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SIMULATION OF PARTICLE MOTION IN STILL A ROUGH SURFACE AND ITS TESTING FOR EXAMPLE INCLINED PLANE

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Annotation. *A analytical, algorithmic and software simulation of a particle on any stationary rough surface as a function of time, the position of the particles on the surface and the direction of its movement. Algorithm worked out by the example of the inclined plane.*

Key words: ***material point, trihedron, inclined plane, rough surface modeling***

In many industrial processes there is motion of material on rough working surfaces of complex shape, particularly the movement of granular materials through pipelines, supply of fertilizers scattering to drive grain separation inclined vibrational planes. Understanding the patterns of movement of particles (as of a point) on the rough surface of an arbitrary position in three dimensions allows purposefully to calculate structural and kinematic parameters of working bodies.

Analytical output law of motion of a particle on the rough surface is reduced to drawing up a system of differential equations of 2nd order desired dependency which is the trajectory of the particle, its velocity, acceleration, length of the path, the force of normal reaction, the move to its stop and other trajectory-kinematic characteristics . The sequence of analytical output of differential equations and ways of its solution is quite time consuming. Over the past decade (during the emergence and development of computer technology) significant changes in research methods formalization of a particle on rough surfaces of complex shape have occurred. In the current study every scientist personally performs differently cumbersome analytic transformation of the formation law of a particle, the complexity of which essentially depends on the shape of the surface. So for surfaces, which were studied motion of a particle is limited by a plane of rotation of the cylinder, cone rotation, screw cone, helicoid.

The purpose of research – to develop computer models of a particle on a fixed plane rough arbitrary position.

Materials and methods of research. Based algorithm for automatic generation of a particle in law random surface *R (u, v)* in media computer mathematics charged trihedron trajectory. It is advisable to use the two accompanying trihedrons (Fig. 1): a) *OuvN* – ort *Ou* and *Ov* is adjacent to *u, v* - coordinate lines surface: b) trihedron Darboux *OTPN* – ort *OT* is a tangent to the trajectory *r* particle sand ort sand *OP* is normal to the trajectory *r* in tangent plane *μ (OuvN≡OTPN)* surface *R (u, v). ON* ort normal to the surface is common to both cover trihedrons, because in every moment of elementary particle carries a tangent plane displacement surface *μ R (u, v).*

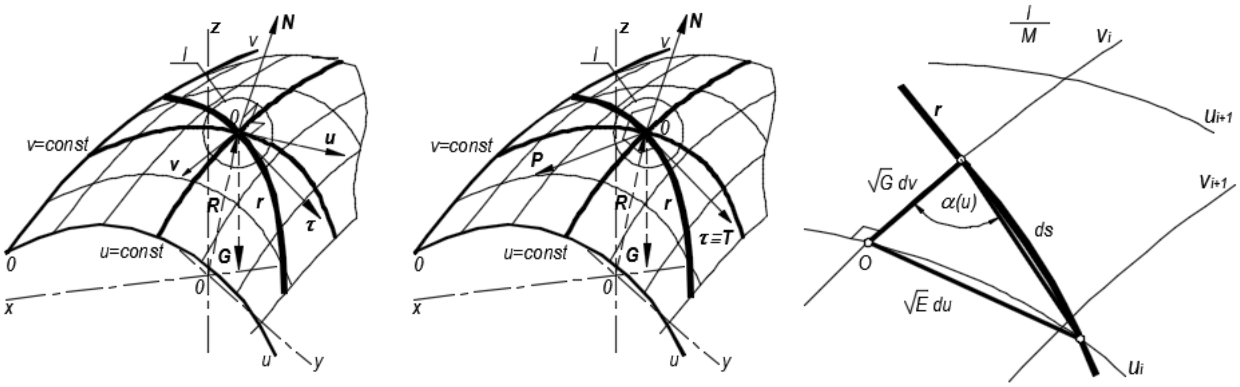


Fig. 1. Cover trihedrons on the surface of the particle trajectory:
a) *OuvN;* b) *OTPN;* c) coordinate differentials surface lines

