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## A SIMPLE APPROACH TO CREATING ARBITRAGE IN OPTION PRICING THEORY

G.M. Vostrov, Dr. of economic sciences, professor

O.R. Alao

*Odessa National Polytechnic University, Odessa, Ukraine*

An option is a contract in which the holder has the right but not the obligation to carry out the terms of the contract. Examples of options include European options and American options [1].

European options: where the holder can exercise his right to buy (a call) or sell (a put) at a specified date.

American options: where the holder has the right to buy or sell at any date up to the one specified in the contract. Note that options can be traded at any time before their exercise date.

To enable the purchasing or selling of an option, we would like to be able to determine its value at any point in time. In particular, we would like to know the value at the time the option is created, before the future behaviour of the underlying asset is known. Determining an option's value is commonly called option pricing.

### Analysis of recent researches and publications

The theory of option pricing is discussed in several works of foreign and domestic scientists. The most well-known authors of this concept: W. F. Sharpe, G. J. Alexander, J. W. Bailey, J. Hull, Y. Lyuu, D. C. Lewis, T. Bjork.

But this theory still requires refinement and detail approaches to the formation of the arbitration what is the relevance of this research.

### Unsolved aspects of the problem

In this article we want to see the mathematical concept about arbitrage in option pricing where we will use a discrete-time set up in order to simplify the mathematics involved; however, the discrete models do capture the fundamental aspect of option pricing in more general continuous time.

*The aim of the article* is to describe simple approach to creating arbitrage in option pricing theory.

### The main part

We now introduce basic financial terms that will be used throughout these papers.

A stock is a security representing partial ownership of a company. A unit of stock is called a share. Stocks are traded in the stock market.

The money market consists of risk-free securities, such as bonds, which accrue interest over time.

*Востров Г.М., Алао О.Р. Простий підхід до створення арбітражу в теорії ціноутворення опціонів.*

У статті розглянуто фінансові ринки і цінні папери, як договори, які мають особливе значення і є об'єктом купівлі-продажу. У статті запропоновано простий підхід до формування арбітражу згідно теорії ціноутворення опціонів.

*Ключові слова:* акції, ціна, опції, арбітраж, ціни опції, біноміальної моделі

*Востров Г.М., Алао О.Р. Техническая политика – основа развития потенциала предприятия.*

В статье рассмотрены финансовые рынки и ценные бумаги, как договоры, которые имеют особое значение и являются объектом купли-продажи. В статье предложен простой подход к формированию арбитража согласно теории ценообразования опционов.

*Ключевые слова:* акции, цена, опции, арбитраж, цены опции, биномиальная модель

*Vostrov G.M, Alao O. R. A simple approach to creating arbitrage in option pricing theory.*

The article discusses financial markets and securities, as agreements that are of particular importance and are the object of sale. This article proposes a simple approach to the formation of the arbitration according to the option pricing theory.

*Keywords:* stocks, price, options, arbitrage, options pricing, binomial model

In this paper, the interest rate  $r > 0$  is defined such that \$1 invested in the money market at time zero will be worth  $\$(1 + r)$  at time one.

An arbitrage is a trading strategy that, beginning with zero wealth, has zero probability of losing money, and has positive probability of making money.

An investor can short sell a stock by borrowing it from the owner and selling it to obtain the proceeds. The investor must repurchase the stock at some point, and return to the owner. If the share price falls after the investor short sells, the investor will make a profit after repurchasing the stock. Mathematically, this is equivalent to purchasing negative shares of stock

A portfolio is a collection of securities.

In short, a stock is a risky asset whereas assets from the money market are riskless.

American versus European options: the variables are relation to early exercise of options.

An American option is exercised at any point to its expiration, while a European option can be exercised only at expiration.

The possibility of early exercise makes an American option more valuable than otherwise similar European option.

However, in most cases, the time premium associated with the remaining life of an option makes early exercise sub-optimal.

Early exercise is generally not optimal, but there are two exceptions where.

The underlying asset pays large dividends, thus reducing the value of the asset, and of call options on it. In this case, a call option may be exercised just before an ex-dividend date, if the time premium on the option is less than the expected decline in asset value.

When an investor holds the underlying asset and deep in the money put on that asset, at a time when interest rates are high. The time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

Before proceeding further in this paper, we should be note that the stock market is currently divided into two segments: the stock market and the stock-exchange market. The volume of transactions on the stock market exceeds the sum of \$60 million and the market is not in stock exchange transactions exceeds \$350 million. The value of transactions on the stock market only significantly exceeds the value of the gross domestic product of all countries in the world. This fact exists as witness that the background of the market has a significant impact on the efficiency of the entire economic system of the world.

In this paper we consider only the exchange options market. Each option represents a package of 100 shares. Everyone who gets the stakes actually invest money in a concrete company. Despite the fact that the purchase of one or a set of options which is carried out in accordance with the contract stipulated for all stages of the transaction, however, because the stock price changes at random and mostly difficult to predict, there is always a significant probability of

losing a significant part of the money spent on the purchase of shares.

