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THE MOVEMENT OF PARTICLE ON A ROUGH SURFACE ELLIPSOID ROTATION

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Annotation. *Given the trajectory and kinematic properties of the motion of a particle on a rough surface of an ellipsoid of rotation.*

Keywords: ***rough surface, ellipsoid of rotation, the motion of particles, differential equations, trajectory, speed***

Justification design parameters of working surfaces or for moving loose material separation involves the study of movement of individual particles on rough surface [1]. For an ellipsoid of revolution, such studies can be done only with the help of modern computer technology.

In [2] the method and analytical description of a particle along rough surface on the basis of supporting trihedron of trajectory.

The purpose of research - development environment for Maple simulation of a particle along rough surface and explore an ellipsoid of revolution trajectory-kinematic properties according to the following initial conditions: 1) parameters of form and position of the surface; 2) initial velocity throwing particles; 3) the angle of the direction of throwing a tangent plane; 4) coefficient of friction.

Materials and methods of research. In the projections for orts *u* and *v* of trihedron OuvN law of motion of the particle can be written [3]:

$$\begin{cases} Ou := m\,W\cos(\widehat{\boldsymbol{R}_u,\boldsymbol{w}}) = F_g\,\cos(\widehat{\boldsymbol{R}_u,\boldsymbol{G}}) - f\,F_N\cos(\widehat{\boldsymbol{R}_u,\boldsymbol{\tau}}) \\ Ov := m\,W\cos(\widehat{\boldsymbol{R}_v,\boldsymbol{w}}) = F_g\,\cos(\widehat{\boldsymbol{R}_v,\boldsymbol{G}}) - f\,F_N\cos(\widehat{\boldsymbol{R}_v,\boldsymbol{\tau}}) \end{cases}, \quad (1)$$

where $\boldsymbol{G} = [0,0,-1]$ - direction of gravity in system $Oxyz$;

$W = |\boldsymbol{w}|$ – the magnitude of acceleration;

$F_N = F_g\cos(\widehat{\boldsymbol{N},\boldsymbol{G}}) \pm F_C\cos(\widehat{\boldsymbol{N},\boldsymbol{n}})$ - normal reaction force;

$F_g = mg$ і $F_C = m\,V^2 k$ - gravity and centrifugal force;

$\boldsymbol{N} = [0, 0, 1]$ - normal to the plane $\boldsymbol{R}(u, v)$ in the points of the trajectory $\boldsymbol{r}$;

$\boldsymbol{n}$ і $\boldsymbol{\tau}$ - main normal and tangent to the trajectory *r* of particles.

Parametric equations uv-grid ellipsoid of revolution with an axis in the plane coordinate system Oyz Oxyz written as:

$$\boldsymbol{R}(u,v) = \boldsymbol{R}\begin{bmatrix} a\ \cos(u)\ \cos(v), \\ a\ \sin(u)\ \cos(v)\ \cos(\xi) - b\ \sin(v)\ \sin(\xi), \\ a\ \sin(u)\ \cos(v)\ \sin(\xi) + b\ \sin(v)\ \cos(\xi) \end{bmatrix}, \tag{2}$$

where a, b – parameters form an ellipse in the plane Oxz;

ξ – the angle between the spin axis of the ellipse and the axis in the plane of *Oz* plane Oyz;

$u \in [0; 2\pi]$, $v \in [-\pi/2; \pi/2]$ – coordinates curved surface.

Taking into account the angle ξ significantly affect the trajectory-kinematic properties of a particle on the rough surface of an ellipsoid of revolution, which are expressed very cumbersome analytical equations. For maple-developed models *Ellipsoid_t* complexity of their output does not matter because in the environment of symbolic algebra Maple [3] are carried out automatically. In the case where the position of the rotation axis of the ellipse coincides with the axis Oz $(\xi = 0)$ parametric equation *uv*-mesh ellipsoid of revolution (2) with the axis *Oz* will have the form:

$$\boldsymbol{R}(u,v) = \boldsymbol{R}[a\ \cos(u)\ \cos(v), a\ \sin(u)\ \cos(v), b\ \sin(v)], \tag{3}$$

