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STATE FUNCTION IN UNSTABLE CONVECTIVE MEDIUM

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Convection in a thin layer of liquid (gas) with temperature independent viscosity between poorly heat conducting boundaries is studied within framework of the Proctor-Sivashinsky model. We have shown by numerical simulation of the Proctor-Sivashinsky model that the state with certain topology can be described by the state function, which is the sum of squared mode of temperature spatial spectrum on the surface. The transitions between these states are characterized by splashes in time-derivative of this function and different meta-stable structures, corresponding to different values of the state function have different visually distinguishable topologies.
KEY WORDS: Rayleigh-Bénard convection, the Proctor-Sivashinsky model structural phase transitions, temperature undependent viscosity

ФУНКЦИИ СОСТОЯНИЯ В НЕСТАБИЛЬНОЙ КОНВЕКТИВНОЙ СРЕДЕ
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Конвекция в тонком слое жидкости (газа) между плохо проводящими тепло поверхностями рассмотрена в условиях применимости модели Проктора- Сивашинского при отсутствии зависимости вязкости от температуры. Мы покажем с помощью численного анализа модели Проктора –Сивашинского, что каждое состояние может быть описано с помощью функции состояния, которая равна сумме квадратов мод спектра пространственного распределения температуры на поверхности. Переходы между состояниями характеризуются изменением производной по времени от этой функции. Различие между метастабильными состояниями, которые отличаются топологией, определяется разными значениями функции состояния.
КЛЮЧЕВЫЕ СЛОВА: конвекция Релея-Бенара, модель Проктора-Сивашинского структурно-фазовых переходов, независимость вязкости от температуры

ФУНКЦІЇ СТАНУ У НЕСТАБІЛЬНОМУ КОНВЕКТИВНОМУ СЕРЕДОВИЩІ
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Конвекцію у тонкому шарі рідини (газу) між поверхнями, що недостатньо гарно проводять тепло, розглянуто в умовах придатності моделі Проктора-Сівашинського, причому при відсутності залежності в'язкості від температури. Ми покажемо з використанням числового моделювання, що кожний стан, може бути представлено фіксованим значенням функції стану, що дорівнює сумі квадратів мод просторового спектра температури на поверхні. Перехід між станами характеризується значенням похідної по часу від функції станів. Різниця між станами, що відрізняються топологією, відповідає різниці між відповідними значеннями функції станів.
КЛЮЧОВІ СЛОВА: конвекція Рєлея-Бенара, модель Проктора-Сівашинського структурно-фазових переходів, незалежність в'язкості від температури

The issues of structural transformations, structural second-order phase transitions, resulting in the changes of the symmetry and some characteristic scales of spatial structures always be of great interest to researchers and developers of technologies for example for description of the convection [1-3].

The Proctor-Sivashinsky model is found to be very attractive [4,5] for studying the processes of pattern formation in systems which possess a preferred characteristic spatial scale of interaction between the elements of future structure. This model was developed for description of convection in a thin layer of liquid between poorly conducting horizontal boundaries. Authors of [6] have found the stationary solutions with a small number of the spatial modes, one of which (convective cells) was steady and the second one turned out to be unstable (convective rolls). A particular future of the model is that it forces a preferred spatial scale of interaction, leaving the system a chance of selecting the symmetry during evolution. It was found, that the type of symmetry and hence the characteristics of the structure are determined by the minima of the potential of interaction between modes lying on a circle in k-space. Even within the Proctor-Sivashinsky model not all processes and the phenomena were studied. The detailed analysis of instability leading to the formation of a metastable structure (convective rolls) will be presented below. Earlier, it was found that at first stage of the instability evolution the metastable long-lived state (the curved quasi-one-dimensional convective rolls) arises. And later, after a lapse of time (which is considerably greater than the reverse linear increment of the process), the system transforms to the steady state (square convective cells) [7,8]. The detailed treatment of the Proctor-Sivashinsky model presented below shows that this structural transition demonstrates all the characteristics of the second order phase

transition (the continuity of the sum of squared mode amplitudes over the spectrum that the same, the continuity of density of this value and discontinuity of its time derivative. The existence of preferred scale (the distance between the regular spatial perturbations) and the possibility to select the type of symmetry (the regular spatial configuration) motivate the interest to this physical model, particularly for description of processes in solid state physics, where the characteristic distance between elements of spatial structures (atoms, molecules) in their condensed state is almost invariable. The objective of this work is investigation of the mechanisms of pattern formation and mode competition in convective medium. The nature and evolution of structural phase transitions between patterns of different topology are considered.

