UDC 514.18

MODEL OF MOTION OF A PARTICLE IN A VERTICAL PLANE IN THE FUNCTION OF ITS POSITION PARAMETER

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Abstract. *The purpose of the research was to develop a computer model of motion of a particle on a rough vertical plane in the environment of the symbolic algebra Maple and with the help of computational experiments to find out its properties.*

Trajectory-kinematic characteristics of motion of particles along a rough vertical plane are established. The particle trajectories and graphs of its velocity are shown, depending on the position and different coefficients of friction and initial velocity.

On the basis of the performed research it can be concluded that the motion of a particle in the vertical plane occurs in the same way as in the motion of a particle under the influence of the force of gravity in space.

Key words: ***motion of a particle, vertical plane, system of differential equations, trajectory, velocity, position***

Topicality. In many agricultural processes, the movement of particles of material on rough planes takes place. Understanding the patterns of motion of a particle (as a material point) on a rough plane of arbitrary position in a three-dimensional space allows purposefully to calculate the structural and kinematic parameters of the working bodies.

Analysis of recent research and publications. An analytical formation of the law of motion of a particle on any rough surface reduces to the compilation of a system of differential equations of the second order, the solution of which is the particle trajectory, its velocity, acceleration, the length of the traversed path, the strength of the normal reaction, the time to its stop, and others trajectory-kinematic characteristics. The sequence of analytic transformations in the derivation of the system of differential equations and the methods of its solution is quite labor-intensive. During the last decades (in the period of the emergence and development of computer technologies), significant changes in the automation of the methods for deriving the laws of motion of a particle on a rough surface of complex form did not occur. In existing studies, each scientist individually performs analytical transformations in order to obtain the law of motion of a particle in the form of systems of differential equations of the second order, the complexity of which essentially depends on the shape of the surface. That is why the list of surfaces on which the motion of the particle was investigated is limited to a plane, a cylinder and a cone of rotation, a screw conoid, an expanding helicoid [1].

Computer simulation of the motion of a particle on rough surfaces allows to discard bulky analytical transformations and provide the scientist with a convenient dialogue mode for carrying out the necessary computational experiments on particle motion analysis under different initial conditions of its throw on any rough surface that is in a certain way located in space [2, 3].

The purpose of the study is to develop, in the environment of the symbolic algebra Maple, a computer model of motion of a particle along a rough vertical plane, and with using computational experiments find out its trajectory-kinematic properties.

Materials and methods of research. In the maple model *PlaneOxzR_u* the formation of the particle motion law in the vertical plane $\boldsymbol{R}(u, v)$ is carried out in the function of the independent parameter *u* - its position on the *u*-coordinate direct plane. To derive this law, we substitute the dependencies *u* = *u* and *v* = *v (u)* of the particle trajectory in the internal u, v coordinates of the vertical plane, from which we obtain its trajectory ***r****(u)* in the system of cartesian coordinates *Oxyz* in the form:

- $$\boldsymbol{r}(u) = \boldsymbol{r}[u, 0, v(u)], \tag{1}$$

where *v(u)* is the desired trajectory in the internal *u, v* coordinates of the plane$\boldsymbol{R}(u, v)$.

According to the obtained equation ***r****(u),* we define:

vector of tangent vector $\tau(u)$ of the trajectory $\boldsymbol{r}(u)$ and velocity $V(u)$:

- $$\tau(u) = \tau\left[1, 0, \frac{d}{du}v(u)\right], \tag{2}$$

- $$V(u) = \left|\frac{d}{du} r(u)\right| = \sqrt{1 + \left(\frac{d}{du} v(u)\right)^2};$$ (3)

acceleration vector ***w(t)*** and its value $W(t)$:

- $$\boldsymbol{w}(u) = \frac{d}{du}\boldsymbol{\tau}(u) = \boldsymbol{w}\left[0, 0, \frac{d^2}{dt^2} v(u)\right],$$ (4)
- $$W(u) = |\boldsymbol{w}(u)| = \frac{d^2}{dt^2} v(u);$$ (5)

the vector of the normal $\boldsymbol{n}$ trajectory $\boldsymbol{r}(u)$ and its curvature $k(u)$:

