

.. ,

[1].

[2]. . . . [3]. . .

[4].

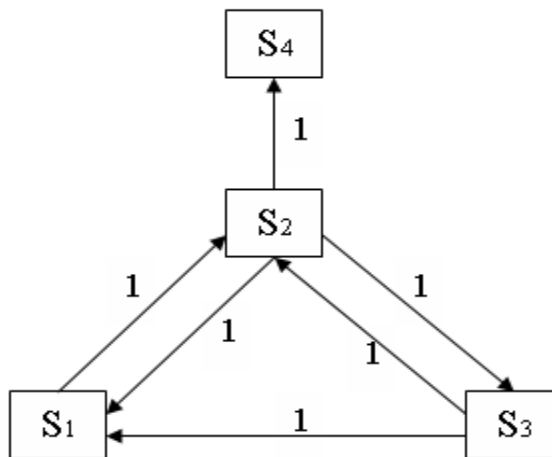
« » « ».

$t=1,$; $t=0$

$S,$: $S_1 -$, $S_2 -$, $S_3 -$, $S_4 -$

S

. 1.



.1.

S

S, S₁, S₂, S₃, S₄,

S,

[6; 7].

$$\lambda = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (1)$$

$p_1(t), p_2(t), p_3(t)$ $t,$

[8]:

$$\begin{cases} \frac{dp_1(t)}{dt} = -p_1(t) + p_2(t) + p_3(t); \\ \frac{dp_2(t)}{dt} = -3p_2(t) + p_1(t) + p_3(t); \\ \frac{dp_3(t)}{dt} = -2p_3(t) + p_2(t); \\ \frac{dp_4(t)}{dt} = p_2(t). \end{cases} \quad (2)$$

$p_4(t),$

$$\begin{cases} \frac{dp_1(t)}{dt} + p_1(t) - p_2(t) - p_3(t) = 0; \\ \frac{dp_2(t)}{dt} + 3p_2(t) - p_1(t) - p_3(t) = 0; \\ \frac{dp_3(t)}{dt} + 2p_3(t) - p_2(t) = 0. \end{cases} \quad (3)$$

(3)

(4):

$$\begin{cases} p_1(t) = \gamma_1 \cdot e^{\lambda t}; \\ p_2(t) = \gamma_2 \cdot e^{\lambda t}; \\ p_3(t) = \gamma_3 \cdot e^{\lambda t}, \end{cases} \quad (4)$$

$$\gamma_1, \gamma_2, \lambda \quad (3)$$

(4).

$$p_1(t) \quad p_2(t) \quad (3):$$

$$\begin{cases} \frac{d(\gamma_1 e^{\lambda t})}{dt} + \gamma_1 e^{\lambda t} - \gamma_2 e^{\lambda t} - \gamma_3 e^{\lambda t} = 0; \\ \frac{d(\gamma_2 e^{\lambda t})}{dt} + 3\gamma_2 e^{\lambda t} - \gamma_1 e^{\lambda t} - \gamma_3 e^{\lambda t} = 0; \\ \frac{d(\gamma_3 e^{\lambda t})}{dt} + 2\gamma_3 e^{\lambda t} - \gamma_2 e^{\lambda t} = 0. \end{cases} \quad (5)$$

$e^{\lambda t} (> 0):$

$$\begin{cases} \gamma_1 \cdot \lambda \cdot e^{\lambda t} + \gamma_1 e^{\lambda t} - \gamma_2 e^{\lambda t} - \gamma_3 e^{\lambda t} = 0 | : e^{\lambda t}; \\ \gamma_2 \cdot \lambda \cdot e^{\lambda t} + 3\gamma_2 e^{\lambda t} - \gamma_1 e^{\lambda t} - \gamma_3 e^{\lambda t} = 0 | : e^{\lambda t}; \\ \gamma_3 \cdot \lambda \cdot e^{\lambda t} + 2\gamma_3 e^{\lambda t} - \gamma_2 e^{\lambda t} = 0 | : e^{\lambda t}. \end{cases} \quad (6)$$

$$\begin{cases} \gamma_1 \cdot \lambda + \gamma_1 - \gamma_2 - \gamma_3 = 0; \\ \gamma_2 \cdot \lambda + 3\gamma_2 - \gamma_1 - \gamma_3 = 0; \\ \gamma_3 \cdot \lambda + 2\gamma_3 - \gamma_2 = 0. \end{cases} \quad (7)$$

$$\begin{cases} \gamma_1 \cdot (\lambda + 1) - \gamma_2 - \gamma_3 = 0; \\ \gamma_2 \cdot (\lambda + 3) - \gamma_1 - \gamma_3 = 0; \\ \gamma_3 \cdot (\lambda + 2) - \gamma_2 = 0. \end{cases} \quad (8)$$

$$\begin{cases} \gamma_1 \cdot (\lambda + 1) - \gamma_2 - \gamma_3 = 0; \\ -\gamma_1 + \gamma_2 \cdot (\lambda + 3) - \gamma_3 = 0; \\ -\gamma_2 + \gamma_3 \cdot (\lambda + 2) = 0. \end{cases} \quad (9)$$