Law of motion of particles in which is expressed somewhat simpler system of differential equations:

$$\begin{cases} Ou := -ma\left(2\sin(v(t))\frac{d}{dt}u(t)\frac{d}{dt}v(t) - \cos(v(t))\frac{d^2}{dt^2}u(t)\right) = \\ \quad -\dfrac{mf\,a^2\cos(v(t))\frac{d}{dt}u(t)\left(b\left(\left(\frac{d}{dt}v(t)\right)^2 + \cos(v(t))^2\left(\frac{d}{dt}u(t)\right)^2\right) + g\,\sin(v(t))\right)}{v\sqrt{(b^2-a^2)\cos(v(t))^2 + a^2}} \\ Ov := ma\left(\left((b^2-a^2)\cos(v(t))^2 + a^2\right)\frac{d^2}{dt^2}v(t) + \sin(v(t))\cos(v(t))\left((a^2-b^2)\left(\frac{d}{dt}v(t)\right)^2 + a^2\left(\frac{d}{dt}u(t)\right)^2\right)\right) = \\ \quad -\dfrac{amf\,\frac{d}{dt}v(t)\left((a^2-b^2)\cos(v(t))^2 - a^2\right)\left(-b\cos(v(t))^2\left(\frac{d}{dt}u(t)\right)^2 - b\left(\frac{d}{dt}v(t)\right)^2 + g\,\sin(v(t))\right)}{v\sqrt{(b^2-a^2)\cos(v(t))^2 + a^2}} - b\,mg\cos(v(t)) \end{cases}. \tag{4}$$

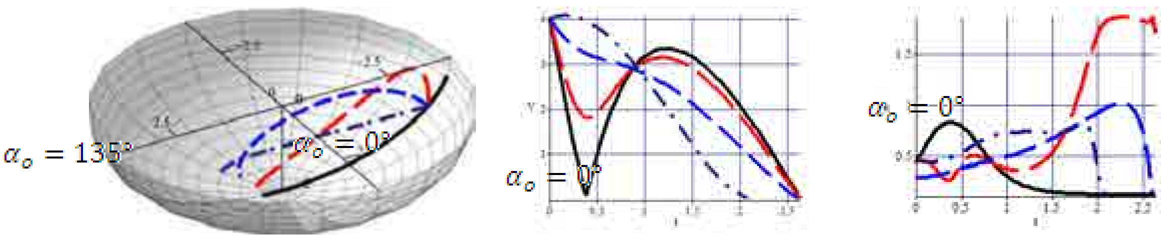
The initial conditions of solutions of differential equations (4) are:

$$Oi := u(0) = u_o, \frac{d}{dt}u(0) = \frac{V_o \sin(\alpha_o)}{a \cos(v_o)}, v(0) = v_o, \frac{d}{dt}v(0) = \frac{V_o \cos(\alpha_o)}{\sqrt{(b^2 - a^2)\cos(v(t))^2 + a^2}}, \quad (5)$$

where α_o – the angle between the speed vector *V(t)* and *u*-coordinate lines;

$[u_o, v_o]$ - its position at the moment $t_0 = 0$.

Results. Desired dependence *u(t)* and *v(t)* of equations *(4) - (5)* can be found only approximately, including the method of Runge-Kutta in discrete form [3]. Figure 1 shows the trajectory *r(t)* the particle, its graphics speed *V(t),* and curvature *k(t)* trajectory on the inner surface of the rotation ellipsoid shape parameters *a=4, b=2*, depending on the angle of throwing $\alpha_o = 0°,45°,90°,135°$, initial position $u_o = \pi, v_o = \pi/6$, the initial velocity $V_o = 4$ м/с and friction $f = 0.3$. For schedules velocity *V (t)* can be argued that all the particles at different time intervals stop in the vicinity of the lowest point (pole) ellipsoid. Moreover, the particle thrown to the side and bottom of the ellipsoid ($\alpha_o = 135°$) fastest stop *t≈2.1c*. In stopping the particle curvature *k (t)* of the trajectory is close to zero.