MODEL DESCRIPTION

When the Rayleigh number Ra exceeds the critical value corresponding to the onset of convective flow, the three-dimensional convection begins in a thin layer of liquid between poorly conducting horizontal plates heated from below [2], which can be described by the Proctor-Sivashinsky equation [4,5]. This equation determines the dynamics of temperature field in the horizontal plane (*x,y*):

$$\dot{\Phi} = \varepsilon^2\Phi + \gamma\cdot\nabla(\Phi\nabla\Phi) - (1-\nabla^2)^2\Phi + \frac{1}{3}\nabla\left(\nabla\Phi|\Phi|^2\right) + \varepsilon^2 f\,, \tag{1}$$

where f is the random function describing the external noise, and the quantity ε determines the convection threshold overriding, which is assumed to be sufficiently small ($0<\varepsilon<1$). The term $\gamma\nabla(\Phi\nabla\Phi)$ describes the temperature dependence of viscosity. Further, we assume $\gamma = 0$ for simplicity. In this case we shall find the solution in the form

$$\Phi = \varepsilon\sum_j a_j \exp(i\vec{k}_j\vec{r}) \tag{2}$$

with $|\vec{k}_j| = 1$. Renormalizing the time units $\propto \varepsilon^2$, we obtain the evolution equation for slow amplitudes a_j :

$$\dot{a}_j = a_j - \sum_{m=1}^{N} V_{mj}\,|a_m|^2\,a_j\,, \tag{3}$$

where interaction coefficients are determined as follows

$$V_{jj} = 1 \tag{4}$$

$$V_{ij} = (2/3)\left(1 - 2\left(\vec{k}_i\vec{k}_j\right)^2\right) = (2/3)\left(1 + 2\cos^2\vartheta\right). \tag{5}$$

Here ϑ is the angle between vectors $\vec{k}_i$ and $\vec{k}_j$.

The instability interval in *k*-space represents a ring with average radius equal to unit and the width is order of relative above-threshold parameter ϵ, i.e. much less than unity. During the development of the instability, the effective growth rate of modes that are localized outside of the very small neighborhood near the unit circle will decrease due to the growth of the nonlinear terms and can change sign which will lead to a narrowing of the spectrum to the unit circle in the *k*-space. Since the purpose of further research will be the study of stability of spatial structures with characteristic size of order $2\pi/k \propto 2\pi$ and the important characteristic for visualization of simulation results will be evidence of these structures, so we restrict ourselves by considering some idealized model of the phenomenon, assuming that the oscillation spectrum is already located on the unit circle in the *k*-space.

SIMULATION RESULTS

Development of perturbations in the system shown by the numerical analysis. Starting from initial fluctuations, the modes over a wide range of ϑ begin grow. The value of the quadratic form of the spectrum $I = \sum_j a_j^2$ can be estimated to obtain as result a value close to 0.75. It was shown that in the absence of temperature dependence of viscosity and when the number of modes is sufficiently large, the system delayed the development while remaining in a dynamic equilibrium. For further development - "crystallization", one of the modes must get a portion of the energy, which excesses some threshold value. That is, in these case, it is necessary a certain level of noise (fluctuations).

If one of the modes gets the proper amount of energy, then the process of formation of a simplest convective structure – rolls begins (Fig. 1a). Note that in the nature, the thin clouds also can form the roll structure. The value of I in this case tends to unity ($I \to 1$). However, this state is not stable and then we can see the next structural transition: convective rolls are modulated along the axis of fluid rotation, and the typical size of this modulation phases down. In this transition state, the system stays for a sufficiently long time (which slightly increases within some limits with increase in the number of modes), and the value $I \approx 1.07$ remains constant during this time. After a rather long time, ten times more than the inverse linear growth rate of the initial instability only the one mode “survives” from newly formed “side” spectrum, which amplitude is comparable with the amplitude of the primary leading mode. In the end, the stable convective structure – square cells is generated (Fig. 1b), and the quadratic form I reaches the value of $I = 1.2$.

Further researches of this process have found the following dynamics of quadratic form $I = \sum_j a_j^2$ with time

(Fig.2). Exact after the first peak of the derivative, the metastable structure – a system of convective rolls is formed, and up to the moment when the second burst have appeared with value of $I \approx 1$ it remains unchanged. The next burst indicates the onset of a secondary metastable structure with a new value of $I \approx 1.07$.

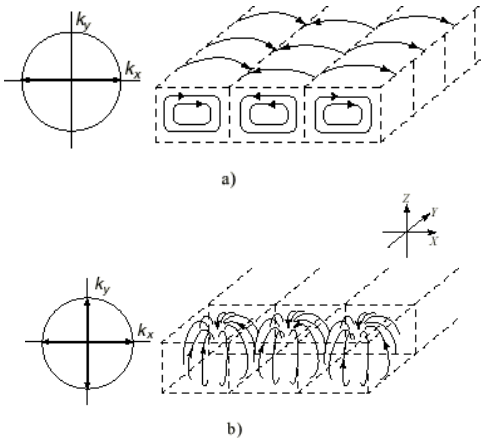

Fig.1. Convective structures: rolls (a) and square cells (b).

