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CLASSIFICATION OF PARTICLES AT ARBITRARY QUANTITY OF GENERATIONS. I. HADRONS

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New classification of particles is proposed. This classification is based on $U(N_f, g)\times SU(3,c)\times SU(4, fs)\times O(3)$ -group, where $U(N_f, g)$ corresponds to the particle generations, $SU(3,c)$ - to the color, $SU(4, fs)$ - to the flavor and the spin (instead of known $SU(6, fs)$ -group), and $O(3)$ - to the orbital excitation with the L -momentum. The N_f -number equals the quantity of fermion generations. From the convergence of the integrals corresponding to the Green functions for generalized Dirac equations and the continuity of these functions it follows that the minimal quantity of the N_f -number equals six. The homogeneous solutions of derived equations are sums of fields, corresponding to particles with the same values of the spin, the electric charge, the parities, but with different masses. Such particles are grouped into the kinds (families, dynasties) with members which are the particle generations. For example, the electronic kind ($e_1 = e,\ e_2 = \mu,\ e_3 = \tau,\ e_4,\ e_5,\ e_6,$), the kind of up-quarks ($U_1 = u,\ U_2 = c,\ U_3 = t,\ U_4,\ U_5,\ U_6, \cdots$), and the kind of down-quarks ($D_1 = d,\ D_2 = s,\ D_3 = b,\ D_4,\ D_5,\ D_6, \cdots$) can exist. Massless neutrino can be one only. The photonic and the gluonic kinds must include massive particles in addition to usual the photon and the gluon. At N_f = 6 the nucleons and $\Delta(1232)$ belong to the $56\times 1\times 20\times 1$ -representation. Lagrangians for the generalized Dirac equations of arbitrary order are derived.
KEY WORDS: generations of particles, symmetry properties, quark models, new particles, Lagrangians

КЛАСИФІКАЦІЯ ЧАСТИНОК ПРИ ДОВІЛЬНІЙ КІЛЬКОСТІ ПОКОЛІНЬ. I. АДРОНИ

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Запропоновано нову класифікацію частинок. Ця класифікація основана на групі $U(N_f, g)\times SU(3,c)\times SU(4, fs)\times O(3)$, де $U(N_f, g)$ відповідає поколінням частинок, $SU(3,c)$ - кольору, $SU(4, fs)$ - аромату та спіну (замість відомої групи $SU(6, fs)$), та $O(3)$ - орбітальному збудженню з моментом L. Число N_f дорівнює кількості поколінь ферміонів. Із збіжності інтегралів відповідних функціям Гріна для узагальненого рівняння Дірака та неперервності цих функцій випливає, що мінімальне значення N_f дорівнює 6. Однорідні розв'язки одержаних рівнянь представляють собою суми полів відповідних частинкам з однаковими значеннями спіну, електричного заряду, парності, але з різними масами. Такі частинки групуються у роди (сім'ї, династії) з номерами, які відповідають поколінням частинок. Наприклад, можуть існувати електронний рід ($e_1 = e,\ e_2 = \mu,\ e_3 = \tau,\ e_4,\ e_5,\ e_6,$), рід верхніх кварків ($U_1 = u,\ U_2 = c,\ U_3 = t,\ U_4,\ U_5,\ U_6, \cdots$), та рід нижніх кварків ($D_1 = d,\ D_2 = s,\ D_3 = b,\ D_4,\ D_5,\ D_6, \cdots$). Безмасове нейтрино може бути тільки одне. Фотонний та глюонний роди повинні містити масивні частинки в доповнення до звичайних фотона та глюона. При N_f =6 нуклони та $\Delta(1232)$ належать до подання $56\times 1\times 20\times 1$. Одержано лагранжиани узагальнених рівнянь Дірака довільного порядку.
КЛЮЧОВІ СЛОВА: покоління частинок, симетрійні властивості, кваркові моделі, нові частинки, лагранжиани

КЛАССИФИКАЦИЯ ЧАСТИЦ ПРИ ПРОИЗВОЛЬНОМ КОЛИЧЕСТВЕ ПОКОЛЕНИЙ. I. АДРОНЫ

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Предложена новая классификация частиц. Эта классификация основана на группе $U(N_f, g)\times SU(3,c)\times SU(4, fs)\times O(3)$, где $U(N_f, g)$ соответствует поколениям частиц, $SU(3,c)$ - цвету, $SU(4, fs)$ - аромату и спину (вместо известной группы $SU(6, fs)$), и $O(3)$ - орбитальному возбуждению с моментом L. Число N_f равно количеству поколений фермионов. Из сходимости интегралов соответствующих функциям Грина для обобщенного уравнения Дирака и непрерывности этих функций слудует, что минимальное значение N_f равно 6. Однородные решения полученных уравнений представляют собой суммы полей соответствующих частицам с одинаковыми значенями спина, електрического заряда, четности, но с разными массами. Такие частицы группируются в роды (семьи, династии) с номерами, соответсвующими поколениям частиц. Например, могут существовать електронный род ($e_1 = e,\ e_2 = \mu,\ e_3 = \tau,\ e_4,\ e_5,\ e_6,$), род верхних кварков

($U_1 = u, U_2 = c, U_3 = t, \quad U_4, \quad U_5, \quad U_6, \cdot\cdot$), и род нижних кварков ($D_1 = d, D_2 = s, D_3 = b, \quad D_4, \quad D_5, \quad D_6, \cdot\cdot$). Безмассовое нейтрино может быть только одно. Фотонный и глюонный рода должны содержать массивные частицы в дополнение к обычным фотону и глюону. При N_f =6 нуклоны и $\Delta(1232)$ принадлежат представлению $56\times1\times20\times1$. Получены лагранжианы обобщеных уравнений Дирака любого порядка.
КЛЮЧЕВЫЕ СЛОВА: поколения частиц, свойства симметрий, кварковые модели, новые частицы, лагранжианы

Known particles are separated on hadrons and leptons. Properties of hadrons and leptons are different. Now it is known greater than 300 hadrons and twelve leptons together with antileptons. Mainly hadrons can be classified in quark models as $q\bar{q}-$ and q^3-systems for mesons and baryons, respectively. From usual point of view there are not any relations between the hadrons and the leptons. Investigations of the Adler – Bell – Jackiw axial anomaly in the electroweak theory showed that a contribution of one spin-$\frac{1}{2}$ particle (a quark or a lepton) leads to the linear divergence [1]. But taking into account of some sets of the leptons and the quark such as e, ν_e, u, d (first generation of the particles) or μ, ν_μ, c, s (second generation) or τ, ν_τ, t, b (third generation) allow to eliminate this divergence. Thus the convergence of the axial anomaly gives some relations between the quarks and the leptons.