Results. Any curvilinear trajectory *r* particles on an arbitrary surface *R (u, v)* can be expressed in analytical dependence *f (u, v) = 0*, which connects the inner *u* and *v* coordinates curved surface *R (u, v)* as a function of a given independent parameter (argument). If this parameter is the time *t*, then the desired internal paths are depending on the type: *u = u (t), v = v (t).* For independent option will also take curvilinear coordinate *u* or *v* - curvilinear coordinate, where the trajectory of particles in the inner *u, v* – surface coordinates *R (u, v)* respectively have the form $u = u(v), v = v$ and $u = u, v = v(u)$. For surface *R (u, v)* of orthogonal *u, v* - coordinate lines related to differentials (Fig. 2) the dependence determined $\frac{\sqrt{G}\,dv}{\sqrt{E}\,du} = ctg(\alpha(u))$, where *E, G* - 1st coefficients quadratic form surface *R (u, v)*, attributed to the orthogonal coordinate grid lines. This allows you to create a trajectory $\boldsymbol{r}(\alpha(u))$ particles on the surface of *R (u, v)* in function of independent variable $\alpha(u)$ - the angle between the tangent $\boldsymbol{\tau}$ trajectory and tangent to the *u* - coordinate lines. Then in the mid *u, v* - coordinates of the trajectory of the particles will look like: *u* = *u*, , $v = \int \frac{\sqrt{G}}{\sqrt{E}} ctg(\alpha(u))du$. If for surface *R (u, v)* with *u, v* - orthogonal coordinate lines related $\sqrt{E}\,du$ differentials and differential to $\sqrt{G}\,dv$ to *ds* arc trajectory is $\frac{\sqrt{E}\,du}{ds} = sin(\alpha(u))$, $\frac{\sqrt{G}\,dv}{ds} = cos(\alpha(u))$, then the trajectory *r (s)* particles as a function of the independent variable s in the inner *u, v* - coordinates are $u(s) = \int \frac{sin(\alpha(u))}{\sqrt{E}} ds$, $v(s) = \int \frac{cos(\alpha(u))}{\sqrt{G}} ds$.

Each of the above approaches formation trajectory *r* particles in the inner *u, v* - curved surface coordinates *R(u, v)* has advantages at the expense of independent control parameter - $t,\ u,\ \alpha,\ s$. For example, the equation trajectory *r (u)* particles as a function of the independent parameter *u*, which determines the position of *v*-coordinate line the surface of the particle on it, you can explore its kinematic properties on a limited part of the $u = [u_o, u_n]$ and $v = [v_o, v_n]$ surface *R (u, v)*, where $[u_o, v_o]$ and $[u_n, v_n]$ - inside the coordinates of the initial and final provisions of the particles on the surface. Formation trajectories $\boldsymbol{r}(\alpha)$ as a function of angle $\alpha(u)$between the vector tangent $\boldsymbol{\tau}$ trajectory and *u*-surface lines coordinate *R(u, v)* allowing the study of a particle in the range of $\alpha = [\alpha_o, \alpha_n]$, where α_o and α_n initial and final viewing direction moving particles. For example, when

$\alpha_o = 0$ particle begins its movement along the *v*-coordinate line, while $\alpha_o = \pi/2$- *u* -coordinate along the lines of the surface *R (u, v)*.

For moving particles on the surface of *R(u, v)* on ort normal vector *N* defined length F_N, the amount of force that characterizes normal reaction:

$$ON := F_N = F_g\cos(\widehat{\boldsymbol{N},\boldsymbol{G}}) \pm F_c\cos(\widehat{\boldsymbol{N},\boldsymbol{n}}). \quad (1)$$

The "+" or "-" characterized by direction vector directed centrifugal force $F_c\cos(\widehat{\boldsymbol{n},\boldsymbol{N}})$ with respect to the surface normal vector N - pressed to the surface of a piece or separates from it. Negative values F_N expression (1) means lead particles from the surface.