- $$\boldsymbol{n}(u) = \boldsymbol{n}\left[-\frac{d^2}{du^2} v(u) \frac{d}{du} v(u), 0, \frac{d^2}{du^2} v(u)\right];$$ (6)
- $$k(u) = \frac{\frac{d^2}{du^2} v(u)}{\left(1+\left(\frac{d}{du} v(u)\right)^2\right)^{3/2}};$$ (7)

the centrifugal force $\boldsymbol{F_c}$ and the force of the normal reaction $\boldsymbol{F_N}$ of the particle:

- $$F_c(u) = m\, V(u)^2\, k(u) = \frac{m \frac{d^2}{dt^2} v(u)}{\sqrt{1+\left(\frac{d}{du} v(u)\right)^2}};$$ (8)
- $$F_N(u) = mg\, \cos(\varepsilon) + F_c \cos(\eta) = 0,$$ (9)

where $\cos(\varepsilon)$ is the cosine of the angle ε between the vectors $\boldsymbol{N}(u)$ and $\boldsymbol{n}(u)$:

- $$C\varepsilon(u) = \cos(\varepsilon) = \cos(\widehat{\boldsymbol{n}, \boldsymbol{N}}) = 0.$$ (10)

To form the motion law of a particle in projections on the orths $\boldsymbol{u}$ i $\boldsymbol{v}$ triedron $\boldsymbol{OuvN}$ we define the cosines of the angles between the vectors $\boldsymbol{w}(u)$ and $\boldsymbol{\tau}(t)$ and the vectors $\boldsymbol{R}'_u$ i $\boldsymbol{R}'_v$:

- $$Cwu(u) = \cos(\widehat{\boldsymbol{w}, \boldsymbol{R}'_u}) = 0,$$ (11)
- $$Cwv(u) = \cos(\widehat{\boldsymbol{w}, \boldsymbol{R}'_v}) = 1,$$ (12)
- $$C\tau u(u) = \cos(\widehat{\boldsymbol{\tau}, \boldsymbol{R}'_u}) = \frac{1}{\sqrt{1+\left(\frac{d}{du} v(u)\right)^2}},$$ (13)
- $$C\tau u(v) = \cos(\widehat{\boldsymbol{\tau}, \boldsymbol{R}'_v}) = \frac{\frac{d}{du} v(u)}{\sqrt{1+\left(\frac{d}{du} v(u)\right)^2}}.$$ (14)

Their substitution into the system of differential equations leads to its following form:

- $$\begin{cases} Ou: = \ 0 = 0 \\ Ov: = \ m\, \frac{d^2}{dt^2} v(u) = -mg \end{cases}$$ (15)

The zero equality of the left-hand side of the resulting differential equation means that the velocity of the particle along the $Ou \equiv Ox$ axis is a constant value and is equal to $V_o \, sin(\alpha_o)$, and therefore the value of the traversed path $s(u)$ is equal to:

- $$s(u) = u\,V_o\,sin(\alpha_o) + u_o, \tag{16}$$

where: u is equal to:

- $$u = \frac{s(u) - u_o}{V_o\,sin(\alpha_o)}. \tag{17}$$

Use *dsolve* as:

- $$dsolve\left(\left\{m\frac{d^2}{dt^2}v(u) = -mg,\ D(v)(0) = V_o\,cos(\alpha_o),\ v(0) = v_o\right\},\ v(u)\right), \tag{18}$$

leads to its solution:

- $$v(u) = -\frac{1}{2}gu^2 + V_o\,u\,cos(\alpha_o) + v_o, \tag{19}$$

and replacing $s(u) \equiv u$ we have:

- $$v(u) = -\frac{g\,(u-u_o)^2}{2\,V_o^2\,sin(\alpha_o)^2} + \frac{cos(\alpha_o)\,(u-u_o)}{sin(\alpha_o)} + v_o. \tag{20}$$

Substitution of the obtained dependence *v(u)* to the equation of the trajectory ***r(u)*** of the particle is brought to its next parametric form:

- $$\boldsymbol{r}(u) = \boldsymbol{r}\left[u, 0, -\frac{g\,(u-u_o)^2}{2\,V_o^2\,sin(\alpha_o)^2} + \frac{cos(\alpha_o)\,(u-u_o)}{sin(\alpha_o)} + v_o\right]. \tag{21}$$