In connection with this the next questions arise:

1. Why do the particle generations exist?
2. How many of the particle generations must exist?
3. What can hadrons and leptons have got common?

In relation with these questions the divergent integrals for the Green functions of the Klein-Gordon and Dirac equations are studied in Refs. [2, 3]. To avoid these divergences the generalizations of the Klein-Gordon and the Dirac equation is proposed in Refs. [2, 3]. In these papers next generalization of the Klein-Gordon equation has been proposed

$$\left(\Box+m_1^2\right)\left(\Box+m_2^2\right)........\left(\Box+m_{N_b}^2\right)\Phi(x)=\eta(x), \tag{1}$$

where $\Phi(x)$ is the field and $\eta(x)$ is the current (the field source). In momentum space the differential operator in (1) is the polynomial of the N_b-degree. We consider the case of the polynomial with real non-negative different zeros at $m_1 < m_2 < m_3 < .... < m_{N_b}$.

The general classical solution $\Phi_{cl}(x)$ of the linear equation (1) is the sum of the general solution of the corresponding homogeneous equation $\Phi(x)_{free}$ and partial solution $\Phi(x)_{nh}$ of non-homogeneous equation:

$$\Phi(x)_{free} = \int d^4q \sum_{k=1}^{N_b} \delta\left(q^2 - m_k^2\right)\left[c_k e^{-iqx} + \tilde{c}_k e^{iqx}\right] \tag{2}$$

$$\Phi(x)_{nh} = \int \bar{G}(x-y)\eta(y)d^4y, \tag{3}$$

where c_k and $\tilde{c}_k$ are the arbitrary constants. Thus $\Phi(x)_{free}$ is the sum on the terms corresponding to particles with the same charges, parity, spin but with different masses. Each term in (2) corresponding to number k is the solution of the homogeneous Klein – Gordon equation as $\left(\Box+m_k^2\right)\left(c_k e^{-iqx} + \tilde{c}_k e^{-iqx}\right)\delta\left(q^2 - m_k^2\right) = 0$. In Ref. [3] it is shown that the case of equal masses in Eq. (1) must be excluded. It was shown that the functions $\Phi(x)_{free}$ can include non- normalizable terms if at least two masses are equal. Thus the masses in the generalized Klein Gordon equation must be different. The N_b-number equals to the quantity of generations for spinless bosons and order of the equation (1) equals 2 N_b.

The Green functions for the generalized Klein-Gordon equations (1) are given by

$$\bar{G}(x) = \frac{1}{(2\pi)^4}\int \frac{e^{-iqx}d^4q}{\left(-q^2+m_1^2\right)\left(-q^2+m_2^2\right)....\left(-q^2+m_{N_b}^2\right)} \tag{4}$$

It is clear that the integrals in (4) can converge at $N_b \geq 3$, i. e., when the order of the equation (1) is equal or greater than six.

For the spin-$\frac{1}{2}$ particles the next generalization of the non-homogeneous Dirac equation is proposed in Refs. [2,3]

$$\left(-i\hat{\partial}+m_1\right)\left(-i\hat{\partial}+m_2\right).....\left(-i\hat{\partial}+m_{N_f}\right)\Psi(x)=\chi(x). \tag{5}$$

The classical solution of the homogeneous equation (5) is given by analogy with (2)

$$\Psi^{\alpha}(x)_{free}=\sum_{s}\sum_{k=1}^{N_f}\int d^4p\delta\left(q^2-m_k^2\right)\left[C_k u^{\alpha}{}_{k,s}(q)e^{-iqx}+\tilde{C}_k v^{\alpha}{}_{k,s}(q)e^{iqx}\right], \tag{6}$$

where α is the bispinor index, s corresponds to spin projection, $u^{\alpha}{}_{k,s}(q)$ and $v^{\alpha}{}_{k,s}(q)$ are the bispinors, C_k and $\tilde{C}_k$ are arbitrary constants. The Green functions (which are 4×4-matrixes) for this equation may be written as

$$\bar{S}(x)=\frac{1}{(2\pi)^4}\int\frac{(\hat{q}+m_1)(\hat{q}+m_2)....\left(\left(\hat{q}+m_{N_f}\right)\right)}{\left(-q^2+m_1^2\right)\left(-q^2+m_2^2\right)...\left(-q^2+m_{N_f}^2\right)}d^4q \tag{7}$$

The N_f-number equals to the quantity of generations for the spin-$\frac{1}{2}$ fermions and order of the equation (5) .The integrals in (7) can converge at $N_f \geq 5$. Note that for the advanced, retarded and causal Green functions we must write the corresponding imaginary infinitesimal term to each m_k^2 in (4), (7).

In Ref. [4] a continuity of the causal Green functions (4) and (7) has been investigated. It has been shown that these functions can be the continuous functions at $N_b \geq 3$, and $N_f \geq 6$, respectively. For these N_b and N_f the causal Green functions (4) and (7) have no any singularities in all the space-time. Thus, each particle must correspond to some generation. The minimal quantities of the generations for the spinless bosons and for the spin-$\frac{1}{2}$ fermions equal three and six, respectively.

Usually classification of hadrons in quark models is related to some unitary symmetries. These symmetries are valid for some particles with equal masses. In reality these symmetries are broken, as the particle masses are different. If the particles masses are nearly equal then results derived in unitary groups are approximately valid. Such situation is for the $SU(3,f)$-group related to the hadrons consisting of the $u,d,s-$ quarks and the $SU(6,fs)-$ group, which is the union of the $SU(3,f)$-flavor group and the $SU(2,s)-$ spin group [5-7]. Some predictions of these groups are in satisfactory agreement with experimental data. However, the agreement of predictions of the $SU(4,f)$- flavor group for the hadrons consisting of the $u,d,s,c-$quarks with experimental data is worse (and similarly for the $SU(8,fs)-$ group). Such agreement is not surprise as $m_u \approx m_d \approx$ 0.36 GeV, $m_s \approx$ 0.5 GeV, $m_c \approx$ 1.5 GeV .(in non-relativistic quark models). But the states of hadrons derived in these unitary groups can be used in quark models. In relation with possible existence of the hadrons consisting of the quarks from all the generations (from the first to the N_f-number generations) the question on the classifications of all the hadrons arises.

