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## LIGHT ABERRATION IN OPTICAL ANISOTROPIC SINGLE-AXIS MEDIUM

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The entrainment of the light flux by a uniaxial anisotropic medium and its influence on the measurement of stellar aberration are analyzed. The influence of the entrainment of the light flux by an isotropic medium on the measurement of stellar aberration was considered by Fresnel early. The absence of such influence was confirmed by Erie's experience when filling the telescope tube with water. The formula itself was perfectly confirmed by Fizeau's experiments with moving water and the repetition of this experiment with an increase in the accuracy of measurements by Michelson, Zeeman, and others. G.A. Lorentz already on the basis of the electromagnetic theory specified the formula with allowance for the frequency dispersion of the light flux. A. Einstein made an analysis of the schemes of experiments for determining the drag coefficient, covering all possible variants of similar experiments. As a result, he obtained Fresnel and Lorentz formulas, taking into account the frequency dispersion of light, starting from the theory of relativity. The entrainment of light and its influence on the measurement of stellar aberration by a uniaxial anisotropic medium have not been considered anywhere. An analysis of such influence is carried out. The results of the analysis indicate the possibility of measuring the current value of stellar aberration using a uniaxial anisotropic medium. The concept of active light aberration is introduced. The proposed schemes of experiments of using the entrainment of a light flux by an anisotropic substance for measuring the current value of stellar aberration are investigated. It is concluded that it is possible to study the determination of the current velocity of an inertial system relative to the light flux.

**KEYWORDS:** light aberration, entrainment of the light flux by the medium, refractive index, optically anisotropic medium, telescope, observer speed, light speed

### АБЕРАЦІЯ СВІТЛА В ОПТИЧНОМУ АНІЗОТРОПНОМУ ОДНОВІСНОМУ СЕРЕДОВИЩІ

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Аналізується захоплення потоку світла одновісним анізотропним середовищем та його вплив на виміри зіркової аберрації. Вплив захоплення потоку світла оптично ізотропним середовищем на виміри зіркової аберрації було розглянуто ще Френелем. Відсутність такого впливу підтверджено дослідом Ері при наповненні труби телескопу водою. Сама формула захоплення була яскраво підтверджена дослідами Фізо з водою яка рухається вздовж потоку світла. Подібні досліди неодноразово повторювалися з підвищенням точності вимірювань Майкельсоном, Зесманом та іншими. Г.А. Лоренц вже на основі електромагнітної теорії уточнив формулу Френеля з урахуванням частотної дисперсії світла. А. Ейнштейн виконав аналіз схем дослідів визначення коефіцієнту захоплення, охоплюючих всі можливі варіанти подібних дослідів. В підсумку він отримав формули Френеля та Лоренца з урахуванням частотної дисперсії світла виходячи з теорії відносності. Захоплення потоку світла та його вплив на виміри зіркової аберрації одновісним анізотропним середовищем з одною оптичною віссю ще ніде не розглядалося. Виконано аналіз такого впливу. Результати аналізу вказують на можливість виміру поточного значення зіркової аберрації з використанням одновісного оптично анізотропного середовища. Введено поняття активної аберрації світла. Досліджені запропоновані схеми дослідів застосування захоплення потоку світла одновісним анізотропним середовищем для виміру поточного значення зіркової аберрації. Зроблено висновок про можливість дослідження визначення поточної швидкості інерціальній системи відносно потоку світла.

**КЛЮЧОВІ СЛОВА:** аберрація світла, захоплення потоку світла середовищем, показник заломлення, оптично анізотропне середовище, телескоп, швидкість світла