A similar result will also be obtained with the application of the ***OTPN*** trihedrone bar of the formation of the particle motion law in the function of the independent parameter *u*, although the analytical calculations will be somewhat different. First we find:

vector $\boldsymbol{P} = \boldsymbol{N} \times \boldsymbol{\tau}$ trihedrone Darboux $\boldsymbol{OTPN}$:

- $$\boldsymbol{P}(u) = \boldsymbol{P}\left[-\frac{d}{du}v(u), 0, 1\right]; \tag{22}$$

cosines of the angles φ and ψ between the vector $\boldsymbol{G}[0, 0, -1]$ gravity and the vectors $\boldsymbol{\tau}$ and $\boldsymbol{P}$:

- $$C\varphi(u) = cos(\varphi) = cos(\widehat{G,\tau}) = -\frac{\frac{d}{du}v(u)}{\sqrt{1+\left(\frac{d}{du}v(u)\right)^2}}; \tag{23}$$

- $$C\psi(u) = cos(\psi) = cos(G,P) = -\frac{1}{\sqrt{1+\left(\frac{d}{du}v(u)\right)^2}}; \quad (24)$$

the differential of the arc ds of the trajectory ***r*** provided $dv = \frac{d}{du}v(u)\,du$:

- $$ds = \sqrt{du^2 + dv^2} = \sqrt{1 + \left(\frac{d}{du}v(u)\right)^2}\,du; \quad (25)$$

normal k_n and geodesic k_g curvature of the trajectory $\boldsymbol{r}(u)$:

- $$k_n(u) = k\cos(\varepsilon) = 0; \quad (26)$$
- $$k_g(u) = k\sin(\varepsilon) = \frac{\frac{d^2}{du^2}v(u)}{\left(1+\left(\frac{d}{du}v(u)\right)^2\right)^{3/2}}. \quad (27)$$

Since the trajectory $\boldsymbol{r}(u)$ of the particle lies in the vertical plane $\boldsymbol{R}(u,v)$, its normal curvature $k_n(u)$ equals zero, and the curve $k(u)$ coincides with its tangential curvature $k_g(u)$.

We obtain:

- $$\begin{cases} OT := mV(u)\frac{d}{du}V(u) = -mg\left(\frac{d}{du}v(u)\right) \\ OP := m\frac{V(u)^2\frac{d^2}{du^2}v(u)}{1+\left(\frac{d}{du}v(u)\right)^2} = -mg \end{cases}. \quad (28)$$

Note that the desired dependencies, in contrast to the previous systems of differential equations of the particle motion law, are the velocity $V(u)$ of the particle and the $v(u)$- coordinate of its vertical position for the variable parameter u. The initial conditions for finding them are:

- $$OI := \frac{d}{du}v(u_o) = ctg(\alpha_o), v(u_o) = v_o, V(u_o) = V_o. \quad (29)$$

The use of the operator $dsolve(\{OT, OP, OI\}, \{V(u), v(u)\})$ allows us to obtain explicit dependences of the velocity $V(u)$ of the particle and the coordinate $v(u)$ of its position in the function of the variable parameter u:

- $$V(u) = V_o\sin(\alpha_o)\sqrt{1+\left(-\frac{g(u-u_o)}{V_o^2\sin(\alpha_o)^2}+ctg(\alpha_o)\right)^2}, \quad (30)$$
- $$v(u) = -\frac{g(u-u_o)^2}{2V_o^2\sin(\alpha_o)^2} + (u-u_o)ctg(\alpha_o) + v_o. \quad (31)$$

Результати дослідження та їх обговорення. In figure the trajectories of the motion of the particle $\mathbf{r}(u)$, their curvature $k(u)$ and the velocity $V(u)$ are constructed, depending on the angle of the throw $\alpha_o = 15°, 45°, 90°, 120°$ at the initial velocity $V_o = 4$ m/s. Note that the value of the independent parameter u lies within $[u_o, u_n]$, where: the parameter u_o determines the v- coordinate line on which the particle is located at the time of its throw. Therefore, the graphs of the velocities $V(u)$ of the particle and the curves $k(u)$ of the trajectory $\mathbf{r}(u)$ are displaced along the axis Ou y the value u_o (figure,b,c). It is clear that the velocity $V(u)$ of a particle thrown at an angle α_o within the limits $]0; 90°]$ will first fall to a certain value V_{min} at its highest point, and then increase with $u \to \infty$.