KINDS OF ELEMENTARY PARTICLES

Consider the distribution of the elementary particles in kinds (dynasties or families). Integrals corresponding to Green functions for the 1-spin diverge too. In Refs. [2, 3] the differential equations similar to Eqs. (1) and (5) for higher spin bosons and fermions have been proposed. Then for the photon and the gluon it can be put $m_1 = 0$. Therefore we can propose that two (or greater) massive members of the photon kind must exist. They must have zero electric charge and $J^p = 1^-$, $C = -1$. These particles must contribute to amplitudes of $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to$ hadrons at high energies and give the resonance behavior. We can expect that the coupling constants for the interactions of these members of the photonic kind with the leptons and the hadrons of the same electric charges must be equal. Similarly in the gluonic kind two (or greater) massive colored particles must exist. Besides two (or greater) massive members must exist in the Z^0 - and $W^{\pm}$ - kinds. In relations with the necessary existence of massive photons and gluons such questions arise: 1) Is the gauge invariance for massive photons and gluons possible or not?; 2) Does the scaling in deep inelastic lepton – hadron scattering at higher energies exist or not?

It has been shown that for the spin-$\frac{1}{2}$ particles the number of the kind members (i.e. generations) must be equal 6 (or greater). We assume that electronic kind ($e_1 = e,\, e_2 = \mu,\, e_3 = \tau,\, e_4,\, e_5,\, e_6, \ldots$), the neutrino kind $\left(\nu_1 = \nu_e, \nu_2 = \nu_\mu,\, \nu_3 = \nu_\tau, \nu_4, \nu_5, \nu_6, ...\right)$, three kinds of the colored $up-$quarks, and three kinds of the colored $down$ - quarks exist (for three colors). Note that in our approach only one neutrino may be masses (i. e., m_1 may equal zero and m_k must be positive at $k \geq 2$).

LAGRANGIANS FOR GENERALIZED DIRAC EQUATIONS

Operators of the generalized Dirac equations (5) are polynomials with respect to $-i\hat{\partial}$. They can be written as

$$\prod_{n=1}^{N} (-i\hat{\partial} + m_n) = \sum_{n=1}^{N} S(m_1, m_2, \cdots, m_N)_{N-n} (-i\partial)^n . \tag{8}$$

The S_k values are elementary symmetric functions [8]. They equal:

$$\begin{aligned}
&S(m_1, m_2, \cdots, m_N)_0 = 1, \\
&S(m_1, m_2, \cdots, m_N)_1 = m_1 + m_2 + \cdots + m_N, \\
&S(m_1, m_2, \cdots, m_N)_2 = m_1 m_2 + m_1 m_3 + \cdots + m_{N-1} m_N, \\
&S(m_1, m_2, \cdots, m_N)_3 = m_1 m_2 m_3 + m_1 m_2 m_4 + \cdots + m_{N-2} m_{N-1} m_N, \\
&S(m_1, m_2, \cdots, m_N)_N = m_1 m_2 m_3 \cdots m_{N-1} m_N.
\end{aligned} \tag{9}$$

For these functions the formula can be written at $k > 1$

$$S(m_1, m_2, \cdots, m_N)_k = \sum_{i_k > i_{k-1} > \cdots > i_2 > i_1 \geq 1}^{N} m_{i_1} m_{i_2} m_{i_3} \cdots m_{i_{k-1}} m_{i_k} . \tag{10}$$

The elementary symmetric functions are related to the binomial coefficients C_N^k

$$S(m, m, \cdots, m_N)_k = m^k C_N^k = m^k \frac{N!}{k!(N-k)!} \tag{11}$$

As the operators of generalized Dirac equations (5) include the partial derivatives of the N order and they are polynomials, Lagrangins for these equations must have polynomial structure. Let us denote

$$\Psi(x)_{\mu_1\mu_2\cdots\mu_k} = \partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_k}\Psi(x), \qquad \bar{\Psi}(x)_{\mu_1\mu_2\cdots\mu_k} = \partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_k}\bar{\Psi}(x) \quad . \tag{12}$$

In the operators of the equations (5) the terms with even and odd degrees of $\hat{\partial}$ must be separated (as $\hat{\partial}^2 = \Box$). Therefore, in the Lagrangians the terms including the derivatives of even and odd orders must be separated also. The Lagrangian for arbitrary N may be written as

$$L(x) = L(x)_{free} - \bar{\Psi}(x)\chi(x) - \bar{\chi}(x)\Psi(x), L(x)_{free} = \alpha_0 L_0 + \sum_{n=1}^{N_e} \alpha_{2n} L_{2n} + \sum_{n=0}^{N_{od}} \alpha_{2n+1} L_{2n+1} , \tag{13}$$

where α_k are numbers related to the elementary symmetric functions, $\chi(x)$ is current. The numbers N_e and N_{od} equal $\frac{N}{2}$ and $\frac{N}{2}$-1 for even N, respectively. Similarly, for odd N $N_e = N_{od} = (N-1)/2$. The terms of Lagrangians including the fields are given by

$$L_0 = \bar{\Psi}(x)\Psi(x), \qquad L_{2n} = \bar{\Psi}(x)_{\mu_1\mu_2\cdots\mu_{2n}} \Psi(x)_{\mu_1\mu_2\cdots\mu_{2n}} . \tag{14}$$

$$L_{2n+1} = \bar{\Psi}(x)_{\mu_1\mu_2\cdots\mu_{2n+1}}\gamma_{\mu_{2n+1}}\Psi(x)_{\mu_1\mu_2\cdots\mu_{2n}} - \bar{\Psi}(x)_{\mu_1\mu_2\cdots\mu_{2n}}\gamma_{\mu_{2n+1}}\Psi(x)_{\mu_1\mu_2\cdots\mu_{2n+1}}.$$