### АБЕРРАЦИЯ СВЕТА В ОПТИЧЕСКОЙ АНИЗОТРОПНОЙ ОДНООСНОЙ СРЕДЕ

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Анализируется увеличение светового потока одноосной анизотропной средой и его влияние на измерение звездной aberrации. Влияние увеличения светового потока изотропной средой на измерение звездной aberrации было рассмотрено еще Френелем. Отсутствие такого влияния было подтверждено опытом Эри при заполнении трубы телескопа водой. Сама формула была прекрасно подтверждена опытами Физо с движущейся водой и повторением этого опыта с повышением точности измерений Майкельсоном, Зесманом и другими. Г.А. Лоренц уже на основании электромагнитной теории уточнил формулу с учетом частотной дисперсии светового потока. А. Эйнштейн сделал анализ схем опытов определения коэффициента увеличения, охватывающих все возможные варианты подобных опытов. В результате он получил формулы Френеля и Лоренца с учетом частотной дисперсии света исходя из теории относительности. Увеличение света и его влияние на измерение звездной aberrации одноосной анизотропной средой еще нигде не рассматривалось. Проведен анализ такого влияния. Результаты анализа указывают на возможность измерения текущего значения звездной aberrации при использовании одноосной анизотропной среды. Введено понятие активной aberrации света. Исследованы предложенные схемы опытов использования увеличения светового потока анизотропным веществом для измерения текущего значения

звездной aberrации. Сделан вывод о возможности исследования определения текущей скорости инерциальной системы относительно светового потока.

**КЛЮЧЕВЫЕ СЛОВА:** aberrация света, увлечение светового потока средой, показатель преломления, оптически анизотропная среда, телескоп, скорость наблюдателя, скорость света

Influence of the light flux carrying away by anisotropic medium on star aberration measurement was considered already by Fresnel. At such measurement light carrying away by anisotropic medium was not taken into account.

Let us analyze the influence of light carrying away by single-axis anisotropic medium on measurement of star aberration.

In 1818, Fresnel proposed theory of partial light flux carrying away by a moving substance having carrying away coefficient

$$\chi = 1 - \frac{1}{n^2}. \quad (1)$$

Based on the simplest mechanical model on full carrying away of the part by bodies (the part which forms its over-density as compared with the surrounding ether) he obtained correct formula [1, 2] confirmed by experiments. This formula was excellently proved by Fizeau experiment (1851) with moving water, repeated by Michelson (1886) and Zeeman (1914) with increase of measurement accuracy.

Base done lecturer magnetic theory G.A. Lorentz (1886) corrected the formula with the account of light flux frequency dispersion [3]. Zeeman experiments with moving rods confirmed the existence of Lorentz dispersion term.

A. Einstein highly appreciated the significance of Fizeau experiment.

«The answer to this problem is given by highly important experiment made more than half a century ago by brilliant physicist Fizeau...».

«Although it should be noted that long before appearance of relativity theory G.A. Lorentz explained the theory of this phenomenon and justified it by purely electrodynamics method by means of certain hypothesis about electromagnetic matter structure.

However this does not diminish the evidential force of Fizeau experiment has experimentum crucis in favors of relativity theory, since Maxwell-Lorentz electrodynamics which was used as original theory do not contradict the relativity theory». «Fizeau experiment is fundamental also for the special relativity theory» [4].

A. Einstein made analysis based on relativity theory of three scheme variants of experiments on determination of carrying away coefficient embracing all possible variants of similar experiments. As a result, he obtained Fresnel and Lorentz formulae with the account of light frequency dispersion. This once again proved correctness of here relativity theory [4].

W.E. Frankfurt, A.M. Frank [5] note that «in the relativity theory results of these experiments are explained simply as consequence of relativity velocity addition formula

$$c' = \frac{\frac{c}{n} \pm V}{1 \pm \frac{V}{nc}}. \quad (2)$$

Taking only terms of the first order, we obtain

$$c' = \frac{c}{n} \pm \left( 1 - \frac{1}{n^2} \right) V, \quad (3)$$

(where  $c'$  is the speed of light with respect to the fixed installation and the observer).

Although the formula has the same appearance here “partial carrying away” is the result of pure metric properties and is not connected with any assumption about substance structure or ether properties» [5].

A. Sommerfeld [6] investigated aberration and crystal optics based on Lorentz transformations however all these investigations and experiments related to longitudinal, as to direction of propagation, light flux carrying away but transverse carrying away was not considered.

At the same time (1818), Fresnel investigated the influence of such transverse carrying away on star aberration measuring. He considered the experiment with filling a telescope tube with water and made the conclusion about absence of influence of such filling on the value of star aberration. «Although this experiment was not yet made, but I have no doubt, that it will support this conclusion ...» he wrote to Arago in 1818.

Eri made such an experiment in 1871, which confirmed permanency of star aberration angle. The analysis of such filling in was made in detail in [7].