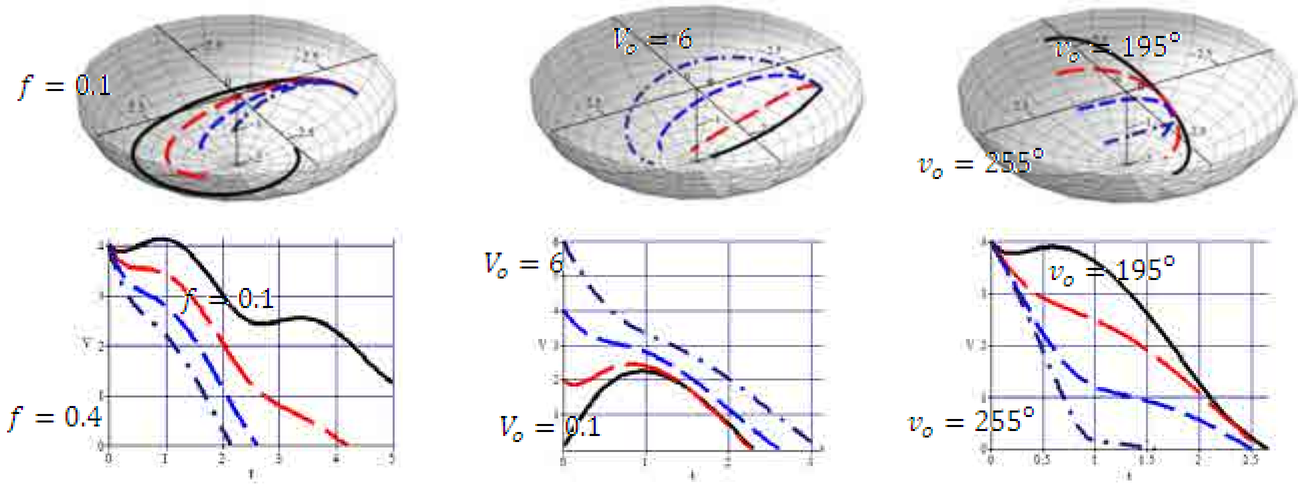


a b c

Fig.1. The trajectories $r(t)$ the particle, its graphics speed $V(t)$ and curvature $k(t)$ depending on the angle of throwing α_o

The trajectories $r(t)$ of the particle and graphics speed $V(t)$ depending on: a) the coefficient of friction $f = 0.1, 0.2, 0.3, 0.4$ and $\alpha_o = 90°$ i $V_o = 4$ м/с; b) the initial velocity $V_o = 0.1, 2, 4, 6$ and $\alpha_o = 90°$ i $f = 0.3$; c) initial position $v_o = 195°, 215°, 235°, 255°$ and $\alpha_o = 90°$ i $V_o = 4$ built in Figure 2. The greater the friction coefficient *f*, the faster the particles stop. Place of stop particles is in the vicinity of the lower pole ellipsoid. Throw particles with initial velocities $V_o = 0.1$ i

$V_o = 2$ м/с respectively at angles $\alpha_o = 0°$ i $\alpha_o = 45°$ though with different trajectories, but stopped because of the same period of time $t \approx 2.2c$. The particles thrown from various parallels $v_o = 195°, 215°, 235°$ ellipsoid stop almost simultaneously $t \approx 2.5c$.



a b c

Fig.2. The trajectories *r (t)* of the particle and graphics speed *V (t)* on the inner surface of the ellipsoid under different conditions of throwing

Let ellipsoid shape parameters equal to $a = 1$ i $b = 2$, angle throwing $\alpha_o = 0°$,-45°,$-90°$,$-135°$, starting position $u_o = 3\pi/2, v_o = 0$, the initial velocity $V_o = 4$ м/с, the friction coefficient $f = 0.3$ (in Figure 3 and shown 1/8 of the surface). The particle thrown up along the prime meridian ($\alpha_o = 0°$) after lapse $t \approx 0.2c$ away from the floor. All other particles are not looking up from the surface will move to the lower pole of the ellipsoid in the vicinity which stop.

A somewhat different character of the particles will be held on the surface of an ellipsoid of revolution when its axis takes a horizontal position. The trajectories $\boldsymbol{r}(t)$ and its graphics speed $V(t)$ particles depending on the angle of throwing $\alpha_o = 0°, 45°, 90°, 135°$ in the initial position $u_o = \pi, v_o = -\pi/6$, the initial velocity $V_o = 4$ м/с and the coefficient of friction $f = 0.3$ are shown in Fig. 3, b. Under these initial conditions, all particles in a confined space ellipsoid of revolution stop at the bottom - the graphs velocity $V(t)$ it begins to happen after lapse $t \approx 3.5$. At the time of their normal reaction stop $F_N(t)$ approaches the value $F_N = mg$. The particle thrown to the side pole ellipsoid early movement has a greater amplitude of normal reaction force $F_N(t)$ due to the greater speed $V(t)$ and lesser curvature $k(t)$ of trajectory.