Using the least action principle (the Ostrogradskij-Hamilton principle) the Ostrogradskij-Euler equations [9, 10], which are generalizations of the Euler-Lagrange equations, can be derived. The equation (5) can be obtained by means of the variation of the Lagrangian *L(x)* with respect to $\bar{\Psi}$. Consider the operators of these equations

$$O_{EL}(\bar{\Psi}) = \frac{\partial}{\partial\bar{\Psi}} + \sum_{n=1}^{N}(-1)^n \partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_n}\frac{\partial}{\partial\bar{\Psi}_{\mu_1\mu_2\cdots\mu_n}}. \quad (15)$$

Then, the Ostrogradskij-Euler equations, which must coincide with the equations (5), may be written as

$$O_{EL}(\bar{\Psi})L(x) = 0. \quad (16)$$

For the terms in the Lagrangian (13) next expressions may be derived

$$O_{EL}(\bar{\Psi})L_0 = \Psi, \quad O_{EL}(\bar{\Psi})L_{2n} = \Box^n\Psi, \quad O_{EL}(\bar{\Psi})L_{2n+1} = -2\hat{\partial}\Box^n\Psi. \quad (17)$$

Using last results, by means of a comparison of the equations (8) and (16), relations between the α_k-coefficients in the Lagrangian (13) and the elementary symmetric functions S_k are obtained. It allows one to derive the Lagrangians in terms of the S_k-functions (9) and the L_k-values (14) at arbitrary N:

1. For even *N*

$$L(x) = -\bar{\Psi}(x)\chi(x) - \bar{\chi}(x)\Psi(x) + S_N L_0 + \sum_{n=1}^{\frac{N}{2}}(-1)^n S_{N-2n}L_{2n} + \frac{i}{2}\sum_{n=1}^{\frac{N}{2}}(-1)^{n+1}S_{N-2n+1}L_{2n-1} \quad (18)$$

In particular, at N = 6 the Lagrangian equals:

$$L(x) = -\bar{\Psi}(x)\chi(x) - \bar{\chi}(x)\Psi(x) + S_6L_0 - S_4L_2 + S_2L_4 - S_0L_6 + \frac{i}{2}(S_5L_1 - S_3L_3 + S_1L_5)$$

2. For odd N

$$L(x) = -\bar{\Psi}(x)\chi(x) - \bar{\chi}(x)\Psi(x) + S_N L_0 + \sum_{n=1}^{\frac{N-1}{2}}(-1)^n S_{N-2n}L_{2n} + \frac{i}{2}\sum_{n=0}^{\frac{N-1}{2}}(-1)^n S_{N-2n-1}L_{2n+1} \quad (19)$$

In particular, at N =7 the Lagrangian equals

$$L(x) = -\bar{\Psi}(x)\chi(x) - \bar{\chi}(x)\Psi(x) + S_7L_0 - S_5L_2 + S_3L_4 - S_1L_6 + \frac{i}{2}(S_6L_1 - S_4L_3 + S_2L_5 - S_0L_7)$$

NEW CLASSIFICATION OF HADRONS

Different quarks have got the quantum numbers related to a spin (s), a color (c), and flavors (f). For description of hadrons consisting of u-, d-, s-quarks usually the $SU(3,c)\otimes SU(6,fs)\otimes O(3)$ group is exploited [7]. The $O(3)$-group is used for a description of orbital excitations of the quarks in some hadrons.

Now consider two quark kinds: U-quarks and D-quarks for each color. The members U_k and D_k of the U-quark and the D-quark kinds can be written as: $U_1 = u, U_2 = c, U_3 = t, \ U_4, \ U_5, \ U_6 \cdots$ and $D_1 = d, D_2 = s, D_3 = b, \ D_4, \ D_5, \ D_6, \cdots$, respectively. Such the k number (which is related to generations) can be considered as new quantum number. Therefore, for description of hadrons we propose the $U(N_f, g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group. In this paper the hadrons consisting of quarks in $1s-$state are considered. Such hadrons are described by singlets of the $O(3)-$group. The U_k- and D_k- quarks from one generation belong the $N_f\times 3\times 4\times 1$-representation of the $U(N_f, g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group.

Usually the symmetry groups such as $SU(3,f), SU(6,fs)$ are considered for particles with equal or near masses, But the differences between the masses for particles of different generations are very large. Indeed, the $U-$ quarks have got masses: $m_u = m(U_1) \approx$ 0.36 GeV, (in non-relativistic quark models), $m_c = m(U_2) \approx$ 1.5 GeV, $m_t = m(U_3) \approx$ 177 GeV. Similarly, the $D-$ quarks have got the masses: $m_d = m(D_1) \approx$ 0.36 GeV, $m_s = m(D_2) \approx$ 0.5 GeV, $m_b = m(D_3) \approx$ 4.5 GeV [11]. Therefore, the use of the $SU(N_f, g)$-group can seem doubtful.

It may be assumed that the laws of a conservation for such quantities as the strangeness , the charm, the beauty, and the truth (by analogy with the law of the conservation of the particle electric charges) do not exist. Indeed, the charm and the truth correspond to the U_k-quarks at $k = 2,3$. Similarly, the strangeness and the beauty correspond to the D_k-quarks for $k = 2,3$. As usual, propose that non-zero anticommutators for creation operators and annihilation operators are given by

$$\{a(\vec{p}_1)_{k_1,\sigma_1}, a(\vec{p}_2)^+_{k_2,\sigma_2}\} = 2p_{10}\delta_{k_1k_2}\delta_{\sigma_1\sigma_2}\delta(\vec{p}_1 - \vec{p}_2),$$

$$\{b(\vec{p}_1)_{k_1,\sigma_1}, b(\vec{p}_2)^+_{k_2,\sigma_2}\} = 2p_{10}\delta_{k_1k_2}\delta_{\sigma_1\sigma_2}\delta(\vec{p}_1 - \vec{p}_2), \tag{20}$$

where $a(\vec{p})_{k,\sigma}$ ($a(\vec{p})^+_{k,\sigma}$) are the annihilation (creation) operators of the quark for the k-generation with σ quantum numbers (spin projection, flower (U or D), color, and other values), $p = (p_0, \vec{p})$ is the 4-momentum of the quark ($p_{0k} = \sqrt{\vec{p}_k^{\,2} + m_k^{\,2}}$). By analogy, $b(\vec{p})_{k,\sigma}$ ($b(\vec{p})^+_{k,\sigma}$) are the annihilation (creation) operators of the antiquark. The anticommutators (20) agree with the orthonormalization condition

$$\langle \vec{p}_1, k_1, \sigma_1 | \vec{p}_2, k_2, \sigma_2 \rangle = 2p_{10}\delta_{k_1k_2}\delta_{\sigma_1\sigma_2}\delta(\vec{p}_1 - \vec{p}_2) \tag{21}$$

In formulae (20, 21) the conditions $p_{k_1} = p_{k_2}$ are equivalent to $\delta_{k_1k_2}$. An inclusion of the $\delta_{k_1k_2}$-factor in the formulae (20, 21) permits to see explicitly the zero results at $k_1 \neq k_2$. Thus, the conservation of such quantities as the strangness , the charm, the beauty, and the truth is the consequence of the anticommutators (20) and the conditions (21).