In 1977 H. Bilger and W. Stawell conducted the experiment, wherein light propagated in the rotating optical disc

of the annular laser interferometer. [8]

Anisotropy of the electromagnetic radiation velocity space under transversal light entrainment in the rotating optical disc was investigated by V.O. Gladyshev, P.S. Tiunov, A.D. Leontiev, T.M. Gladysheva, E.A. Sharandin. By anisotropy in this case we mean the dependence of the propagation velocity of light in the optical medium on the velocity and direction of motion of the medium [9]

All this investigations are related to the light propagation in the isotropic mediums.

The main purpose of this work is to study the effect of the partial entrainment of a light flux by an optically anisotropic uniaxial substance on the measurement of stellar aberration.

### THE TRANSVERSE ENTRAINMENT OF THE LIGHT FLUX BY AN ANISOTROPIC SUBSTANCE

Examine consider influence of the partial light flux carrying away by the anisotropic substance on the star aberration with different refraction indices along telescope axis and in transverse direction for an extraordinary beam.

Assume a star  $S$  be observed through an unfilled telescope (Fig. 1).

Due to star aberration because of telescope motion at speed  $V$ , its axis is directed towards  $O_1S_0$  at the angle  $\alpha_0 = \angle S - O_1 - S_0 = \angle O_2 - O_1 - B_1$  to beam  $S - O_1$  of the star  $S$ . Here  $S_0$  is aster position being watched,  $S$  – true star position. At small angles  $\alpha_0$   $\alpha_0 = \frac{V}{c} \sin \psi$ , where  $c$  is light velocity in vacuum,  $V$  – observer's speed,  $\psi$  – angle between direction  $O_1 - S$  onto star  $S$  and vector of observer's speed  $V$ .

Let us fill the telescope with optically anisotropic substance with refraction indices of an ordinary light  $n_o$  along axis  $O_1O_2$  and radius  $O_2B$  of the telescope tube, extraordinary beam  $n_e = n_o$  along  $O_1O_2$  and  $n_e \neq n_o$  along telescope tube radius  $O_2B$  in the direction of velocity  $V$   $\Delta n = n_o - n_e \neq 0$  (Fig. 2).

Now the light flux  $SO$  of the star  $S$  will get refracted on the medium border  $MN$ . Refraction angle of an ordinary light beam will be  $r_o = \frac{\alpha}{n_o}$ .

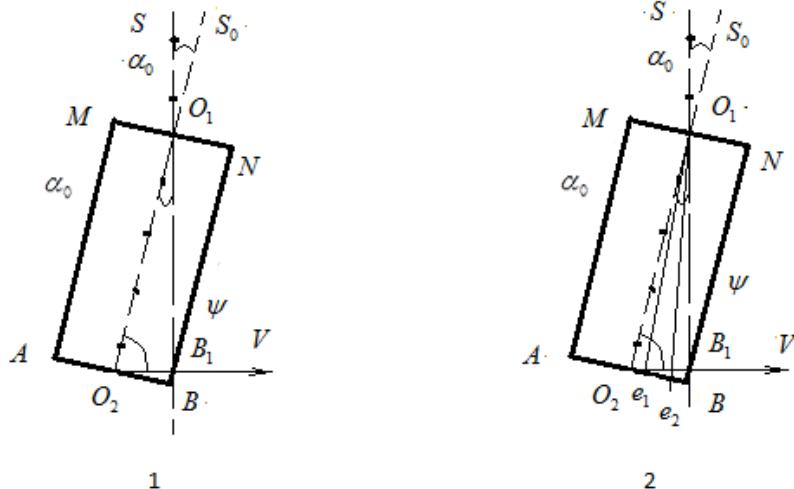


Fig. 1  
Unfilled telescope

Fig. 2  
Telescope filled with anisotropic medium

If to align the exit of an ordinary beam with the telescope cross sight reticule, then as shown by Fresnel and confirmed by Eri experiment, the aberration angle will remain as previous –  $\alpha_0$ .

