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Estimating consumption and changes in stock of firewood and fuel oils applying household expenditure data

Abstract

Information on demand response is key information in evaluating the effectiveness of governmental environmental and energy policies. However, the consumption of storable goods such as firewood and fuel oils does not necessarily imply purchases during a period because of changes in stock. In many cases, we have information about expenditures only, not consumption. A method is developed to obtain an estimate of consumption and changes in stock when only expenditure data are available. In addition to expenditure data, the method requires discrete information about the utilization of available equipment complementary to the storable good in consumption. Household energy consumption is used as an illustration, applying data from the Norwegian Survey of Consumer Expenditure.

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JEL classification: C34, D12, Q41.

Introduction

Household consumption and production activities contribute to a significant share of climate gas emissions, and are hence an important target in climate policy. A range of policy instruments are implemented to move household energy consumption from electricity and fossil fuels to renewable energy, and increase energy efficiency. The effectiveness of these instruments depends on the households' responses to these instruments. To assure the efficiency of current and future policy efforts, analysis of energy demand responses are of great importance.

For nonstorable goods, such as electricity, the value of consumption during a period is equal to the expenditures. However, due to changes in stock, this is not necessarily true for storable goods. For some goods, such as firewood and fuel oils, the stock may last for years. For that reason, a consumer may have small expenditures on a storable good during a period with a relatively large consumption, or a positive expenditure without consumption.

In theory, the optimal stock and changes in stock are thoroughly discussed. However, empirical analyses of micro behavior with respect to changes in stock are rare[1]. The reason is that we seldom have consumption data or information about changes in stock, only expenditure data. Thus, we do not know how consumers actually allocate spending on storable goods on consumption and stock changes, or how consumers change their *consumption* of storable goods in response to price and income changes. The aim of this paper is to develop a method for estimating the effect of price, income and other characteristics of the household on consumption and changes in stock on storable goods when only expenditure data are available.

The literature on infrequency of purchases analyses the case where we are not able to observe all purchases due to a limited observation period, e.g. one or two weeks (Meghir and Robin, 1992; Robin, 1993). Since most purchases are not consumed immediately, but stored for a shorter or longer period, this will result in too many zero observations in the data. A method is developed to correct the estimation based on the purchase frequency. Even if infrequency of purchases occurs because goods are stored, our problem is somewhat different. We have information from the Norwegian Consumer Expenditure Survey (CES), where respondents are asked about their *annual* energy consumption. This means that we are able to observe all purchases during the year, but we do not know the purchase frequency. Secondly, we know that some households have chosen not to consume (corner solution) even if they have the opportunity to consume the good[2]. Finally, the infrequency of purchase models does not divide expenditures into consumption and changes in stock, which is our main aim.

Thus, we need to develop a method for estimating consumption and changes in stock when only information about annual expenditures are available. To be able to identify how expenditures are divided on consumption and changes in stock, we need additional information. In general, energy goods are used in combination with equipment to produce services. In our data, we are able to identify who owns this equipment, and thus has the opportunity to use firewood or/and fuel oils for heating purposes,

[1] The estimation of consumption and changes in stock when only expenditure data are available has, to this author's knowledge, not previously been discussed.

[2] In the literature on infrequency of purchases, corner solutions are often assumed not to exist (Meghir and Robin, 1992).

and who is utilizing this opportunity. We apply discrete information about whether the consumer utilizes the available heating equipment or not (i.e., chooses a corner solution) to identify the share of expenditures that is consumed and/or stored. For consumers who have the necessary equipment to consume the good, we model the choice of corner solutions in terms of a stochastic Kuhn-Tucker optimization problem similar to the specification in Wales and Woodland (1983). An Almost Ideal Demand System (AIDS) is used to describe the structure of the consumption functions (Deaton and Muellbauer, 1980). To distinguish between zero expenditure because the consumer does not own the necessary equipment (limited consumption opportunities) and corner solutions, we apply a Double Hurdle (DH) model (Garcia and Labeaga, 1996; Smith, 2002). The DH model is modified to fit the Kuhn-Tucker conditions; changes in stock are included and the model is extended to a multivariate simultaneous estimation.