The classical free field for the quarks is presented in (6). It can be seen that for quantized fields scalar products (where $\overline{\Psi(x)_\alpha} = (\Psi(x)^+\gamma_0)_\alpha$) are not changed under unitary transformations

$$\Psi(x)^\alpha \to \Psi(x)^{\alpha\prime} = \exp(i\theta_i\Gamma_i)\Psi(x)^\alpha, \tag{22}$$

$$\overline{\Psi(x)_\alpha} \to \overline{\Psi(x)_\alpha}' = \overline{\Psi(x)_\alpha}\exp(-i\theta_i\Gamma_i),$$

where Γ_i are the generators of the $U(N_f, g)$-group, θ_i are the parameters of the transformation. In particular, the vacuum means of the Lagrangians (18, 19) do not change under transformations (22). Indeed, the S_k-functions (9) are constants and the vacuum means of the L_k-values (14) do not change under the transformations (22). In particular, the transformation (22) leading to permutation of generations in one kind, can be considered. The generation numbers are related to particle masses. The equations (5), the solution (6), and the S_k-functions (9) do not change at the permutations of the masses. Therefore, the scalar products $\overline{\Psi(x)_\alpha}\Psi(x)^\alpha$ are invariant under the generation permutation. Thus the use of the $U(N_f, g)$-group can be valid for a classification of particle generations.

CLASSIFICATION OF MESONS

A representation of the $SU(4, fs)$-group for $q\overline{q}-$system is reducable: $4\times 4 = 15 + 1$. A dimension of a representation of the $U(N_f, g)$-group for the $q\overline{q}-$system is equal to $N_f^{\,2}$. Therefore, the colorless states of the $q\overline{q}-$system belong to next representations of the $U(N_f, g)\times SU(3,c)\times SU(4, fs)\times O(3)$-group: $N_f^{\,2}\times 1\times 15\times 1$ and $N_f^{\,2}\times 1\times 1\times 1$. The $SU(4, fs)$-group is a subgroup of the $SU(6, fs)$-group. Therefore, known expansion of the representations $SU(6, fs)$-group with respect to the representations of the

$SU(3,f)\times SU(2,s)$ -group can be used to find similar expansion for the representations $SU(4,fs)$ -group with respect to the representations of the $SU(2,f)\times SU(2,s)$ -group in the case of the U_1 – and D_1 -quarks. For the $15-$plet of the $SU(4,fs)$ -group such expansion can be written as $15 = 3\times 1 + 3\times 3 + 1\times 3$. The first term in this expansion corresponds to pseudoscalar mesons (π^+,π^0,π^-), the second - to charged and neutral vector mesons (ρ^+,ρ^0,ρ^-), and third – to neutral vector mesons (ω^0). This expansion can be generalized on the case of different generations.

1. *Pseudoscalar mesons.* At first consider the charged pseudoscalar mesons. The mesons with positive and negative charges will be denoted as

$$P^+{}_{k_1k_2} = [U_{k_1}\bar{D}_{k_2}], \qquad P^-{}_{k_1k_2} = [D_{k_1}\bar{U}_{k_2}], \qquad \bar{P}^+_{k_1k_2} = P^-{}_{k_2k_1}. \tag{23}$$

The spin states of the pseudoscalar mesons are chosen according to Ref [7], for example

$$\left|P^+{}_{k_1k_2}\right\rangle = \frac{1}{\sqrt{2}}\left|U\uparrow_{k_1}\bar{D}\downarrow_{k_2} - U\downarrow_{k_1}\bar{D}\uparrow_{k_2}\right\rangle = \left|U_{k_1}\bar{D}_{k_2}\uparrow\downarrow\right\rangle_- \tag{24}$$

Consider some well known pseudoscalar mesons: $P^+{}_{11} = \pi^+(140)$, $P^+{}_{22} = D^+{}_s(1969)$, $P^+{}_{12} = K^+(494)$, $P^+{}_{21} = D^+(1869)$, $P^+{}_{13} = B^+(5278)$.

In the $SU(4,fs)$ -group the neutral mesons can be obtained from the charged mesons by means of the isospin symmetry operators. For $k\ge 2$ a use of the isospin symmetry is not adequate. But the quarks can participate in weak interactions. It is possible to choice the phases for transitions with $W^\pm-$bosons such as for the isospin operators, i.e., $U\to W^+D$, $\bar{D}\to -W^+\bar{U}$. Using these transitions it is possible to obtain the states for the neutral mesons from states for the mesons with the positive charge. Such, although the isotopic invariance is not valid for $k\ge 2$, relations between the matrix elements of transitions at $k=1$ can be derived the same as in the isotopic symmetry.