$\gamma_1, \gamma_2, \gamma_3 \quad \lambda$

$$\gamma_1 = \gamma_2 = \gamma_3 = 0,$$

$$p_1(t) = \gamma_1 \cdot e^{\lambda t} = 0; \quad p_2(t) = \gamma_2 \cdot e^{\lambda t} = 0; \quad p_3(t) = \gamma_3 \cdot e^{\lambda t} = 0,$$

$$p_i(t) = 1.$$

$$\begin{vmatrix} \lambda + 1 & -1 & -1 \\ -1 & \lambda + 3 & -1 \\ 0 & -1 & \lambda + 2 \end{vmatrix} = 0. \quad (10)$$

$$\begin{aligned} & \lambda: \\ & (\lambda + 1) \cdot (\lambda + 3) \cdot (\lambda + 2) - 1 - \lambda - 2 - \lambda - 1 = 0; \\ & \lambda^3 + 4\lambda^2 + 3\lambda + 2\lambda^2 + 8\lambda + 6 - 4 - 2\lambda = 0; \\ & \lambda^3 + 6\lambda^2 + 9\lambda + 2 = 0. \end{aligned} \quad (11)$$

$$: a = 1, b = 6, c = 9, d = 2.$$

$$\lambda = y - \frac{b}{3a}. \quad (12)$$

$$y: \quad y^3 + py + q = 0, \quad (13)$$

$$p = -\frac{b^2}{3a^2} + \frac{c}{a}, \quad q = \frac{2b^3}{27a^3} - \frac{b^2}{3a^2} + \frac{d}{a} \quad p = -3, \quad q = 0.$$

$$Q: \quad Q = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2 \Rightarrow Q = -1. \quad (14)$$

$Q:$

$Q > 0$ - ;
 $Q < 0$ - ;
 $Q = 0$ -
 $p = q = 0$, .

$$y_1 = \alpha + \beta; y_2 = -\frac{\alpha + \beta}{2} + i\frac{\alpha - \beta}{2}\sqrt{3}; y_3 = -\frac{\alpha + \beta}{2} - i\frac{\alpha - \beta}{2}\sqrt{3}, \quad (15)$$

$$\alpha = \sqrt[3]{-\frac{q}{2} + \sqrt{Q}}, \beta = \sqrt[3]{-\frac{q}{2} - \sqrt{Q}}.$$

$$\alpha \cdot \beta = -p/3 \quad (\alpha \beta).$$

$\alpha \beta$ ():

$$\alpha_1 = (-1)^{\frac{1}{6}}, \alpha_2 = (-1)^{\frac{1}{6}}(-0,5 + \frac{i}{2}\sqrt{3}), \alpha_3 = (-1)^{\frac{1}{6}}(-0,5 - \frac{i}{2}\sqrt{3});$$

$$\beta_1 = -(-1)^{\frac{5}{6}}, \beta_2 = -((-1)^{\frac{5}{6}}(-0,5 + \frac{i}{2}\sqrt{3})), \beta_3 = -((-1)^{\frac{5}{6}}(-0,5 - \frac{i}{2}\sqrt{3})).$$

$$\alpha \quad \beta. \quad \lambda: \lambda_1 = -2, \lambda_2 = -3.73,$$

$\lambda_3 = -0,27.$

$$(9) \quad \lambda_1 = -2: \begin{cases} \gamma_1 \cdot (-2+1) - \gamma_2 - \gamma_3 = 0; \\ -\gamma_1 + \gamma_2 \cdot (-2+3) - \gamma_3 = 0; \\ -\gamma_2 + \gamma_3 \cdot (-2+2) = 0. \end{cases} \quad (16)$$

$$\begin{cases} -\gamma_1 - \gamma_2 - \gamma_3 = 0; \\ -\gamma_1 - \gamma_2 - \gamma_3 = 0; \\ -\gamma_2 = 0. \end{cases} \Rightarrow \gamma_2 = 0 \Rightarrow -\gamma_3 = \gamma_1. \quad (17)$$