Really, if light flux carrying away by the medium with refraction index  $n_0$  of the telescope moving at the speed  $V$  is not taken into account, then considering light flux  $\bar{c}$  speeding down along axis  $O_1O_2$   $\bar{c} = \frac{c}{n_o}$  the telescope

should be inclined at the angle  $\alpha$   $\tan \alpha = \frac{O_2B}{O_1O_2} \sin \psi$  ( $\psi$  is an angle between the telescope axis and the direction of its motion speed  $V$ ). Since  $O_1O_2 = t\bar{c}$ ,  $O_2B = tV$  and  $\alpha$  is small, then

$$\alpha = n_o \frac{V}{c} \sin \psi = n_o \alpha_0, \quad (4)$$

where  $t$  is time of the light flow passing along the telescope axis  $O_1O_2$ ,  $\bar{c}$  – being light velocity in the medium.

On the other hand, since refraction angle  $r_o = \frac{\alpha}{n_o}$  and we align the exit of the ordinary light beam with the cross sight reticule then the new aberration angle  $\alpha$  could be

$$\alpha = n_o^2 \alpha_0. \quad (5)$$

However, the light flux inside the telescope tube together with the medium is carried away at the speed  $V_u$  in the direction of speed  $V$  with coefficient  $(1 - \frac{1}{n_1^2})$ .  $V_u = V(1 - \frac{1}{n_1^2})$ , where  $n_1$  is the refraction index in the direction of speed  $V$ . This is equivalent to the decrease of speed  $V$  by the value  $V$  with coefficient  $(1 - \frac{1}{n_1^2})$  [7,8].

Speed  $V_n$ , with the account of carrying away will become  $V_n = V - V(1 - \frac{1}{n_1^2}) = V \frac{1}{n_1^2}$ . Substituting  $V_n = V \frac{1}{n_1^2}$  in  $\alpha_0$  we will obtain the value of the new aberration angle  $\alpha$ .

$$\alpha = n_o^2 \frac{V_n}{c} \sin \psi = \frac{n_o^2}{n_1^2} \frac{V}{c} \sin \psi, \quad (6)$$

$$\alpha = \frac{n_o^2}{n_1^2} \alpha_0. \quad (7)$$

When filling in with isotropic l substance  $n_o = n_1$  and  $\alpha = \alpha_0$ . For a no ordinary light beam also  $n_o = n_1$ , that is why  $\alpha = \alpha_0$  is for it too.

For a next, no ordinary light beam  $n_e \neq n_o$ , so if to align the exit of the extraordinary light beam with the cross sight reticule, then the aberration angle will be different:

$$\alpha = \alpha_e = \frac{n_o^2}{n_e^2} \alpha_0. \quad (8)$$

Let us denote  $k = \frac{n_o^2}{n_e^2}$  and  $\alpha = k\alpha_0$  then the aberration angle for an extraordinary light beam will be

$$\alpha_e = k \frac{V}{c} \sin \psi;$$

$$\alpha_e = k\alpha_0. \quad (9)$$

For is anisotropic substances  $k = 1$  and  $\alpha = \alpha_0$ , but for anisotropic substances  $k \neq 1$  and  $\alpha_e$  differs from  $\alpha_0$ .

#### STUDY OF THE SCHEMES OF EXPERIMENTS USING AN ANISOTROPIC SUBSTANCE TO MEASURE THE CURRENT ABERRATION VALUE

Let us consider possible variants of using this difference for determination of the star aberration angle. Then for the sake of simplicity we will regard the angle  $\psi$  being equal  $90^\circ$ .

1. Let us measure the star position  $\gamma_1$  by means of the “empty” telescope (fig.1)  $\gamma_1 = \gamma_3 + \alpha_0$  (where  $\gamma_3$  being

real star coordinate) and by means of  $\gamma_2$  telescope filled with anisotropic ( $n_e \neq n_o$ ) substance with known  $k$  (Fig.2), by the extraordinary light beam (aligning the exit of the extraordinary light beam  $e_2$  with the telescope cross sight reticule)  $\gamma_2 = \gamma_3 + \alpha_e$ . The optical axis of the anisotropic substance is parallel to the telescope axis  $Oo$ . Determine the difference  $\gamma_2 - \gamma_1 = \alpha_e - \alpha_0$ ; substituting  $\alpha_e = k\alpha_0$  we obtain

