КОМП’ЮТЕРНО-ІНТЕГРОВАНІ ТЕХНОЛОГІЇ, СИСТЕМИ ТА ЗАСОБИ АВТОМАТИЗАЦІЇ

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RELIABILITY ANALYSIS OF "SIBLING" COMPONENTS

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Statistical procedure is described for the use for the security analysis of "family" components.

SUMMARY & CONCLUSIONS

Engineering systems often contain some identical components (parts), the so–called "siblings". In the case of an automobile, these would be engine spark plugs, light bulbs, wheels, etc. These sibling components are typically coded with the same part number. When field (warranty) data are analyzed, a dilemma arises as to how to interpret a recurrent replacement of a sibling component belonging to a given system: as a secondary failure of the component that has already been replaced once, or as the first failure of the component's sibling(s)?

From the stand point of root–cause analysis, the task is to understand whether recurrent failures are related to 1) a particular sibling, which might be operating in inauspicious conditions relative to other siblings, or 2) to any other siblings on the vehicle. One could attribute Scenario 1) to a system–level (e.g., system interaction) problem, and Scenario 2) to a component–level (supplier quality) problem. This is also critical in selecting an appropriate probabilistic model for predicting the reliability of sibling components. In this paper, we propose a statistical approach that helps to resolve the above formulated dilemma.

1 INTRODUCTION

Without the loss of (engineering) generality, consider a 4–cylinder petroleum engine with four identical spark plugs, which hereafter will be referred to as *sibling parts (components)*, see Figure 1. All four spark plugs, for warranty tracking purposes, are coded with *the same* part number. This is unlike some other sibling components in the vehicle that may have special suffixes differentiating them (e.g., left and right mirrors, front and rear brake calipers, etc.).

Now consider two consecutive warranty claims coming from the same vehicle and containing the same part number. The question becomes how to interpret the second claim: as 1) a *repeat failure* of the spark plug that has already been replaced once, or 2) as the *first failure* of its siblings? One could attribute Scenario 1 to a system–level (e.g., system interaction) problem repeatedly causing the failure of a spark plug on a particular cylinder that might be creating stressful operating conditions – see Figure 2.

Scenario 2 could be interpreted as a component–level (e.g., supplier quality) problem "universally" affecting all spark plugs of the engine – see Figure 3.

Obviously, the problem becomes more statistically complex, when warranty claims are coming from a *population* of vehicles each containing sibling components. The correct choice between the two possible scenarios identified above provides an important engineering insight into the root–cause analysis of field failures. It also becomes critical in selecting an appropriate probabilistic model for predicting the reliability of sibling components.

The rest of the paper is structured as follows. Section 2 discusses probabilistic models suitable to model the two discussed scenarios for sibling components. Section 3 reviews a statistical procedure helping to decide between the two scenarios both for single and multiple repairable systems. Finally, Section 4 discusses numerical examples that illustrate the use of the proposed approach.

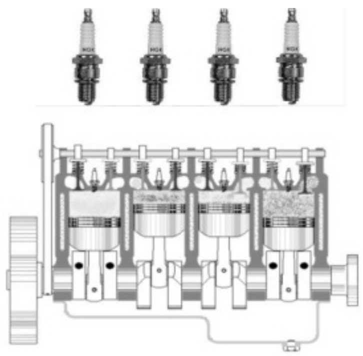

Figure 1 - An example of sibling components (spark plugs) coded with the same part number

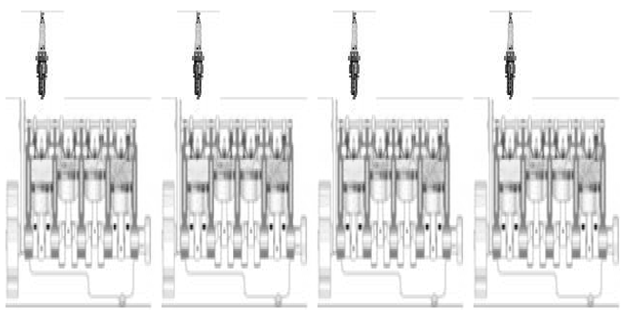

Figure 2 - A system–level problem repeatedly causing the failure of a spark plug of the 1st cylinder that might be creating stressful operating conditions for that spark plug

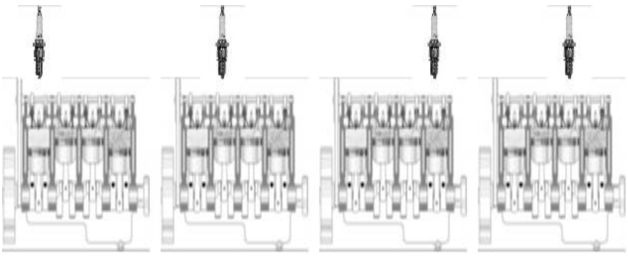

Figure 3 - A component–level (e.g., supplier quality) problem “universally” affecting spark plugs of various cylinders

2 PROBABILISTIC MODEL

Under Scenario 1, related to one particular sibling, the incremental (inter–arrival) time to a recurrent failure is an independently and identically distributed (IID) random variable, which results in the *ordinary renewal process* (ORP) – see Figure 4.