$$(4), \quad \gamma_2, \gamma_1, \gamma_3. \quad \gamma_1 = 1, \quad \gamma_3 = -1.$$

$$(4), \quad : \begin{aligned} p_1^{[1]}(t) &= e^{-2t}; \\ p_2^{[1]}(t) &= 0; \\ p_3^{[1]}(t) &= -e^{-2t}. \end{aligned} \quad (18)$$

$$\lambda_2 = -3,73. \quad (9): \begin{cases} \gamma_1 \cdot (-3,73+1) - \gamma_2 - \gamma_3 = 0; \\ -\gamma_1 + \gamma_2 \cdot (-3,73+3) - \gamma_3 = 0; \\ -\gamma_2 + \gamma_3 \cdot (-3,73+2) = 0. \end{cases} \Rightarrow \begin{cases} -2,73\gamma_1 - \gamma_2 - \gamma_3 = 0; \\ -\gamma_1 - 0,73\gamma_2 - \gamma_3 = 0; \\ -\gamma_2 - 1,73\gamma_3 = 0. \end{cases} \Rightarrow \begin{cases} \gamma_1 = 0,26\gamma_3; \\ \gamma_2 = -1,73\gamma_3. \end{cases} \quad (19)$$

$$\gamma_3 = 1, \quad \gamma_1 = 0,26, \gamma_2 = -1,73 \quad (4): \begin{aligned} p_1^{[2]}(t) &= -0,26e^{-3,73t}; \\ p_2^{[2]}(t) &= -1,73e^{-3,73t}; \\ p_3^{[2]}(t) &= e^{-3,73t}. \end{aligned} \quad (20)$$

$$\lambda_3 = -0,27. \quad (9):$$

$$\begin{cases} \gamma_1 \cdot (-0,27 + 1) - \gamma_2 - \gamma_3 = 0; \\ -\gamma_1 + \gamma_2 \cdot (-0,27 + 3) - \gamma_3 = 0; \\ -\gamma_2 + \gamma_3 \cdot (-0,27 + 2) = 0. \end{cases} \Rightarrow \begin{cases} 0,73\gamma_1 - \gamma_2 - \gamma_3 = 0; \\ -\gamma_1 + 2,73\gamma_2 - \gamma_3 = 0; \\ -\gamma_2 + 1,73\gamma_3 = 0. \end{cases} \Rightarrow \begin{cases} \gamma_1 = 3,72\gamma_3; \\ \gamma_2 = 1,73\gamma_3. \end{cases} \quad (21)$$

$$\gamma_3 = 1, \quad \gamma_1 = 3,72, \gamma_2 = 1,73 \quad (4):$$

$$\begin{cases} p_1^{[3]}(t) = 3,72e^{-0,27t}; \\ p_2^{[3]}(t) = 1,73e^{-0,27t}; \\ p_3^{[3]}(t) = e^{-0,27t}. \end{cases} \quad (22)$$

$$(18), (20) \quad (22) \quad (2):$$

$$\begin{cases} p_1(t) = C_1 \cdot p_1^{[1]}(t) + C_2 \cdot p_1^{[2]}(t) + C_3 \cdot p_1^{[3]}(t); \\ p_2(t) = C_1 \cdot p_2^{[1]}(t) + C_2 \cdot p_2^{[2]}(t) + C_3 \cdot p_2^{[3]}(t); \\ p_3(t) = C_1 \cdot p_3^{[1]}(t) + C_2 \cdot p_3^{[2]}(t) + C_3 \cdot p_3^{[3]}(t), \end{cases} \quad (23)$$

$C_1, C_2, C_3 -$

$$\begin{cases} p_1(t) = C_1 \cdot e^{-2t} + C_2 \cdot (-0,26e^{-3,73t}) + C_3 \cdot 3,72e^{-0,27t}; \\ p_2(t) = C_1 \cdot 0 + C_2 \cdot (-1,73e^{-3,73t}) + C_3 \cdot 1,73e^{-0,27t}; \\ p_3(t) = C_1 \cdot (-e^{-2t}) + C_2 \cdot e^{-3,73t} + C_3 \cdot e^{-0,27t}. \end{cases} \quad (24)$$

$$\begin{cases} p_1(t) = C_1 \cdot e^{-2t} - 0,26 \cdot C_2 \cdot e^{-3,73t} + 3,72 \cdot C_3 \cdot e^{-0,27t}; \\ p_2(t) = -1,73 \cdot C_2 \cdot e^{-3,73t} + 1,73 \cdot C_3 \cdot e^{-0,27t}; \\ p_3(t) = -C_1 \cdot e^{-2t} + C_2 \cdot e^{-3,73t} + C_3 \cdot e^{-0,27t}. \end{cases} \quad (25)$$