For a $U\bar{D}-$system the relation $\left|U\bar{D}\right\rangle \to W^+\frac{1}{\sqrt{2}}\left|-U\bar{U}+D\bar{D}\right\rangle$ can be derived. Let us denote

$$P^0{}_{k_1k_2,U} = [U_{k_1}\bar{U}_{k_2}], \qquad P^0{}_{k_1k_2,D} = [D_{k_1}\bar{D}_{k_2}], \qquad P^0{}_{k_1k_2,UD} = \frac{1}{\sqrt{2}}(P^0{}_{k_1k_2,D} - P^0{}_{k_1k_2,U}). \tag{25}$$

Then for some neutral pseudoscalar mesons it can be written: $P^0{}_{11,UD} = \pi^0(140)$, $P^0{}_{12,D} = K^0(498)$, $P^0{}_{12,U} = \bar{D}^0(1864)$, $P^0{}_{21,D} = \bar{K}^0(498)$ $P^0{}_{21,U} = D^0(1864)$, $P^0{}_{13,D} = B^0(5279)$, $P^0{}_{31,D} = \bar{B}^0(5279)$. Possibly for neutral mesons, corresponding to different k_1 and k_2 such expressions $(P^0{}_{k_1k_2} - P^0{}_{k_2k_1})/\sqrt{2}$ may be considered. These expressions are similar to the expression for π^0-meson ($(d\bar{d}-u\bar{u})/\sqrt{2}$). For $k_1=1, k_2=2$ it corresponds to $K^0{}_2 = (K^0(493) - \bar{K}^0(493))/\sqrt{2}$.

2. *Vector mesons.* In the $SU(4,fs)$ -group the vector mesons correspond two representations of the $SU(2,f)$-group. One of them includes charged and neutral mesons but second – neutral mesons only. For the representation including charged meson (the 3×3 representations of the $SU(2,f)\times SU(2,s)$-group) we use the denotations similar to (11) and (12) for charged and neutral mesons, respectively (with substitutions V instead of P). Some charged vector mesons are: $V^+{}_{11} = \rho^+(770)$, $V^+{}_{22} = D_s^{*+}(2112)$, $V^+{}_{12} = K^{*+}(892)$. The neutral vector mesons from the 3×3 representations of the $SU(2,f)\times SU(2,s)$-group are: $V^0{}_{11} = \frac{1}{\sqrt{2}}(-u\bar{u}+d\bar{d}) = \rho^0(770)$, $V^0{}_{12,D} = K^{*0}(892)$, $V^0{}_{21,D} = \bar{K}^{*0}(892)$, $V^0{}_{12,U} = D^{*0}(2010)$, $V^0{}_{21,U} = \bar{D}^{*0}(2010)$.

States of neutral vector mesons from the 1×3 representations of the $SU(2,f)\times SU(2,s)$-group denote as

$$\omega_{k_1k_2,U} = [U_{k_1}\bar{U}_{k_2}] \qquad \omega_{k_1k_2,D} = [D_{k_1}\bar{D}_{k_2}] \tag{26}$$

The states of some known neutral mesons can be written as: $\frac{1}{\sqrt{2}}\left|\ \omega_{11,U}+\omega_{11,D}\ \right\rangle=\left|\omega^0(783)\ \right\rangle$, $\left|\omega_{22,D}\ \right\rangle=\left|\varphi(1020)\ \right\rangle$, $\left|\omega_{22,U}\ \right\rangle=\left|J/\psi(3097)\ \right\rangle$, $\left|\omega_{33,D}\ \right\rangle=\left|\Upsilon(9460)\ \right\rangle$.

3. *Singlet states of the* $SU(4,fs)$ *-group.* Denote the states corresponding to the $N_f^{\,2}\times1\times1\times1$-representations of the $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group as

$$\left|\ \eta_{k_1k_2}\ \right\rangle=\frac{1}{2}\left|U\uparrow_{k_1}\bar{U}\downarrow_{k_2}+U\downarrow_{k_1}\bar{U}\uparrow_{k_2}+D\uparrow_{k_1}\bar{D}\downarrow_{k_2}+D\downarrow_{k_1}\bar{D}\uparrow_{k_2}\ \right\rangle. \tag{27}$$

At $k_1=k_2=k$ the η_{kk} pseudoscalar mesons are totally neutral. For $k_1\neq k_2$ the $\eta_{k_1k_2}$-mesons have got the zero electric charge, but they are not totally neutral. For example,

$$\left|\ \eta_{12}\ \right\rangle=\frac{1}{2}\left|u\uparrow\bar{c}\downarrow+u\downarrow\bar{c}\uparrow+d\uparrow\bar{s}\downarrow+d\downarrow\bar{s}\uparrow\ \right\rangle \tag{28}$$

and antiparticle of the η_{12}-meson is the η_{21}-meson.

4. $U(N_f,g)$ *-group or* $SU(N_f,g)$ *-group?* The $q\bar{q}-$systems with quarks of N_f generations are described in the $SU(N_f,g)$-group by means of representations with the ($N_f^{\,2}-1$) and 1 dimensions. The representation of the 1 dimension corresponds to totally neutral pseudoscalar meson. The representation of the $N_f^{\,2}-1$ dimension is described by traceless tensor of second rank. Therefore, these traceless tensors give the representations of the $SU(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group which distinguish from the representations of the $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group for neutral and vector mesons. In particular, for positively charged pseudoscalar mesons in the $SU(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$ group next states

$$\frac{1}{\sqrt{2}}(P^+{}_{11}-P^+{}_{22}),\qquad \frac{1}{\sqrt{6}}(P^+{}_{11}+P^+{}_{22}-2P^+{}_{33}), \tag{29}$$

$$\frac{1}{\sqrt{N_f(N_f-1)}}[P^+{}_{11}+P^+{}_{22}+\cdots+P^+{}_{N_f-1,N_f-1}-(N_f-1)P^+{}_{N_fN_f}]$$

must exist instead the $P^+{}_{11}$, $P^+{}_{22}$, $P^+{}_{33}\cdots$ -states. Probably the states (29) do not exist. Therefore, the $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group ought to be used for the classification of the $q\bar{q}-$mesons. This group can be exploited for the baryons also.

CLASSIFICATION OF BARYONS

Consider q^3-baryonic systems with quarks in $1s-$state. It is known (e.g., Refs. [7], [12], [13]) that the representations for q^3-system in the spaces of flavors and spin are reducible and can be expanded as sums of the symmetric (S), the antisymmetric (A) representations, and two representations of the mixed symmetry (M_S,M_A). Such expansions have to exist in the spaces of the color and the generations also. From the Pauli principle the total antisymmetry of the tensors for q^3-systems follows. The $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group gives representations with determined properties of symmetry (related to the space of generations) in addition to representations of .the $SU(3,c)\times SU(4,fs)\times O(3)$-group. It allows one to assume an existence of some baryons which are non-antisymmetric representations of the $SU(3,c)\times SU(4,fs)\times O(3)$-group.