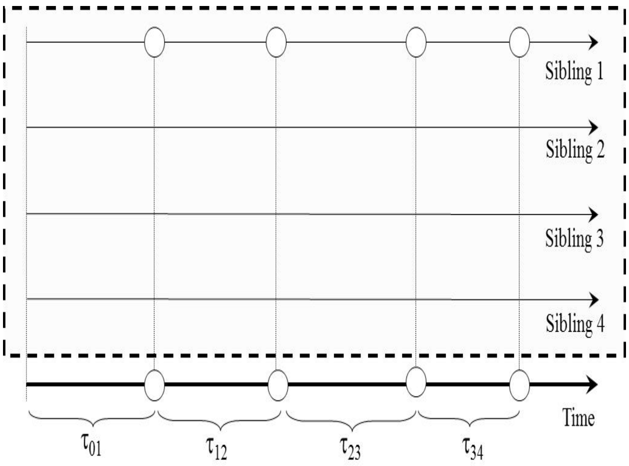

Figure 4 - Scenario 1 (single sibling failures) results in an Ordinary Renewal Process at the system–level

Under Scenario 2, of multiple sibling failures, the system–level failure process is the superposition of individual ORP’s corresponding to each sibling – see Figure 5. The incremental time to a recurrent failure, in this case, is *not* an IID, as the superimposed process, in general, is no longer a renewal process.

Therefore, if the time between recurrent failures is an IID random variable, then it’s an indication of Scenario 1; otherwise – Scenario 2. Alternatively expressed, in case of one particular sibling, the time to a recurrent failure is a realization from the underlying renewal process, whereas in the case of multiple siblings, the time to a recurrent failure is the difference between consecutive realizations from the underlying failure time distribution.

The power of the respective statistical procedure (to distinguish between the two scenarios) would be inversely proportional to the “proximity” of the siblings’ renewal processes to the Homogeneous Poisson Process (HPP), i.e., when the times between failures are exponentially distributed. This is because the superposition of individual HPP’s is also an HPP, i.e., a renewal process, in which case it becomes impossible to distinguish between Scenarios 1 and 2. Kaminskiy and Krivtsov (Ref. 1) discuss a Gini–Type index that helps to assess the proximity of a given renewal process to HPP.

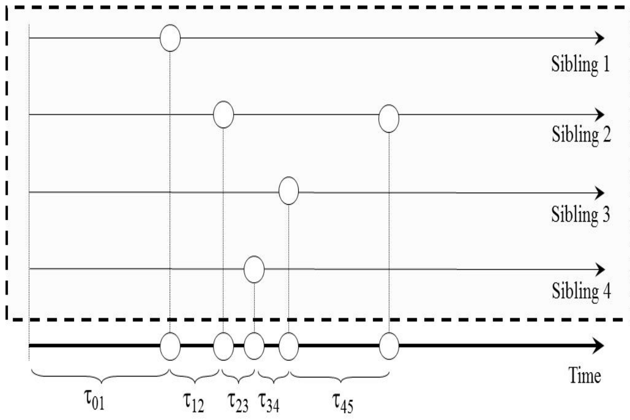

Figure 5 - Scenario 2 (multiple sibling failures) results in superposition of Ordinary Renewal Processes at the system–level, with the resulting process no longer being a renewal process

Assuming the nominally identical siblings, the probability of system failure is related to the probability of a sibling’s failure through the competing risk model:

$$p_s = 1 - (1 - p_c)^n \quad (1)$$

where: p_s and p_c are probabilities of system and sibling component failure, respectively, and n is the number of sibling components in the system.

3 STATISTICAL PROCEDURE

3.1 Single Repairable System

In the case of a single repairable system (e.g., a submarine’s engine) the test statistic can be based on whether or not the times between successive recurrent failures is an IID random variable. There are several tests designed to verify the IID assumption against specific alternatives, such as structural breaks, serial correlation, or autoregressive conditional heteroskedasticity (Ref. 2).

