Poisson distribution and its corresponding stop-loss premium. This discretization is an important step in the numerical evaluation of the distribution of aggregate claims, because recent results on recurrence relations for probabilities only apply to discrete distributions. The discretization technique is efficient in a certain sense, because a properly chosen discretization gives raise to numerical upper and lower bounds on the stop-loss premium, giving the possibility of calculating the numerically estimates for BNs correspond to another GM structure known as a directed acyclic graph (DAG) that is popular in the statistics, the machine learning, and the artificial intelligence societies. BNs are both mathematically rigorous and intuitively understandable.

They enable an effective representation and computation of the joint probability distribution (JPD) over a set of random variables. The structure of a DAG is defined by two sets: the set of nodes (vertices) and the set of directed edges. The nodes represent random variables and are drawn as circles labeled by the variable names. An extension of these genealogical terms is often used to define the sets of "descendants" – the set of nodes that can be reached on a direct path from the node, or "ancestor" nodes – the set of nodes from which the node can be reached on a direct path.

The structure of the acyclic graph guarantees that there is no node that can be its own ancestor or its own descendent. Such a condition is of vital importance to the factorization of the joint probability of a collection of nodes as seen below. Note that although the arrows represent direct causal connection between the variables, the reasoning process can operate on BNs by propagating information in any direction.

A BN reflects a simple conditional independence statement. Namely, that each variable is independent of its non-descendents in the graph given the state of its parents. This property is used to reduce, sometimes significantly, the number of parameters that are required to characterize the JPD of the variables. This reduction provides an efficient way to compute the posterior probabilities given the evidence. In addition to the DAG structure, which is often considered as the "qualitative" part of the model, one needs to specify the "quantitative" parameters of the model.

The parameters are described in a manner which is consistent with a Markovian property, where the conditional probability distribution (CPD) at each node depends only on its parents. For discrete random variables, this conditional probability is often represented by a table, listing the local probability that a child node takes on each of the feasible values – for each combination of values of its parents.

The joint distribution of a collection of variables can be determined uniquely by these local conditional probability tables (CPTs). Following the above discussion, a more formal definition of a BN can be given. A Bayesian network is an annotated acyclic graph that represents a JPD over a set of random variables. The graph encodes independence assumptions, by which each variable Xi is independent of its non-descendents. The second component denotes the set of parameters of the network.

It is well-known that the analysis of activity of an insurance company in conditions of uncertainty is of great importance [2, p. 14-17]. Starting from the classical papers of Cramer and Lundberg which first considered the ruin problem in stochastic environment, this subject has attracted much attention. Recall that, in the classical Cramer-Lundberg model satisfying the Cramer condition and, the positive safety loading assumption, the ruin probability as a function of the initial endowment decreases exponentially [3, p. 47-48]. The problem was subsequently extended to the case when the insurance risk process is a general Levy process.

It considers a person who might suffer from a back injury, an event represented by the variable Back (denoted by B). Such an injury can cause a backache, an event represented by the variable Ache (denoted by A). The back injury might result from a wrong sport activity, represented by the variable Sport (denoted by S) or from new uncomfortable chairs installed at the person's office, represented by the variable Chair (denoted by C). In the latter case, it is reasonable to assume that a co-worker will suffer and report a similar backache syndrome, an event represented by the variable Worker (denoted by W). All variables are binary; thus, they are either true (denoted by "T") or false (denoted by "F").

The CPT of each node is listed besides the node. In this example the parents of the variable Back are the nodes Chair and Sport. The child of Back is Ache, and the parent of Worker is Chair. Following the BN independence assumption, several independence statements can be observed in this case. For example, the variables Chair and Sport are marginally independent, but when Back is given they are conditionally dependent. This relation is often called explaining away.

When Chair is given, Worker and Back are conditionally independent. When Back is given, Ache is conditionally independent of its ancestors Chair and Sport. The conditional independence statement of the BN provides a compact factorization of the JPDs. Note that the BN form reduces the number of the model parameters, which belong to a multinomial distribution in this case, from 25 – 1 = 31 to 10 parameters. Such a reduction provides great benefits from inference, learning (parameter estimation), and computational perspective. The resulting model is more robust with respect to bias-variance effects. A practical graphical criterion that helps to investigate the structure of the JPD modeled by a BN is called d-separation.

It captures both the conditional independence and dependence relations that are implied by the Markov condition on the random variables. Inference via BN Given a BN that specified the JPD in a factored form, one can evaluate all possible inference queries by marginalization, i.e. summing out over "irrelevant" variables. Two types of inference support are often considered: predictive support for node Xi , based on evidence nodes connected to Xi through its parent nodes (also called top-down reasoning), and diagnostic support for node Xi , based on evidence nodes connected to Xi through its children nodes (also called bottom-up reasoning).

One might consider the diagnostic support for the belief on new uncomfortable chairs installed at the person's office, given the observation that the person suffers from a backache. In many practical settings the BN is unknown and one needs to learn it from the data. This problem is known as the BN learning problem, which can be stated informally as follows: Given training data and prior information, estimate the graph topology (network structure) and the parameters of the JPD in the BN.

Learning the BN structure is considered a harder problem than learning the BN parameters. Moreover, another obstacle arises in situations of partial observability when nodes are hidden or when data is missing.