Note that a type of the symmetry for the representation of the q^3-system in some space can be determined by means of a quantity of equal quantum numbers for the quarks. If in a representation for the q^3-system the state with three equal numbers of the quark exists, then this representation is symmetric. If maximal number of equal quantum number of the quarks is equal to two in a representation of a q^3-system, then this representation has got mixed symmetry. If in a representation for the q^3-system all the states have got different quantum numbers of the quarks, then this representation is antisymmetric. For quantum numbers of a quark which correspond to variables in a space of the n-dimension, the representations for the q^3-system of the symmetric, the antisymmetric, and mixed symmetry types have got the dimensions: $N_S(n)=n(n+1)(n+2)/6$, $N_A(n)=n(n-1)(n-2)/6$, and $N_{M_A}(n)=N_{M_S}(n)=n(n^2-1)/3$, respectively.

As Δ^{++}, Δ^{-}, and Ω^{-} consist of the same quarks, nucleons and other baryons (which belong to 56-plet of the $SU(6,fs)$ and similar baryons including the quark $U_2=c$ instead the $D_2=s$) are described by symmetric 20-plet of the $SU(4,fs)$ and symmetric representation of the $U(N_f,g)$ at k=1 and 2. For k=3 we obtain the ttt-baryon (similar to Δ^{++}) and the bbb-baryon (similar to Δ^{-} and Ω^{-}). Thus, these baryons belong to the $N_S(N_f)\times 1\times 20\times 1$-representation of the $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group (for $N_f=6$ the number $N_S(N_f)$ equals 56). These baryons have got the spin-parity $J^P=\frac{1}{2}^+,\frac{3}{2}^+$. Note that known results derived in the $SU(6,fs)$-symmetry (such as $\frac{\mu_p}{\mu_n}=-\frac{3}{2}$ and $\mu_z(\gamma p\to\Delta^+)=\frac{2\sqrt{2}}{3}\mu_p$) must remain valid in the $SU(4,fs)$-symmetry.

The baryon resonances with negative spatial parity corresponding to the quarks with an orbital moment $L=1$ and mixed symmetry of a q^3-system (such as $D_{13}(1520), S_{11}(1535)$) belong to the $N_S(N_f)\times 1\times 20\times 3$-representation of the $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group. These baryon resonances have got the spin-parity $J^P=\frac{1}{2}^-,\frac{3}{2}^-,\frac{5}{2}^-$.

So we can see that the baryons with low masses belong to the symmetric representation of the $SU(4,fs)$-group for a q^3-system (with $J^P=\frac{1}{2}^+,\frac{3}{2}^+$) and the representation of mixed symmetry (with $J^P=\frac{1}{2}^-,\frac{3}{2}^-,\frac{5}{2}^-$). A question arises: what objects correspond to the antisymmetric representation of the $SU(4,fs)$-group for a q^3-system. The antisymmetric representation of the $SU(4,fs)$-group corresponds to the 4-dimension (for two particles of the $\frac{1}{2}$-spin). The representations of the $SU(4,fs)$-group can be classified with respect to the representations of the $SU(2,f)\times SU(2,s)-$group. For the representations of the $SU(2,s)-$group the designation J^p will be used. In particular, the expansion of the antisymmetric representation of the $SU(4,fs)$-group with respect to the representations of the $SU(2,f)\times SU(2,s)-$group for the q^3-system with quarks in the $1s-$states may be written as $4=2\times\frac{1}{2}^+$.

ON COLORED HADRON-LIKE SISTEMS

It is well known that hadrons are colorless. But at consideration of any q^3-system (or $q\bar{q}$-system) with definite J^p colored states appear together with uncolored states. In a relation with an absence of the color in observed hadrons the questions arises: 1) Why are observed hadrons uncolored? What principles ensure an absence of the color in hadrons? Consider the mass formula corresponding to the approximation of the one-gluon exchange:

$$M=M_0-\alpha_s\sum_{i>j}\mu_{mag}(m_i,m_j)(\vec{\sigma}_i\lambda_i^a)(\vec{\sigma}_j\lambda_j^a)+\alpha_s\sum_{i>j}\mu_{el}(m_i,m_j)\lambda_i^a\lambda_j^a \quad (30)$$

The second term corresponds to the color-magnetic interaction and third term- to the color-electric interaction. The

$M_0 -$ value includes the sum of quark masses and it can include some additional terms (equal for all baryons of the representation). The $\sum_{i>j} \lambda_i^a \lambda_j^a$ -value can be calculated using the Kasimir operator C_2 for the $SU(3,c)$ -group (e.g., Refs. [7], [14]). In the color space a q^3 -system can be a singlet or an octet or a decuplet. The Kasimir operator for the singlet, the octet, and the decuplet equals 0, 3, and 6, respectively. For q^3 -baryons it can be derived

$$\sum_{i>j} \lambda_i^a \lambda_j^a = 2C_2 - \frac{3}{2}\vec{\lambda}^2 = 2C_2 - 8. \tag{31}$$

From (30) and the values of the Kasimir operator it follows that indeed the colored baryon-like states have bigger masses in a comparison with uncolored baryon. Similar result can be derived for $q\bar{q}$ -mesons. Colored baryon-like states with small masses may belong to the representation of the $SU(N_f, g) \otimes SU(3,c) \otimes SU(4,sf) \otimes O(3)$ -group corresponding to the total symmetric representation of the $SU(N_f, g)$ -group (i.e. they can consist of the quarks for k =1), the octet of the $SU(3,c)$ -group, the mixed representation of the $SU(4,sf)$ -group at L =0. Possibly colored $q^3 -$ baryon-like states are not stable, as the color octets can decay into a gluon and a colorless baryon. Similarly the hadrons of color decuplet can decay into two gluons and colorless hadrons. Thus, observed hadrons have got lower masses than masses of corresponding colored hadron-like states. The color is conserved. A conservation of the color is a consequence of a gauge invariance. In collisions of hadrons colored states and uncolored states are produced. In a final state the color must be absent. Colored states must decay rapidly into uncolored hadrons. Therefore, in reality colored hadron-like states are not observed.