The simplest way to see if the inter–arrival times are identically distributed is to plot the cumulative number of recurrent failures as a function of time. Nonlinearity of such a plot would be an indication that inter–arrival times are *not* identically distributed. More formal is the Lewis–Robinson procedure (Ref. 3), which uses the following test–statistic:

$$Z = Z' \frac{\mu_\tau}{\sigma_\tau} \quad (2)$$

where μ_τ and σ_τ are the sample mean and standard deviation of the inter–arrival times, and

$$Z' = \frac{\sum_{i=1}^{\gamma}\left(\frac{t_i}{\gamma}\right) - \frac{\tau}{2}}{\tau\sqrt{\frac{1}{12\gamma}}} \quad (3)$$

is the test–statistic of the Laplace test (Ref. 5), where t_i denotes cumulative failure–arrival times, T – the length of the observation period and r – total number of failures. If failure times originate from a renewal process, then Z approximately follows the standard normal distribution, based on which a respective hypothesis testing can be conducted. If the observation period is terminated at the last failure, then in the above equation for the Laplace test, T is replaced with t_r , and r is replaced with $(r-1)$.

When assessing the adequacy of a renewal process, one has to also verify the assumption of *independence* of inter–arrival times. One method is to calculate a first–order serial correlation coefficient between the adjacent inter–arrival times (Ref. 4):

$$\rho_1 = \frac{\sum_{i=1}^{\gamma-1}(\tau_i - \mu_\tau)(\tau_{i+1} - \mu_\tau)}{\sqrt{\sum_{i=1}^{\gamma-1}(\tau_i - \mu_\tau)^2 \sum_{i=1}^{\gamma-1}(\tau_{i+1} - \mu_\tau)^2}} \quad (4)$$

When ρ_1 =0 and r is large, $\hat{\rho}\sqrt{r-1}$ approximately follows the standard normal distribution, based on which a respective hypothesis testing can be conducted.

3.2 Multiple Repairable Systems

In the case of multiple repairable systems (e.g., a population of automobiles) the test statistic can be based on comparing the distribution of times to the 1st occurrence to the distribution of inter–arrival times to the 2nd occurrence, 3rd occurrence, etc. This can be done both non–parametrically and parametrically.

For complete (non–censored) samples one can use the Kolmogorov–Smirnov test (Ref. 6–7) with the test statistic being based on the *supremum* of the set of distances between the two CDF's:

$$D_{n,n'} = \sup\left|F_{1,n}(t) - F_{2,n'}(t)\right| \quad (5)$$

where $F_1(t)$ and $F_2(t)$ are empirical CDF's of the first and the second samples of inter–arrival times, respectively, with the sample sizes of n and n', respectively.

For censored samples, we propose to construct the distribution of differences between empirical CDF's at a time cross–section of interest (say, t_0) :

$$D_{n,n'}(t_0) - F_{2,n'}(t_0) \quad (6)$$

by repeatedly bootstrapping from the two distributions under comparison. If this distribution of differences includes zero with a given probability (significance level), then the two distributions are not IID, and vice versa.

Alternatively, one can use parametric procedures to compare k distributions of failure inter–arrival times. One example is the likelihood ratio test (Ref. 8). The test statistic, for a two–parameter distribution, is

$$\chi = 2(L_1+L_1+\cdots+L_k-L_\beta) \quad (7)$$

where $L_1 \ldots L_k$ are the log–likelihood functions of the distributions of failure inter–arrival times to the 1st, 2nd, and k^{th} failures, respectively, and L_β is the combined log–likelihood function for a common shape parameter β, and scale parameters α_i:

$$L_\beta = L_1(\alpha_1\beta+\alpha_2\beta+\cdots=\alpha_k\beta) \quad (8)$$

Under the assumption of the equality of shape parameters, test statistic, χ approximately follows chi–square distribution with k–1 degrees of freedom, based on which respective hypothesis testing can be conducted.

4. NUMERICAL EXAMPLE

4.1 Single Repairable System

Consider a sample of system–level failure arrival times associated with sibling components, graphically depicted in Figure 6. As it follows from the figure, the failure times appear to have no trend, which would be an indication of a renewal process with identically distributed failure inter–arrival times.

The Laplace test statistic for this data set is Z = –1.07, and the Lewis–Robinson test statistic is Z = –1.34 with the associated p–value of 0.18 thus confirming a renewal process. The first–order serial correlation coefficient between the adjacent inter–arrival times is –0.011 and $\hat{\rho}\sqrt{r-1}$ = –0.035 with the associated p–value of 0.97. All this numerical evidence is an indication of Scenario 1: failures of one particular sibling, i.e., a system interaction (not a component–level) problem.