The vector equation (1) of a particle on time t in projections for orts $\boldsymbol{u} \equiv \boldsymbol{R}'_u$ and $\boldsymbol{v} \equiv \boldsymbol{R}'_v$ trihedron *OuvN* will look like:

$$\begin{cases} Ou := mW\cos(\widehat{\boldsymbol{R}'_u,\boldsymbol{w}}) = F_g\cos(\widehat{\boldsymbol{R}'_u,\boldsymbol{G}}) - (fF_N + qF_V)\cos(\widehat{\boldsymbol{R}'_u,\boldsymbol{\tau}}) \\ Ov := mW\cos(\widehat{\boldsymbol{R}'_v,\boldsymbol{w}}) = F_g\cos(\widehat{\boldsymbol{R}'_v,\boldsymbol{G}}) - (fF_N + qF_V)\cos(\widehat{\boldsymbol{R}'_v,\boldsymbol{\tau}}) \end{cases}, \quad (2)$$

and the projections for T and P ort trihedron Darboux *OTPN*:

$$\begin{cases} OT := mW_\tau = F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{\tau}}) - f(F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{N}}) \pm F_C\cos(\widehat{\boldsymbol{n},\boldsymbol{N}})) - qF_V \\ OP := mW_n\sin(\widehat{\boldsymbol{n},\boldsymbol{N}}) = F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{P}}) \end{cases}, \quad (3)$$

where W = | W | - the value of acceleration of particles; $W_\tau = \frac{d}{dt}V(t)$, $W_\tau = V\frac{d}{ds}V(s)iW_\tau = V\frac{d}{d\alpha}V(\alpha)k$ - tangential acceleration respectively independent parameters function of time t, the trajectory and arc $s\alpha$angle to the movement of particles; $W_n = V^2k$- normal acceleration.

Given the expressions W_τ tangential acceleration expression (3) of the movement of the particles can be rewritten as a function of arc length s trajectory:

$$\begin{aligned} OT &:= mV\frac{d}{ds}V(s) = F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{\tau}}) - f\left(F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{N}}) \pm F_C\cos(\widehat{\boldsymbol{n},\boldsymbol{N}})\right) - qF_V \\ OP &:= F_C\sin(\widehat{\boldsymbol{n},\boldsymbol{N}}) = F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{P}}) \end{aligned}, \quad (4)$$

and a function of the angle α direction of the particles:

$$\begin{aligned} OT &:= mV\frac{d}{d\alpha}V(\alpha)k = F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{\tau}}) - f\left(F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{N}}) \pm F_C\cos(\widehat{\boldsymbol{n},\boldsymbol{N}})\right) - qF_V \\ OP &:= F_C\sin(\widehat{\boldsymbol{n},\boldsymbol{N}}) = F_g\cos(\widehat{\boldsymbol{G},\boldsymbol{P}}) \end{aligned}. \quad (5)$$

The choice of reference *OuvN* or *OTPN* laws in the formation of a particle on the surface *R (u, v)* only affects the appearance of differential equations (2) - (5), but the results of their solution.

The proposed linear algorithm for determining components of expressions that are part of equation (1) - (5) differential equations laws of a particle simultaneously projected on orts accompanying trihedron *OuvN* and *OTPN*, independent of the equation *R (u, v)* surface (of form) .