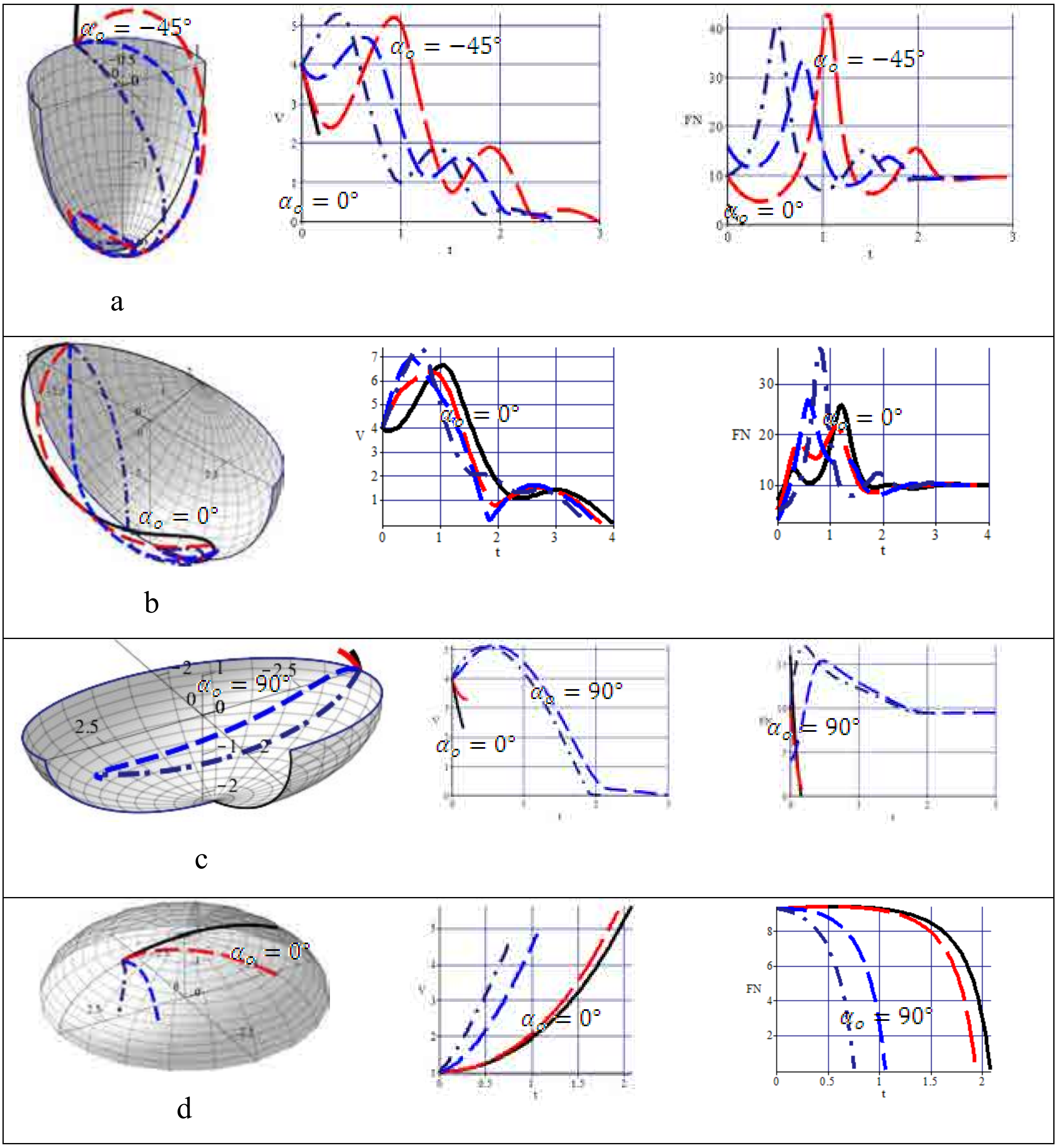


Fig.3. The trajectories of the particle, its speed and graphics power normal reaction

If you turn the vertical axis of the ellipsoid of revolution (1) Ox axis at an angle $\xi = 15°$, then the trajectory $r(t)$ the particle surface in the middle will not go in the direction of the lower pole (Figure 3, c). The graphics speed $V(t)$ and normal reaction force $F_N(t)$ particles depending on the angle of throwing $\alpha_o = 0°,45°,90°,135°$ at $u_o = \pi, v_o = 0, V_o = 4$ м/с and $f = 0.3$ show that only particles with throwing angle $\alpha_o = 90°, 135°$ does not come off from the surface.

One of the results of a particle on the outer surface of an ellipsoid of revolution are given in Figure 3. You can see that all particles with throwing angle $\alpha_o = 0°,45°,90°,135°$ at $f = 0.3$ i $V_o = 1$ м/с will come off from the surface before reaching its equator.

Conclusions

Options shape and position of the ellipsoid of revolution, initial velocity, position and coefficient of friction particles have complex effects on the movement of loose material on the surface.

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РУХ ЧАСТИНКИ ПО ШОРСТКІЙ ПОВЕРХНІ ЕЛІПСОЇДА ОБЕРТАННЯ

А. В. Несвідомін

Анотація. *Наведено траєкторно-кінематичні властивості руху частинки по нерухомій шорсткій поверхні еліпсоїда обертання.*

Ключові слова: ***шорстка поверхня, еліпсоїд обертання, рух частинки, диференціальні рівняння, траєкторія, швидкість***

ДВИЖЕНИЕ ЧАСТИЧКИ ПО ШЕРОХОВАТОЙ ПОВЕРХНОСТИ ЕЛЛИПСОИДА ВРАЩЕНИЯ

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Аннотация. *Приведены траекторно-кинематические свойства движения частицы по шероховатой поверхности эллипсоида вращения.*

Ключевые слова: ***шероховатая поверхность, эллипсоид вращения, движение частицы, дифференциальные уравнения, траектория, скорость***