In general, four BN learning cases are often considered, to which different learning methods are proposed. The log-likelihood scoring function decomposes according to the graph structure; hence, one can maximize the contribution to the log-likelihood of each node independently. Another alternative is to assign a prior probability density function to each parameter vector and use the training data to compute the posterior parameter distribution and the Bayes estimates.

To compensate for zero occurrences of some sequences in the training dataset, one can use appropriate (mixtures of) conjugate prior distributions, Such an approach results in a maximum a posteriori estimate and is also known as the equivalent sample size (ESS) method.

Let us investigate the problem of consistency of risk measures with respect to usual stochastic order and convex order. It is shown that under weak regularity conditions risk measures preserve these stochastic orders. This result is used to derive bounds for risk measures of portfolios.

As a by-product, we extend the characterization of coherent, law-invariant risk measures with the property to unbounded random variables. A surprising result is that the trading strategy yielding the optimal asymptotic decay of the ruin probability simply consists in holding a fixed quantity (which can be explicitly calculated) in the risky asset, independent of the current reserve. This result is in apparent contradiction to the common believe that 'rich' companies should invest more in risky assets than `poor' ones. The reason for this seemingly paradoxical result is that the minimization of the ruin probability is an extremely conservative optimization criterion, especially for 'rich' companies [2, p. 351].

In general, the other learning cases are computationally intractable. In the second case with known structure and partial observability, one can use the EM (expectation maximization) algorithm to find a locally optimal maximum-likelihood estimate of the parameters. MCMC is an alternative approach that has been used to estimate the parameters of the BN model. In the third case, the goal is to learn a DAG that best explains the data. This is an NP-hard problem, since the number of DAGs on N variables is super-exponential in N. One approach is to proceed with the simplest assumption that the variables are conditionally independent given a class, which is represented by a single common parent node to all the variable nodes.

This structure corresponds to the naive BN, which surprisingly is found to provide reasonably good results in some practical problems. To compute the Bayesian score in the fourth case with partial observability and unknown graph structure, one has to marginalize out the hidden nodes as well as the parameters. Since this is usually intractable, it is common to use an asymptotic approximation to the posterior called Bayesian information criterion (BIC) also known as the minimum description length (MDL) approach. In this case one considers the trade-off effects between the likelihood term and a penalty term associated with the model complexity.

An alternative approach is to conduct local search steps inside of the M step of the EM algorithm, known as structural EM, that presumably converges to a local maximum of the BIC score. BN and Other Markovian Probabilistic Models It is well known that classic machine learning methods like Hidden Markov models (HMMs), neural networks, and filters can be considered as special cases of BNs Specific types of BN models were developed to address stochastic processes, known as dynamic BN, and counterfactual information, known as functional BN.

However it is not always possible to produce absolute bounds.

The surplus process of an insurance portfolio is defined as the wealth obtained by the premium payments minus the reimbursements made at the times of claims. When this process becomes negative (if ever), we say that ruin has occurred. The general setting is the Gambler's Ruin Problem. We address the problem of estimating derivatives (sensitivities) of ruin probabilities with respect to the rate of accidents. Estimating probabilities of rare events is a challenging problem, since naive estimation is not applicable.

It is clear that, risky investment can be dangerous: disasters may arrive in the period when the market value of assets is low and the company will not be able to cover losses by selling these assets because of price fluctuations. Regulators are rather attentive to this issue and impose stringent constraints on company portfolios. Typically, junk bonds are prohibited and a prescribed (large) part of the portfolio should contain non-risky assets (e.g., Treasury bonds) while in the remaining part only risky assets with good ratings are allowed. The common notion that investments in an asset with stochastic interest rate may be too risky for an insurance company can be justified mathematically.

Solution approaches are very recent, mostly through the use of Importance Sampling techniques. Sensitivity estimation is an even harder problem for these situations. We study different methods for estimating ruin probabilities: one via importance sampling (IS), and two others via indirect simulation: the storage process (SP), which restates the problems in terms of a queuing system, and the convolution formula (CF).

The weak development of insurance market in Ukraine is explained by the low incomes of Ukrainians and their disinterest in spending money on insurance, although some cases. The analyzed economic and mathematical models are recommended to be used in for Ukrainian insurance companies for increasing profitability and diversification of ruin risks. Although there are several methods for calculation the ruin probabilities for insurance companies, this study may enrich existing methods, for cases of investment activities for Ukrainian insurance companies.

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РЕЗЮМЕ

Іллічевський Сергій

Аналіз українського страхового ринку за допомогою байєсівських мереж

Стаття присвячена дослідженню і розробці нових методів розрахунку ймовірності розорення страхової компанії на основі байєсівських мереж. Новизна даної статті полягає в тому, що ми використовуємо байєсівської мережі для вимірювання ймовірності банкрутства страхової компанії.

РЕЗЮМЕ

Илличевский Сергей

Анализ украинского страхового ринка с помощью байесовский сетей.

Статья посвящена исследованию и разработке новых методов расчета вероятности разорения страховой компании на основе байесовских сетей. Новизна данной статьи состоит в том, что мы используем байесовской сети для измерения вероятности банкротства страховой компании.

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