The movement of the particles on rough plane is the easiest. For the inclined plane $\boldsymbol{R}(u,v) = [u, -v\,sin(\xi)\,, v\,cos(\xi)]$,, where ξ - angle vertical plane around the axis Ox (ξ-90 ° angle of the plane) we obtain the following laws of differential equations of a particle projected on orts:

u and *v* trihedron *OuvN* (Fig. 1, a):

$$\begin{cases} Ou := \ m\frac{d^2}{dt^2}u(t) = -\dfrac{\frac{d}{dt}u(t)\left(fmg\,sin(\xi)+q\left(\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2\right)\right)}{\sqrt{\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2}} \\ Ov := m\frac{d^2}{dt^2}v(t) = -\dfrac{fmg\,sin(\xi)\frac{d}{dt}v(t)}{\sqrt{\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2}} - mg\,cos(\xi) - q\frac{d}{dt}v(t)\sqrt{\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2} \end{cases}; \quad (6)$$

T and *P* trihedron *OTPN* (Fig. 2b):

$$\begin{cases} OT := \dfrac{m\left(\frac{d}{dt}u(t)\frac{d^2}{dt^2}u(t)+\frac{d}{dt}v(t)\frac{d^2}{dt^2}v(t)\right)}{\sqrt{\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2}} = -\dfrac{mg\,cos(\xi)\frac{d}{dt}v(t)}{\sqrt{\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2}} - fmg\,sin(\xi) - q\left(\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2\right) \\ OP := m\left(\frac{d}{dt}v(t)\frac{d^2}{dt^2}u(t) - \frac{d}{dt}u(t)\frac{d^2}{dt^2}v(t)\right) = -mg\,cos(\xi)\frac{d}{dt}u(t) \end{cases} \quad (7)$$

The initial conditions the solution of differential equations (6) and (7) determine, provided that the direction of the initial velocity in the inner $V_o u$, *v* -plane coordinates *R (u, v)* asked α_o angle, for example, the tangent *u* - coordinate line. Then the arc length *ds* trajectory *r (t)* at the beginning of a particle is:

$$ds = V_o\,dt, \quad (8)$$

and the ratio differentials coordinate lines arc length *ds* are:

$$\frac{\sqrt{E}\frac{d}{dt}u(t)}{ds} = \frac{\sqrt{E}\frac{d}{dt}u(t)}{V_o dt} = sin(\alpha_o), \quad \frac{\sqrt{G}\frac{d}{dt}v(t)}{ds} = \frac{\sqrt{G}\frac{d}{dt}v(t)}{V_o dt} = cos(\alpha_o). \quad (9)$$

From the expression (10) we obtain the initial velocity and position of a particle in the inner *u*, *v*-surface coordinates *R (u, v):*

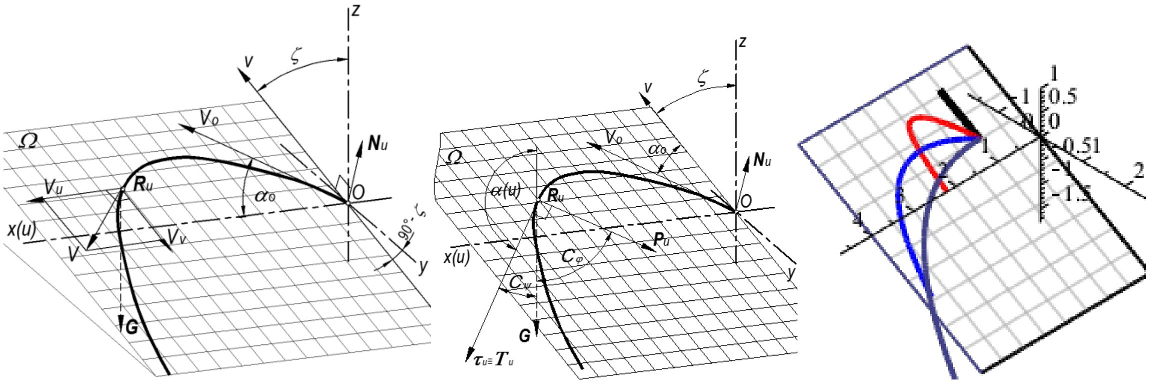
$$\frac{d}{dt}u(t) = \frac{V_o sin(\alpha_o)}{\sqrt{E}}, \quad \frac{d}{dt}v(t) = \frac{V_o cos(\alpha_o)}{\sqrt{G}}, \quad (10)$$

$$u(t_o) = u_o, \quad v(t_o) = v_o, \quad (11)$$

where $E = 1$, $G = 1$ - coefficient of 1st plane quadratic form $R\ (u, v)$; u_o, v_o- internal coordinates of the initial position the particles in the plane.