$$\gamma_2 - \gamma_1 = (k - 1)\alpha_0. \quad (10)$$

Hence we will determine the aberration angle  $\alpha_0$  of the star S

$$\alpha_0 = \frac{\gamma_2 - \gamma_1}{k - 1}. \quad (11)$$

2. Let us measure the star position by means of the telescope, filled with anisotropic substance ( $n_e \neq n_o$ ) with the known  $k$  (Fig. 2) by the ordinary light beam (with the ordinary light beam exit  $e_1$  being in the cross sight reticule)  $\gamma_4$  and by the extraordinary light beam (with the extraordinary light beam exit  $e_2$  being in the cross sight reticule)  $\gamma_5$ . Optical axis of the anisotropic substance is parallel to the telescope axis. Determine their difference  $\gamma_5 - \gamma_4$ . By analogy we obtain

$$\alpha_0 = \frac{\gamma_5 - \gamma_4}{k - 1}. \quad (12)$$

3. Let us install a single-axis doubly refracting crystal 1, with crystal faces being perpendicular to its optical axis, so that optical axis of the crystal was parallel to the telescopic axis 4 (Fig. 3). We'll observe the star through the telescope aligning the star image by means of the ordinary light beam with telescope cross sight reticule 6. The star light flux will fall on the crystal face at the aberration angle  $\alpha_0$ . In the crystal the light flux will divide into ordinary and extraordinary flows having different direction of the linear polarization. Due to different refract ion indices  $n_e \neq n_o$ , they will get refracted at different refraction angles.

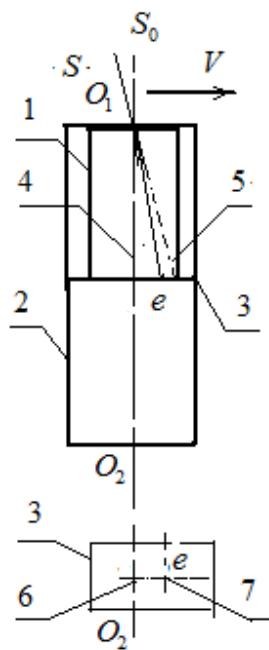


Fig.3  
Telescope with single-axis crystal

The ordinary light beam refraction angle will be  $r_o = \frac{\alpha_0}{n_o}$ . For the extraordinary light beam refraction angler of

the single-axis doubly refracting crystal in the direction of the optical axis is equal to the ordinary light beam refraction index no and changes depending on the extraordinary light beam direction angle to the optical axis  $0 \rightarrow \frac{\pi}{2}$  within

$$n_o \rightarrow n_e.$$

Thus, the extraordinary light beam  $r_e$  fraction angle depends upon the angle of incidence  $\alpha$  of the falling light beam. At the same time speed  $V_n$  with the account of the extraordinary light beam carrying away also depends on  $n_e^2$   $V_n = \frac{1}{n_e^2} V$ , so  $r_e$  depends upon the direction with respect to the optical axis and upon the value of the crystal motion speed (Fig. 1,2).

$$r_e = f_1(\alpha) \text{ and } r_e = f_1(V). \quad (13)$$

For a crystal with ace perpendicular to the optical axis the refraction index of the ordinary light beam can be  $r_e = r_o$  demonstrate das asp here with radius  $n_o$ , and the refraction index of the extraordinary light beam as a revolution ellipsoid with the revolution axis  $n_o = n_e$  along optical axis and with axis radius  $n_e$  in the face plane. The incidence anglers small (aberration angle  $\leq 200''$ ), so small is the refraction angle and the difference between the extraordinary light beam refraction index and the ordinary light beam refraction indexes small so it is possible to assume that

$$n_e(r_e) = n_e(\alpha) = n_o. \quad (14)$$

The main influence on the extraordinary light beam refraction angle at small  $r_e$  will be exerted by the extraordinary light beam carrying away during crystal motion perpendicular to the optical axis in the angle  $r_e$  plane where the difference  $n_o - n_e$  is maximum, i.e., the difference between angles of propagation of the ordinary and extraordinary light beams will be determined in this case by the lateral speed of the telescope motion (Fig.3).

If to take into account  $(n_e(\alpha) = n_o)$  and minuteness of angles  $r_e, r_o$  (Fig.3), then

$$r_e - r_o = \frac{O_2 e}{O_1 O_2}; O_2 e = t V_n; V_n = V \frac{1}{n_e^2(\alpha)}; O_2 e = t V \frac{1}{n_e^2(\alpha)}; O_2 O_1 = t \bar{c}; r_e - r_o = \frac{V}{\bar{c}} \frac{1}{n_e^2(\alpha)}; \bar{c} = \frac{c}{n_o};$$

$$\Delta_1 = \frac{n_o}{n_e^2} \alpha_0, \quad (15)$$

where  $\bar{c}$  – is the speed of the ordinary light beam in the crystal,  $t$  – time of passing of the ordinary light beam along the crystal axis.