1. Expenditure, corner solutions and changes in stock

Since the demand function does not necessarily equal the consumption function for storable goods, we need to model the relationship between consumption, changes in stock and expenditures on storable energy goods by consumers with the opportunity to consume.

1.1. The Kuhn-Tucker problem. We start by modeling the choice between utilizing the available heating equipment and choosing a corner solution. We assume that consumer h derives utility (U) from the consumption of a vector of goods (q^h), including energy goods (i = 1, 2, 3) and all other goods (i = 4) that are available to the consumer at a vector of prices (p^h). The utility function is assumed to be continuously differentiable, quasiconcave and increasing in the consumption of all goods. From the consumer's point of view, utility is assumed to be nonstochastic. However, from the researcher's point of view, utility is stochastic, as we assume differences in individual tastes to be randomly distributed across consumers. We also assume that consumers consider buying only those energy goods they have the opportunity to consume and that their choice of heating technology is not affected by changes in income and prices. That is, we study short-term effects on the utilization of the preexisting heating equipment.

The consumer is assumed to maximize utility with respect to the consumption of all available goods subject to his budget. We assume the total expenditure (x^h) to be less than or equal to total income (m^h); $x^h = p^{h\prime} q^h \leq m^h$, and that the consumer cannot have negative consumption of any good; $q^h \geq 0$. This gives the following optimization problem:

$$\max_{q^h} U^h(q^h) : p^{h\prime} q^h \leq m^h, \ q^h \geq 0. \quad (1)$$

As the utility function is increasing in the consumption of all goods, the consumer will use the entire income, and at least one good will be consumed. In this study, we focus on energy goods: electricity (i = 1), fuel oils (paraffin and fuel oil) (i = 2) and firewood (i = 3). Thus, we assume that the fourth good (i = 4), which contains consumption of all goods other than energy goods, is used as a reference good in this analysis. The necessary and sufficient conditions for this optimization problem may be written as follows (see e.g. Wales and Woodland, 1983):

$$\begin{aligned} &\frac{p_4^h U_i^{h\prime}}{m^h} - \frac{p_i^h U_4^{h\prime}}{m^h} \leq 0 \leq q_i^h, \quad i = 1,2,3, \\ &p^{h\prime} q^h = m^h. \end{aligned} \quad (2)$$

The consumption of good i equals zero if the marginal rate of substitution is less than the price ratio for all units of consumption: $U_i^{h\prime} / U_4^{h\prime} < p_i^h / p_4^h$. Otherwise, the consumer has a positive consumption of good i, and optimal consumption is characterized by equality between the marginal rate of substitution and the price ratio: $U_i^{h\prime} / U_4^{h\prime} = p_i^h / p_4^h$. That is, the household will have positive consumption of good i only if the marginal utility of consuming the first unit relative to the marginal utility of increasing other consumption exceeds the relative cost of this consumption. This leads to the consumption of all goods that the consumer has the opportunity to consume, as a function of all prices and income:

$$q_i^{h*} = q_i^h \left(p^h, m^h \right) \geq 0. \quad (3)$$

Specifying the choice of corner solutions stochastically, we follow the approach in Wales and Woodland (1983) and assume that marginal utility comprises common deterministic ($\underline{U}_i^{h\prime}$) and random ($\varpi_i^h$) components: $U_i^{h\prime} = \underline{U}_i^{h\prime} + \varpi_i^h$. The stochastic component is assumed to be independent and identically normally distributed with zero expectation and a constant variance $\left(\varpi_i^h \sim IIN\left(0, \sigma_{\varpi_i^h}^2\right), \forall h = 1, \ldots, H \right)$.