Note that expressions (10) and(11) set the initial conditions for the solution of differential equations laws of motion of particles on any surface $R\ (u, v)$.

Find the desired dependence $u\ (t)$ and $v(t)$ of the system of differential equations can only be approximated, such as Runge-Kutt method. There were a set of computational experiments to investigate the movement of particles on an incline, vertical and horizontal planes depending on the initial velocity and direction V_o, α_o particles throwing its initial position u_o, v_o, friction coefficient f and $\xi - 90°$ angle inclination plane. Fig. 2, as one of many experiments built trajectory $r\ (t)$ particles depending on the angle $\alpha_o = 1°, 30°, 60°, 90°$of throwing it at constant values $V_o = 4$m/s, $f = 0.3$, $\xi = 60°$.



a) b) c)

Fig. 2. Cover orts:

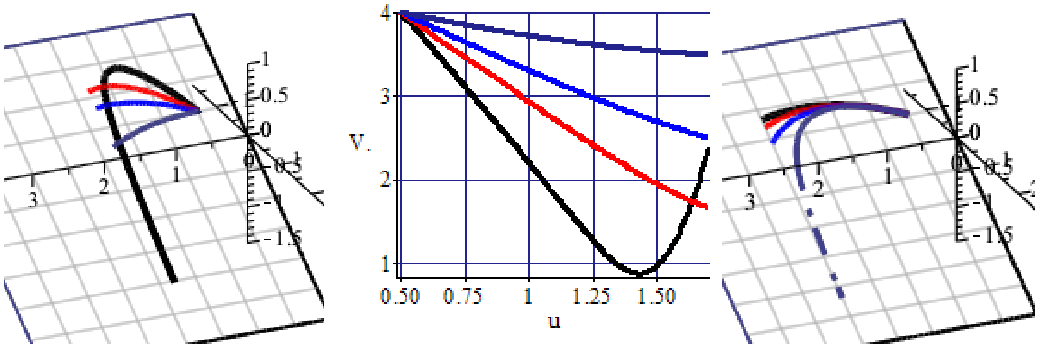
a) *u* and *v* system *OuvN*; b) *T* and *P* trihedron Darboux *OTPN*; c) trajectory *r(t)*

Displaying law of motion of the particles on rough inclined plane as a function of the independent parameter u in the projections for *T* and *P* ort trihedron *OTPN* as:

$$\begin{cases} OT := \ mV(u)\frac{d}{du}V(u) = -mg\left(\frac{d}{du}v(u)\cos(\xi) + f\sin(\xi)\sqrt{1+\left(\frac{d}{du}v(u)\right)^2}\right) \\ OP := \ mV(u)^2 = -\dfrac{mg\left(1+\left(\frac{d}{du}v(u)\right)^2\right)\cos(\xi)}{\frac{d^2}{du^2}v(u)} \end{cases}. \quad (12)$$

Unlike (6) - (7), the desired dependency equations (12) are speed $V\ (u)$ and coordinate $v(u)$. Fig. 3, b is constructed trajectory $r(u)$ and graphics speeds $V(u)$

particles on an inclined plane with an angle $\xi = 60°$ tilt at it from a vertical position, depending on the angle of throwing $\alpha_o = 30°, 45°, 60°, 90°$ at the original speed $V_o = 4$ м/cand friction $f = 0.3$. All particles under these conditions in an inclined plane not stop - they are first rate drops to a certain size and then grow. The lowest speed *V = 0.88 m/s* particles will point its position *u = 1.42*, then it accelerates almost a straight line. Built trajectory *r (u)* in Fig. 4, confirm the fact that the greater the friction coefficient $f = 0, 0.1, 0.2, 0.3, 0.4$ $(V_o = 4\frac{m}{s}$ and $\alpha_o = 60°)$, the faster the particle starts to slide down the plane. Note that the trajectory *r (u)* is based on the range of parameter *u = 0..2.5*, which localizes a given region $[u_o..u_n]$, which investigated plane motion of a particle.