On the other hand  $\Delta_1 = \frac{O_2 e}{O_1 O_2}$  and

$$\alpha_0 = \frac{n_e^2}{n_o} \frac{O_2 e}{O_1 O_2}. \quad (16)$$

Knowing  $O_1 O_2$ ,  $n_o$ ,  $n_e$  and having measured  $O_2 e$ , aberration angle  $\alpha_0$  can be determined.

Thus, observing the star through the telescope with a single-axis doubly refracting crystal, align the star image by means of ordinary light beams with the cross sight reticule and measure the distance  $O_2 e$  between star images by the ordinary and extraordinary light beams. Knowing  $n_o$ ,  $n_e$  and crystal length  $O_1 O_2$  we determine  $\alpha_0$ , without changing the direction of the observer's motion. The aberration direction is determined by the direction of the star image shifting by extraordinary light beams in the focal plane of the telescope.

Lorentze transformation defined movement effect (second-order quantities  $\Delta t$ ,  $\Delta r$  proportional to  $\frac{V^2}{c^2}$ ) may be

ignored if measured quantities are proportional to  $\frac{V}{c}$ .

In all variants under consideration the current value of the star aberration can be determined without changing the direction of the observer's motion.

When observing star through the unfilled telescope the light flow does not change its direction when entering the telescope tube. To hold the star image in the cross hairs of the focal plane it has to be deflected by the aberration angle towards the telescope motion relative to the light flux of the star being observed.

In the telescope filled with isotropic medium the light flux gets refracted at the exit in to the telescope, but because of the transverse carrying away of the medium moving which is moving together with the telescope the refraction angle gets compensated, the light flux does not change the direction of its propagation and the telescope has to be deflected by the same aberration angle.

In both cases, the light flux does not change the propagation direction, and the telescope passively gets adjusted to its motion.

In the telescope filled with anisotropic medium with optical axis parallel to the telescope axis the extraordinary light beam changes its propagation direction as compared with the direction of the entry star light flux and that of the ordinary light beam in the telescope.

Active change of the extraordinary light beam direction is caused by different refraction indices of the medium along its optical axis, coinciding with the telescope axis and the direction of its motion together with the telescope.

This active aberration differs from passive aberration by the value and coincides by the direction.

Among abundance of stars one can find stars having at the given moment speed equal to observer's speed.

Aberration of their light flow for the observer is of no importance, which proves independence of the light velocity from the speed of its source. At the same time, the star and the observer can be considered as an inertial system with a source. Measuring active and passive aberration, we measure by their difference the system speed without changing the direction of the observer's motion. This gives an opportunity to investigate practical realization of devices for measuring speed of inertial systems with respect to the light flux.

## CONCLUSIONS

1. When observing through the telescope filled with optically anisotropic medium by means of extraordinary light beams the aberration angle differs from the aberration angle when observing through then filled telescope or telescope filled with isotropic medium.
2. Different proportion of transverse carrying away speed of the extraordinary light beam flux and its longitudinal speed in the moving anisotropic medium, as compared with their proportions in the isotropic medium, cause difference in aberration angles in these media
3. Since anisotropic medium refraction index depends on the direction of the light flux propagation, this direction changes at the transverse motion of the anisotropic medium. The value of this change depends on the direction and speed of the medium motion.
4. In contrast to star aberration of the unfilled telescope, where there is no change of the light flow direction but passive adjustment of the moving telescoped expending on its speed takes place, when filling it with anisotropic medium the direction of the light flow propagation of the extraordinary light beams is actively changing.
5. The difference of passive and active aberration angles gives an opportunity to measure current values of star aberrations of various types (daily, yearly, century-old) and hence of various components of the telescope speed.
6. Measure mentor various speed components is possible without changing the direction of the telescope motion.
7. Active aberration gives an opportunity to investigate measurement of the transverse speed of the inertial system with respect to the light flux without change of direction of the observer's motion.

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