ON MASSIVE PARTNERS OF PHOTONS AND GLUONS

From convergence of integrals for generalized wave equation (which is a partial case of generalized Klein-Gordon equation (1) at $m_1 = 0$) it follows that the photon and the gluons have to be massless members of the photonic and the gluonic kinds, respectively. Other members of these kinds must be massive. It is clear that a convergence of integrals for the Green functions of generalized Klein-Gordon equations do not depends on magnitudes of particle masses in the kinds.

At first consider the photonic kind. Denote the members of the photonic kind as γ_k ($\gamma_1 = \gamma$ is the photon, γ_k for *k > 1* are massive partners of the photon). As the processes of the photon interactions with charged leptons are described well in frameworks of the quantum electrodynamics, it of interest to investigate the interactions of massive partners of the photon with quarks. The massive partners of the photon have to be coupled with quarks and antiquarks in the state of electrical neutral uncolored vector mesons with negative spatial and charge parities. Now a lot of such mesons are known. Next questions arise in relation with massive partners of the photon: 1) Are the massive partners known or unknown particles? 2) What properties must have the massive partners of the photon, which allow to distinct them from other particles? Consider requirements for the massive partners of the photon.

1. The members of the photonic kind can be emitted and absorbed by charged leptons, similarly to the photon. The amplitudes for interactions of massive partners of photon with quarks and leptons are determined by the electric charge of a quark or a lepton. Therefore, the coupling constants of some γ_k with all the *U*-quarks are the same, and similarly for *D*-quarks. The ratio of the coupling constants for the interactions of γ_k with the *U*-quarks and the *D*-quarks equals – *2*.

2. Some γ_k can have got enough big masses and can decay into mesons. The massive partners of the photon can be produced in the $e^+e^- -$ interaction and can be observed as resonances. Each virtual massive partners of the photon can interact with all the *U*-quarks and the *D*-quarks. But the massive partners of the photon decay into such final states that their total energies do not exceed the energies of these γ_k . Therefore, the mesons including heavy quarks cannot be observed in decay products of some γ_k with low masses.

The massive partners of the photon decay into mesons through a production of quark-antiquark pair. As quark have got some color an additional quark-antiquark pair must appear in a final state. These additional quark-antiquark pairs make complicated an analysis of decays of the massive partners of the photon. Therefore, it is of importance the consideration of relations of the massive partners of the photon with leptons.

It is known that the $\rho^0(770)-$, the $\omega^0(783)-$, the $\varphi^0(1020)-$, the $J/\Psi(3097)-$, the $\Upsilon(9460)-$ mesons, and other similar vector mesons consisting of a quark and corresponding antiquark are excited in the $e^+e^- \to hadrons$ and the $e^+e^- \to e^+e^-$. Therefore it may be assumed that these vector mesons are related to the massive partners of the photon. Then the quantity of the massive partners of the photon equals $2N_f$ (N_f partners related to the uncolored $U_k\bar{U}_k$-pairs and N_f partners -to $D_k\bar{D}_k$-pairs, *k=1, 2, …* N_f). This assumption on the massive partners of the photon is related to the model of vector dominance. Indeed it can be shown that some formulae for reaction amplitudes coincide in these two approaches. Therefore, we can hope that this assumption on the massive partners of the photon will not lead to new contradictions with experimental data. Note that at this assumption the masses of the massive partners of the photon can be calculated in quark models for uncolored systems.

Now consider the massive partners of the gluons. Similarly to the massive partners of the photon it can be assumed that the massive partners of the gluons are electrically neutral vector states of $q\bar{q}-$systems with negative spatial parity, which are the color octet. The masses of the massive partners of the gluons can be calculated in the same quark models similarly to the massive partners of the photon. Indeed, the massive partners of the photon can be presented as uncolored $q\bar{q}-$systems and the massive partners of the gluons as colored $q\bar{q}-$systems. Then it can be expected that the masses of the massive partners of the gluons will exceed the masses of the massive partners of the photon for each number of generation. Note that the mass formula (30) must be modified for $q\bar{q}-$systems. The $\sum_{i>j}\vec{\lambda}_i\vec{\lambda}_j$-value in (30) must be changed by the $\vec{\lambda}_1\vec{\lambda}_2$-value, which is equal to *-16/3* for the massive partners of the photon (color singlet) and *2/3* for the massive partners of the gluons (color octet).

CONCLUSION

In present paper it is proposed to classify all the quarks by means two numbers: the electric charge and the generation number. The quarks with the $\frac{2}{3}Q_p$-charge belong to the $U-$quarks and the quarks with the $-\frac{1}{3}Q_p$-charge – to the $D-$quarks. Thus, it is proposed that the quarks belong to two flavors: *up* and *down*. The flavors of all the quarks are described by the $SU(2,f)$-group. Then the states of all the quarks in the spin space and in the flavor space are described by the $SU(4,fs)$-group. The conservations of the strangeness, the charm, the beauty, the truth, and other similar possible numbers in the strong and the electromagnetic interactions are the consequences of conservations of the electric charge and the number of generations.

In Ref. [4] it is shown that minimal quantity of the quark generations equals six (i.e., $N_f \geq 6$). Note that now it is known a half of necessary quantity of quarks. Therefore, it may be proposed that the quarks and hadrons can be classified as representations of the $U(N_f,g)\otimes SU(3,c)\otimes SU(4,sf)\otimes O(3)$-group. The representations of the $U(N_f,g)$-group for q^3-systems have got different symmetry properties. According to Pauli principle the representations of the $U(N_f,g)\otimes SU(3,c)\otimes SU(4,sf)\otimes O(3)$-group for q^3-systems must be antisymmetric as well as the representations of known $SU(3,c)\otimes SU(6,sf)\otimes O(3)$-group. Taking into account of the symmetry properties for the representations of the $U(N_f,g)$-group in the $U(N_f,g)\otimes SU(3,c)\otimes SU(4,sf)\otimes O(3)$-group for q^3-systems allow derive the states, which are impossible in the $SU(3,c)\otimes SU(6,sf)\otimes O(3)$-group for u,d,s-quarks (or in the $SU(3,c)\otimes SU(8,sf)\otimes O(3)$-group for u,d,s,c-quarks).

The Lagrangians (18) give the generalized Dirac equations (5) as consequence of the least action principle. These Lagrangians depend on the particle masses by means of the elementary symmetric functions (9, 10). Therefore, the Lagrangians (18, 19) are invariant under the generation permutations.

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