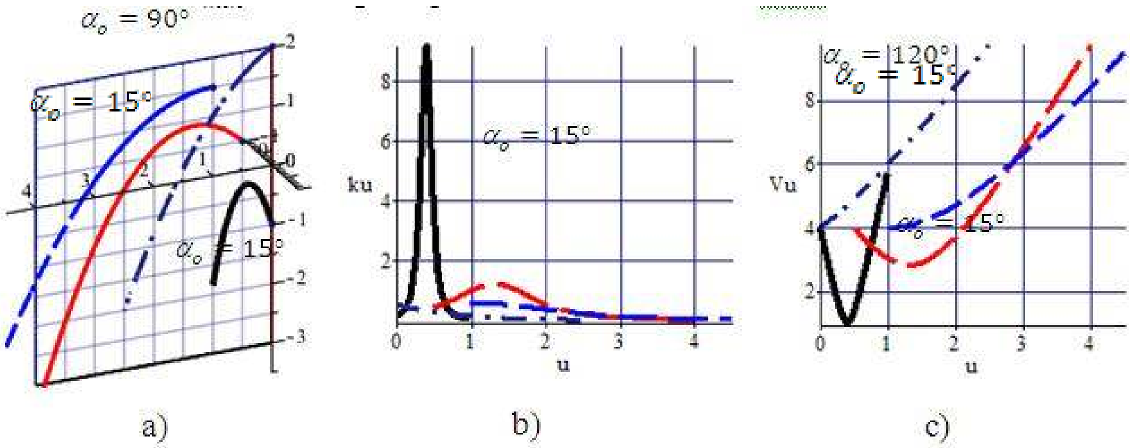

Figure. Trajectories $\mathbf{r}(u)$ curves of $k(u)$ and velocity $V(u)$ depending on the angle α_o of throwing in the function of the independent parameter u

Conclusions and perspectives. On the basis of the performed research it can be concluded that the motion of a particle in the vertical plane occurs in the same way as in the motion of a particle under the influence of the force of gravity in space.

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МОДЕЛЮВАННЯ РУХУ ЧАСТИНКИ ПО ВЕРТИКАЛЬНІЙ ПЛОЩИНІ У ФУНКЦІЇ ЇЇ ПОЛОЖЕННЯ

А. В. Несвідомін

Анотація. *Метою дослідження була розробка комп'ютерної моделі руху частинки по шорсткій вертикальній площині в середовищі символьної алгебри Maple та за допомогою обчислювальних експериментів з'ясовування її властивостей.*

Встановлено траєкторно-кінематичні характеристики руху частинок по шорсткій вертикальній площині. Наведено траєкторії частинки та графіки її швидкості залежно від положення та різних коефіцієнтів тертя і початкової швидкості.

На основі проведених досліджень можна зробити висновок, що рух частинки у вертикальній площині відбувається так само як під час руху частинки під дією сили земного тяжіння у просторі.

Ключові слова: ***рух частинки, вертикальна площина, система диференціальних рівнянь, траєкторія, швидкість, положення***

МОДЕЛИРОВАНИЕ ДВИЖЕНИЯ ЧАСТИЦЫ ПО ВЕРТИКАЛЬНОЙ ПЛОСКОСТИ В ФУНКЦИИ ЕЕ ПОЛОЖЕНИЯ

А. В. Несвидомин

Аннотация. *Целью исследования была разработка компьютерной модели движения частицы по шероховатой вертикальной плоскости в среде символьной алгебры Maple и с помощью вычислительных экспериментов выяснение ее свойств.*

Установлены траекторно-кинематические характеристики движения частиц по шероховатой вертикальной плоскости. Приведены траектории частицы и графики ее скорости в зависимости от положения и различных коэффициентов трения и начальной скорости.

На основе проведенных исследований можно сделать вывод, что движение частицы в вертикальной плоскости происходит так же, как и во время движения частицы под действием силы земного притяжения в пространстве.

Ключевые слова: ***движение частицы, вертикальная плоскость, система дифференциальных уравнений, траектория, скорость, положение***