In the stock markets, the condition to reduce the risks for investors is the process of buying and selling shares of stock. One of the promising methods in the reign of pricing processes in the stock markets is the theory of arbitration which is based on a strategy to obtain a risk-free or very low risk strategy of pricing for identical financial instruments. It can be easily said that arbitration is a common investment that consist of sale options and any other securities at the highest price possible in a market situation where such securities are bought at the lowest price [2].

Arbitrage opportunities are determined based on an analysis of factors that affect the rate of stock options or any other securities which are listed on the stock markets. As noted in [2], the factor model implies that securities with the same sensitivity to influence factors behave the same way in the stock market, except in a no risk factor. Such securities with the same sensitivity must have the same expected return, otherwise will have to close arbitrage opportunities. Experience in equity markets suggests that as soon as these conditions occur, the activities of investors lead to the extinction of the arbitrage. This assumption underlies the theory of arbitration pricing.

Creating a replicating portfolio. The objective in creating a replicating portfolio is to use a combination of risk free borrowing/lending and the underlying asset to create the same cash flows as the option being valued.

$$\text{Call} = \text{Borrowing} + \text{Buying } D \text{ of the underlying Stock} \quad (1)$$

$$\text{Put} = \text{Selling Short } D \text{ on underlying asset} + \text{Lending} \quad (2)$$

The number of shares bought or sold is called the option delta.

The principles of arbitrage then apply, and the value of the option has to be equal to the value of the replicating portfolio.

The Binomial option pricing model. Binomial option pricing model is a simple but powerful technique that can be used to solve many complex option-pricing problems. In contrast to the Black-scholes and other complex option-pricing models that require solutions to stochastic differential equations, the binomial option-pricing model is mathematically simple. It is based on the assumption of no arbitrage.

The assumption of no arbitrage implies that all risk-free investments earn the risk-free rate of return and no investment opportunities exist that require zero dollars of investment but yield positive returns.

The binomial option pricing model limits the price movement to two choices, simplifying the mathematics tremendously at some expense of realism. The binomial option pricing model uses an iterative procedure, allowing for the specification of nodes, or points in time, during the time span between the valuation date and the option's expiration date. We first introduce the one-period binomial model and then discuss the more general multi-period model.

The one-period binomial model. The one-period or one-step binomial model can be used to price the option with the assumption that arbitrage opportunities do not exist. If we are given an example of this type of model, we could argue that because the portfolio has no risk, the expected return must be equal to the risk-free interest rate.

We consider a single stock with a price per share of  $S_0 > 0$  time zero. We can imagine the price at time one to be the result of a coin toss, either heads or tails, with probabilities  $p$  and  $q = 1 - p$ , respectively. At time one, the price per share will be either  $S_1(H)$  or  $S_1(T)$ , with probabilities  $p$  and  $q$ .

Let:

$$u = \frac{S_1(H)}{S_0}, d = \frac{S_1(T)}{S_0} \tag{3}$$

The one-period and multi-period process can form option pricing in a simplified form shown in fig. 1.

The basic assumption to simplify is that, at any level of the multi-step pattern with discrete time indices time  $t$  varies from that  $t_0 = 0$  to  $t = T$ . The value of  $T$  determines the time in multi period model. Recently, the stock price at time  $t$  equals  $S_t$ , and the dynamics of change in the stock price or options is defined by the relation:

$$S_{n+1} = S_n * Z_n \tag{4}$$

where  $Z_0, \dots, Z_{T-1}$  are independent and identical random variables, taking only  $u$  and  $d$  we have:

$$\begin{aligned} P(Z_n = u) &= Pu \\ P(Z_n = d) &= Pd \end{aligned} \tag{5}$$

Assume that both  $d$  and  $u$  are positive, and without loss of generality,  $d < u$ . The situation can then be represented with the following diagram.

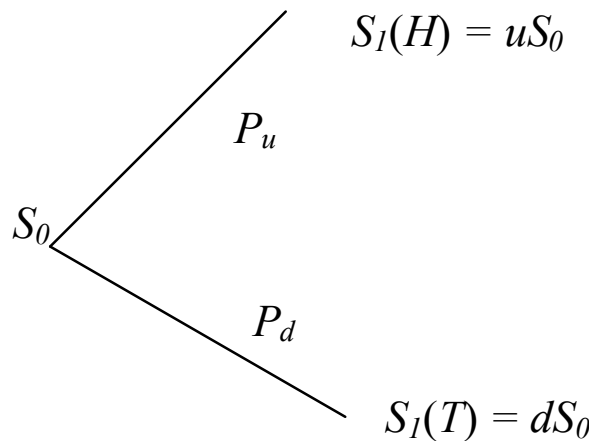


Fig. 1. The one-period binomial model for  $t = 0$  and  $T = t_0 + \Delta t$

In a one-period model,  $P_u + P_d = 1$  and  $\Delta$  is the ratio of the change in the option price to the change in the stock price as we move between the nodes at time  $T$ .