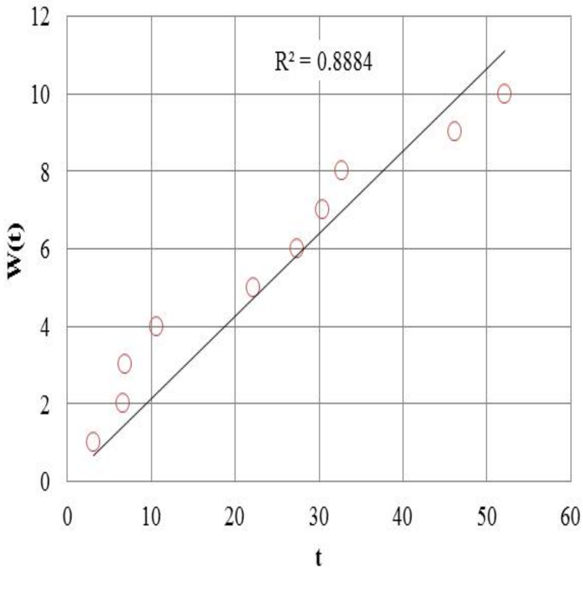

Figure 6 - Cumulative number of failures plotted as a function of the failure arrival times of a single repairable system

4.2 Multiple Repairable Systems

In Figure 7, consider the Kaplan-Meier estimates of the empirical CDF's of inter–arrival times to 1^{st} and 2^{nd} failures, respectively, coming from a population of repairable systems with sibling components.

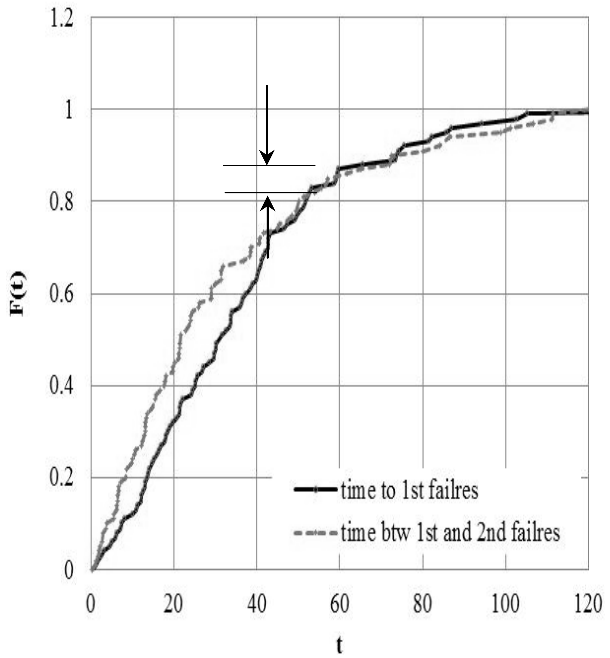

Figure 7 - Kaplan-Meier estimate of the empirical CDF's of the inter–arrival times to 1st and 2nd failures for a population of repairable systems

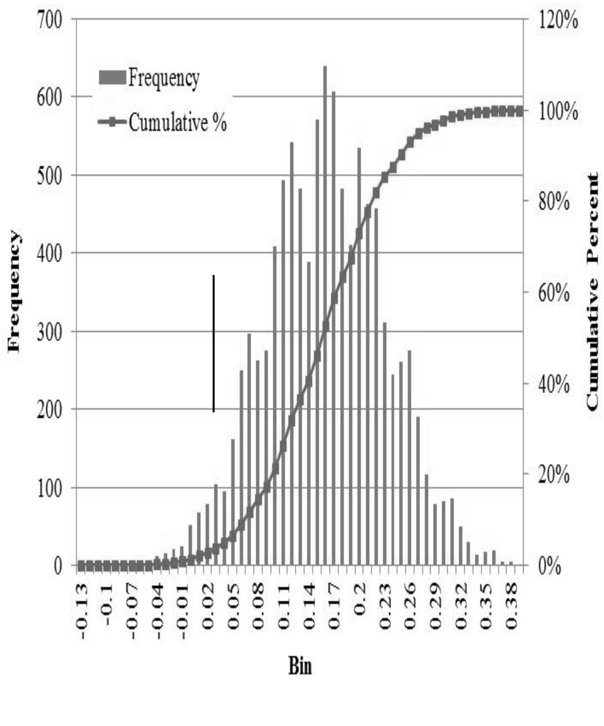

Figure 8 - Empirical distribution of differences between the inter–arrival times to 1^{st} and 2^{nd} failures to at t_0=40, based on n=10,000 bootstrap simulations

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Аннотация

АНАЛИЗ НАДЕЖНОСТИ "РОДСТВЕННЫХ" КОМПОНЕНТ

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Описывается статистическая процедура, рекомендуемая для использования при анализе надежности "родственных" компонентов

Анотація

АНАЛІЗ НАДІЙНОСТІ "СПОРІДНЕНИХ" КОМПОНЕНТ

Кривцов В. В., Франкштейн М. Я.

Описується статистична процедура, яка рекомендується для використання при аналізі надійності "споріднених" компонентів