After the second burst of the quadratic form derivative, a stable structure of squared convective cells is started to build up. Such behavior proves the existence of structural-phase transitions in the system.

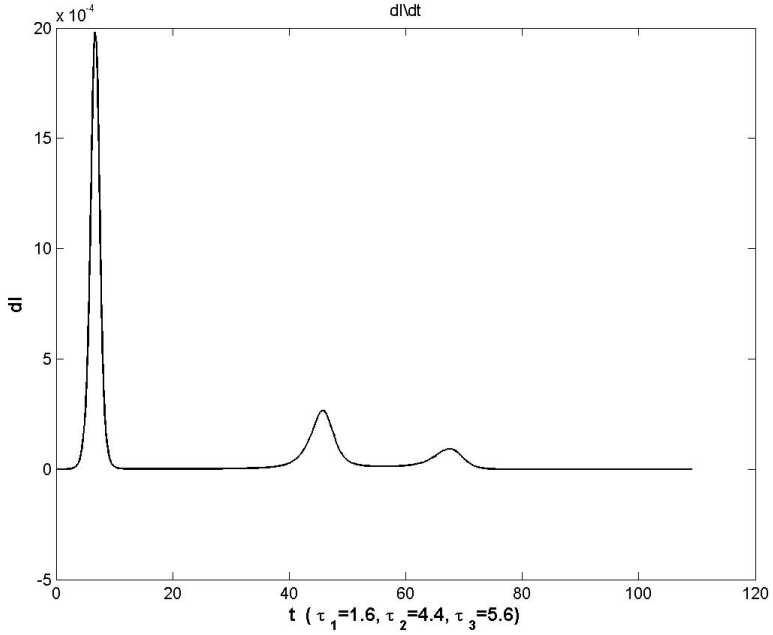

Fig. 2. The evolution of the derivative dI/dt (in relative measuring units) of the integral quadratic form $I = \sum_j a_j^2$

After the second splash of the time-derivative, a stable structure of squared convective cells is started to build up. Such behavior proves the existence of structural- phase transitions in the system. Generally speaking, the characteristic times of relaxation processes during evolution of the system to more equilibrium state are determined as usual by the difference of the state function values before the transition and after it. The greater this difference, the faster the transition from one state to another. It is important to keep in mind that the sequence of state transitions is determined by the characteristic times of instabilities (which play the role of relaxation processes) that provide a cascade evolution of the system to the most equilibrium state. Initially, the most fast relaxation processes take place that associated with large difference of the state function values corresponding to different equilibrium states.

Let us verify that in this case all the phenomena occur in the same order and within the framework of the foregoing scenario. The numerical analysis of the model allows confirming these considerations.

It can be seen that the times of state formation τ_n are inversely proportional to $I = \sum_i A^2{}_i$ the difference between the values $I_n^{(+)} = (\sum_i A^2{}_i)_n^{(+)}$ after *n*-th structural phase transition $I_n^{(+)} = (\sum_i A^2{}_i)_n^{(+)}$ and before it $I_n^{(-)} = (\sum_i A^2{}_i)_n^{(-)}$

$$\tau_n \sim \{(\sum_i A^2{}_i)_n^{(+)} - \sum_i A^2{}_i)_n^{(-)}\}^{-1} = \Delta I_n^{-1} \quad (6)$$

It follows from this that

$$\tau_3 / \tau_2 \approx \Delta I_2 / \Delta I_3, \quad (7)$$

Thus, we have shown by numerical simulation of the Proctor-Sivashinsky model that the state with certain topology can be described by the state function, which is the sum of squared mode amplitudes. The transitions between these states are characterized by splashes in time-derivative of this function and different meta-stable structures, corresponding to different values of the state function have different visually distinguishable topologies.

The fact that the metastable states are characterized by fixed values of the state function was highlighted in our earlier works [9-11]. The numerical study, presented in this paper, confirmed two observations: 1. the difference between the values of the state function before and after the structural phase transition is inversely proportional to the characteristic time of the corresponding structural-phase transition; 2. the evolution of the planar convective structure under consideration demonstrates all the features of a relaxation process, i.e. the fast structural-phase transition is succeeded by more slow ones. Thus, a fuller picture of the process becomes clear.

CONCLUSION

The special feature of the Proctor-Sivashinsky model with temperature independent viscosity is the existence of three possible metastable states. The times of structural transitions between these metastable states are much less than the times of their existence. Each state has a definite topology and can be characterized by definite steady value of the state function. The metastable states are destroyed with time for the instabilities, the growth rate of which can be evaluated from the amplitude of splashes of time-derivative of the state function. It is shown, that the characteristic times of the instabilities, which destroy the previous state and form a new one are inversely proportional to the difference between the values of the state function before and after the structural phase transition. In addition, we show that the faster relaxation processes, i.e. structural phase transitions take priority over more slow ones.

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