The multi-period binomial model. In the one-period model, we assumed that, given an initial price of  $S_0$ , the price of a stock could increase by a factor of  $u$  or decrease by a factor of  $d$  at time one. Now assume that at time two, the stock price can again increase or decrease by the multiplicative factors  $u$  and  $d$ , respectively. Then at time two, the possible stock prices are:

$$\begin{aligned} S_2(HH) &= uS_1(H) = u^2S_0, S_2(HT) = dS_1(H) = duS_0 \\ S_2(TH) &= uS_1(T) = udS_0, S_2(TT) = dS_1(T) = d^2S_0 \end{aligned} \tag{6}$$

Fig. 2 is the binomial multi-period mode. The results shown on fig. 3 show a rough analysis and comparison of dynamic prices of different models being compared to the real option price and how they are formed over time.

Continuing this pattern for multiple steps gives a binomial tree of stock prices.

So, the final tools available to use in the model consists of the stock described as well as the money market with interest rate  $r$ , a key assumption of our model is that it does not allow any arbitrage situations, as the possibility of a riskless profit could lead to contradictory results from the model.

Furthermore, any arbitrages in the real world quickly disappear as people take advantage of them. A simple condition on  $d$  and  $u$  will ensure the no-arbitrage requirement.

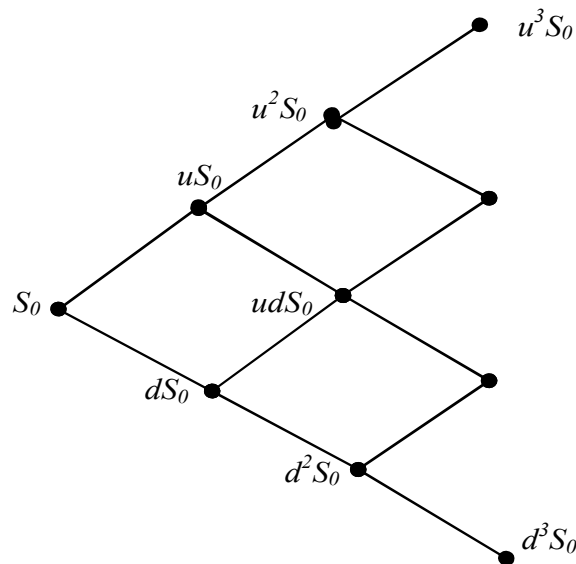


Fig. 2. The multi-period binomial model

The model reduces possibilities of price changes, removes the possibility for arbitrage, assumes a perfectly efficient market, and shortens the duration of

the option. Under these simplifications, it is able to provide a mathematical valuation of the option at each point in time specified.

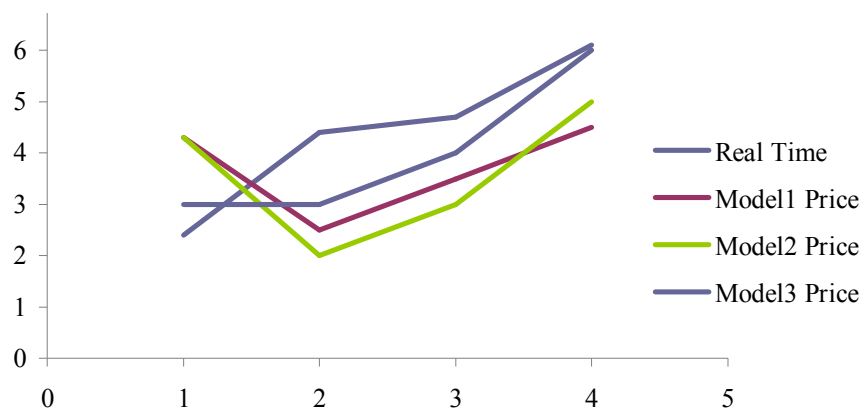


Fig. 3. Real and model dynamic price

### Conclusions

The process used to price the option the same procedure or concept used to price all options, whether with the simple binomial option model or the more complicated Black-scholes model. The assumption is that we find and form a risk-free hedge and then price the option off of that risk-free hedge. The key assumption is that the risk-less hedge will be priced in such a way that it earns exactly the risk-free

rate of return, which is where arbitrageurs come in to play. It is the activity of these individuals, looking for opportunities to invest in a risk less asset and earn more than the risk-free rate of return that insures that options are priced according to the no-arbitrage conditions. In general, there are two approaches to using the binomial model, the riskless hedge and the risk-neutral approach. Either approach will yield the same answer, but the underlying approach differs.

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Востров Георгій Миколайович / Georgiy M. Vostrov

Алао Олуватойм Рут / Oluwatoym R. Alao  
*oluwatoym.alao@gmail.com*

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