$$p_1(0) = 1, p_2(0) = 0, p_3(0) = 0, p_4(0) = 0,$$

$C_1, C_2, C_3:$

$$\begin{cases} p_1(t) = C_1 \cdot e^{-2 \cdot 0} - 0,26 \cdot C_2 \cdot e^{-3,73 \cdot 0} + 3,72 \cdot C_3 \cdot e^{-0,27 \cdot 0} = C_1 - 0,26 \cdot C_2 + 3,72 \cdot C_3 = 1; \\ p_2(t) = -1,73 \cdot C_2 \cdot e^{-3,73 \cdot 0} + 1,73 \cdot C_3 \cdot e^{-0,27 \cdot 0} = -1,73 \cdot C_2 + 1,73 \cdot C_3 = 0; \\ p_3(t) = -C_1 \cdot e^{-2 \cdot 0} + C_2 \cdot e^{-3,73 \cdot 0} + C_3 \cdot e^{-0,27 \cdot 0} = -C_1 + C_2 + C_3 = 0. \end{cases}$$

$$\begin{cases} p_1(t) = C_1 - 0,26 \cdot C_2 + 3,72 \cdot C_3 = 1; \\ p_2(t) = -1,73 \cdot C_2 + 1,73 \cdot C_3 = 0; \\ p_3(t) = -C_1 + C_2 + C_3 = 0. \end{cases} \Rightarrow \begin{cases} 0,74 \cdot C_2 + 4,72 \cdot C_3 = 1; \\ C_2 = C_3; \\ C_1 = C_2 + C_3. \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 5,46 \cdot C_3 = 1; \\ C_2 = C_3; \\ C_1 = C_2 + C_3. \end{cases} \Rightarrow \begin{cases} C_3 = 0,18; \\ C_2 = 0,18; \\ C_1 = 0,36. \end{cases} \quad (25)$$

$$\begin{cases} p_1(t) = 0,36 \cdot e^{-2t} - 0,26 \cdot 0,18 \cdot e^{-3,73t} + 3,72 \cdot 0,18 \cdot e^{-0,27t}; \\ p_2(t) = -1,73 \cdot 0,18 \cdot e^{-3,73t} + 1,73 \cdot 0,18 \cdot e^{-0,27t}; \\ p_3(t) = -0,36 \cdot e^{-2t} + 0,18 \cdot e^{-3,73t} + 0,18 \cdot e^{-0,27t}. \end{cases} \quad (26)$$

$p_4(t)$

$$p_4(t) = 1 - p_1(t) - p_2(t) - p_3(t).$$

$$p_i(t) \leq 1.$$

$$S \quad t=1, \quad p_1(1), p_2(1), p_3(1), p_4(1):$$

$$\begin{cases} p_1(1) = 0,36 \cdot e^{-2} - 0,26 \cdot 0,18 \cdot e^{-3,73} + 3,72 \cdot 0,18 \cdot e^{-0,27} = 0,559 \\ p_2(1) = -1,73 \cdot 0,18 \cdot e^{-3,73} + 1,73 \cdot 0,18 \cdot e^{-0,27} = 0,23 \\ p_3(1) = -0,36 \cdot e^{-2} + 0,18 \cdot e^{-3,73} + 0,18 \cdot e^{-0,27} = 0,093 \\ p_4(1) = 1 - 0,559 - 0,23 - 0,093 = 0,118 \end{cases} \quad (27)$$

0,23; 0,559; 0,093; 0,118.

0,6
0,1

1. , 2010. – 164 .
2. , 2004. – 154 .
3. , 2008. – 37 .
4. , 2009. – 363 .
5. : 2- . / . . – 2000. – 368 .
6. 1964. – 765. : 2. / . . – ,
7. / . . –
8. , 2001. – 368 . / . . ,
2000. – 480 .

519.174

519.174

UDC 519.174

Skrylnyk Irina Ivanivna, first assistant of professor of economical cybernetics speciality of Poltava national technical university. **Budaykov Myhailo Yuriovitch**, the student of economical cybernetics speciality of Poltava national technical university. **Innovative methods of modelling of quality control system.** The article is devoted to the study of an innovative approach of the management of industrial processes and of a running efficiency of quality control system. For the modelling purposes the authors uses the theory of random processes in discrete event system with continuous time flow, as well as graph theory.

Keywords: system, graph states, Markov process.

30.12.2011 .