a) b) c)

Fig. 3. Trajectories *r (u)* speed *V(u)* particles on an inclined plane

If the option to take an independent angle α (movement direction), then the law of a particle on an inclined plane in the projections for *T* and *P* ort trihedron *OTPN* will be as follows:

$$\begin{gathered} OT := \; mV(\alpha)\frac{d}{d\alpha}V(\alpha)k(\alpha) = -mg\,cos(\alpha)\,cos(\xi) - fmg\,sin(\xi) \\ OP := \; mV(\alpha)^2 k(\alpha) = mg\,sin(\alpha)\,cos(\xi) \end{gathered} . \quad (13)$$

Desired dependency equations (13) are speed $V(\alpha)$ and curvature $k(\alpha)$ path that can be explicitly (which is not to be found in the function of independent parameters *t* and *u*:

$$V(\alpha) = \frac{V_o\,sin(\alpha_o)\,(cosec(\alpha) - ctg(\alpha))^{-f\,tan(\xi)}}{sin(\alpha)\,(cosec(\alpha_o) - ctg(\alpha_o))^{-f\,tan(\xi)}}, \quad (14)$$

$$k(\alpha) = \frac{g\ sin(\alpha)^3 \left((cosec(\alpha_o) - ctg(\alpha_o))^{-f\ tan(\xi)}\right)^2}{V_o^2\ sin(\alpha_o)^2\ ((cosec(\alpha) - ctg(\alpha))^{-f\ tan(\xi)})^2}. \qquad (15)$$

Finding the boundary *(limit)* functions (14) and (15), provided that the tangent is ξ friction coefficient *f* ($f\ tan(\xi) = 1$),, leads to the expression:

$$V_\pi = limit\,(V(\alpha), \alpha = \pi) = \frac{Vo}{2}(1 - cos(\alpha_o)),, \qquad (16)$$

$$k_\pi = limit\,(k(\alpha), \alpha = \pi) = 0. \qquad (17)$$

Assertion. When a loose material with an initial velocity perpendicular V_o to the line of greatest slope rough plane set at an angle to the horizontal friction, each piece after the transition to the straight path will move with constant velocity $V_o/2$. Particles thrown along the greatest slope of the plane (angle $\alpha_o = \pi$), will move at a constant initial velocity V_o. If you throw it in the opposite direction (angle $\alpha_o = 0$), it will stop.

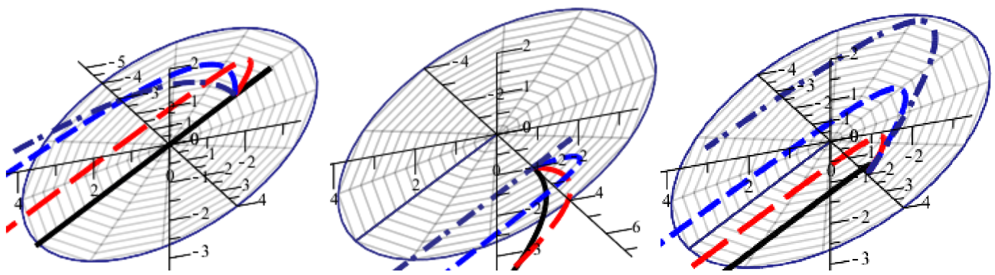
For example, the plane shows that setting surfaces *R (u, v)* in different parameterization does not affect the trajectory-kinematic characteristics of a particle, although analytical calculations differ significantly. In particular, if a flat disc parametric equations written as:

$$\boldsymbol{R}(u,v) = \boldsymbol{R}[v\,cos(u)\,cos(\xi)\,, vsin(u), -vsin(u)\,sin(\xi)], \qquad (18)$$

then the law of a particle on it projected on ort *u* and *v* trihedron *OuvN* compared to (6) will have a different analytical form:

$$\begin{cases} Ou := m\left(2\frac{d}{dt}u(t)\frac{d}{dt}v(t) + v(t)\frac{d^2}{dt^2}u(t)\right) = -mg\,sin(u(t))\,sin(\xi) - \dfrac{mf\,v(t)\frac{d}{dt}u(t)\,cos(\xi)}{\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}} \\ Ov := m\left(\frac{d^2}{dt^2}v(t) - v(t)\left(\frac{d}{dt}u(t)\right)^2\right) = mg\,cos(u(t))\,sin(\xi) - \dfrac{mf\frac{d}{dt}v(t)cos(\xi)}{\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}} \end{cases}. \qquad (19)$$

Fig. 4, built a trajectory *r (t)* particles on rough inclined disc ξ = 30 ° with an initial velocity $V_o = 4\frac{m}{s}$ initial position $u_o = \pi, v_o = 2$, friction *f = 0.3*, depending on the angle $\alpha_o = 0°, 30°, 60°, 90°$ throwing particles. Since $f = 0.3 < \tan(\xi = 30°)$, then none of the particles does not stop on the drive. Rectilinear trajectory of particles thrown at an angle $\alpha_o = 0°$ (up from the center of the disc) will pass through the origin *O*. Changing the initial provisions ($u_o = 1.5\pi$, $v_o = 2$) particles leads to change provisions of their trajectories *r (t)* (Fig. 4, b). Particles thrown in one direction $\alpha_o = 120°$, but with different initial velocity $V_o = 2, 4, 6, 8$ *m/s*, different form the trajectory *r (t)* with respect to the center of the disk (Fig. 4,). But in all cases the trajectory of the particles on rough disk is congruent to the trajectories particles on rough inclined plane under the same initial conditions of throwing.



a) b) c)

Fig. 4. The trajectories *r (t)* particles on rough inclined disc

Conclusions

The method of computer modeling of a particle along rough surface which is based on: 1) the inner surface parameterization: 2) supporting trihedron curves on the surface; 3) independent parameters forming curves on the surface, which is the time the particle position and direction of its movement on the surface.

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МОДЕЛЮВАННЯ РУХУ ЧАСТИНКИ ПО НЕРУХОМІЙ ШОРСТКІЙ ПОВЕРХНІТА ЙОГО ТЕСТУВАННЯ НА ПРИКЛАДІ ПОХИЛОЇ ПЛОЩИНИ

А. В. Несвідомін

Анотація. *Створено аналітичне, алгоритмічне та програмне забезпечення моделювання руху частинки по будь-якій нерухомій шорсткій поверхні у функції часу, положення частинки на поверхні та напряму її переміщення. Опрацьований алгоритм на прикладі похилої площини.*

Ключові слова: ***матеріальна точка, супровідний тригранник, похила площина, шорстка поверхня, моделювання***

МОДЕЛИРОВАНИЕ ДВИЖЕНИЯ ЧАСТИЦЫ ПО НЕПОДВИЖНОЙ ШЕРОХОВАТОЙ ПОВЕРХНОСТИ И ЕГО ТЕСТИРОВАНИЕ НА ПРИМЕРЕ НАКЛОННОЙ ПЛОСКОСТИ

А. В. Несвидомин

Аннотация. *Создано аналитическое, алгоритмическое и программное обеспечение моделирования движения частицы по любой неподвижной шероховатой поверхности в функции времени, положение частицы на поверхности и направления ее перемещения. Отработан алгоритм на примере наклонной плоскости.*

Ключевые слова: ***материальная точка, сопроводительный трехгранник, наклонная плоскость, шероховатая поверхность, моделирование***