Using the Kuhn-Tucker condition for optimization, we express the probability of observing zero consumption of good i for consumer h as a function of whether the marginal rate of substitution is less than the price ratio, as follows:

$$P\left(q_i^{h*}=0\right)=P\left(p_4^h U_i^{h'} - p_i^h U_4^{h'} < 0\right)= = P\left(p_4^h \varpi_i^h - p_i^h \varpi_4^h \leq p_i^h \underline{U_4^{h'}} - p_4^h \underline{U_i^{h'}}\right) = \Phi\left(\underline{\psi_i^h}\right), \quad (4)$$

where $\underline{\psi_i^h} = p_i^h \underline{U_4^{h'}} - p_4^h \underline{U_i^{h'}}$. The probability of consuming good i is given by $P\left(q_i^{h*} > 0\right) = 1 - \Phi\left(\underline{\psi_i^h}\right)$. If the marginal rate of substitution between good i and good 4 increases, so that good i gives more utility relative to other consumption, both $\underline{\psi_i^h}$ and the probability of observing zero consumption of the good decrease.

1.2. Consumption. We assume that the value of the consumption on good i may be described by the functional form of an AIDS-model (Deaton and Muellbauer, 1980). Consumer h's budget share on the consumption of good i, $w_i^{h*} = \frac{q_i^{h*} p_i^h}{x^h}$, is given by[1]:

$$w_i^{h*} = \alpha_i^h + \sum_j^{J_h} \gamma_{ij}^h \log\left(p_j^h\right) + \beta_i^h \left[\log\left(x^h\right) - \log\left(P^h\right)\right], \quad (5)$$

where

$$\log P^h = \sum_{k=1}^{J_h} \alpha_k^h \log\left(p_k^h\right) + \tfrac{1}{2} \sum_{k=1}^{J_h} \sum_{j=1}^{J_h} \gamma_{jk}^h \log\left(p_k^h\right) \log\left(p_j^h\right),^2$$

P^h is a price index, x^h is the total budget of consumer h, and p_j^h is the price of good j for consumer h. Note that we sum over all goods that the consumer has the opportunity to consume, i.e., $k = 1, ..., J_h$ and $j = 1, ..., J_h$, where J_h is the number of goods that consumer h has the opportunity to consume. In this study, we focus on energy goods only, regarding the consumption of other goods ($i = 4$) as a residual consumption good.

The value of the consumption embedded in equation (5), $y_i^{h*} = q_i^{h*} p_i^h$, is assumed to be the sum of a deterministic component measuring the expected value of the consumption on good i (μ_i^h) and a stochastic component (ε_i^h), given by:

$$y_i^{h*} = \mu_i^h + \varepsilon_i^h = = \left[\alpha_i + \sum_j^3 \gamma_{ij} \log\left(p_j^h\right) OE_j^h + \beta_i \left(\log\left(x^h\right) - \log\left(P^h\right)\right)\right] x^h + \varepsilon_i^h. \quad (6)$$

In equation (5), only prices of goods that the household has the opportunity to consume enter the budget share function. To adjust the AIDS model for differences in consumption opportunities, we multiply the logarithms of all prices by a dummy variable (OE_j^h), indicating whether the consumer has the opportunity to consume energy good j. As the dummy equals zero for those who cannot consume good j, the prices of goods that cannot be consumed are excluded from the expenditure function in equation (6). Differences in demand response across consumers are represented by the stochastic term, ε_i^h. We assume that the stochastic term is independent and identically distributed with zero expectation, $E\left(\varepsilon_i^h\right) = 0$, but that the variance may vary across households, $E\left(\varepsilon_i^h \varepsilon_i^h\right) = \sigma_{h\varepsilon_i^h}$. Furthermore, we assume that the stochastic terms are uncorrelated across households and goods, $E\left(\varepsilon_i^h \varepsilon_j^r\right) = 0$.

As the consumption of a storable good does not always equal purchases during a period, we cannot lay restrictions of symmetry, homogeneity or additivity on the demand structure in equation (6). Thus, we cannot interpret our estimation results as we would in an ordinary AIDS estimation. This also implies that we cannot calculate the properties of the consumption function for the residual good ($i = 4$).

1.3. Expenditures on storable goods. We assume that the purchased quantity of the storable good i $\left(q_i^h\right)$ equals the consumption (q_i^{h*}) plus net changes in stock ($\Delta\ddot{q}_i^h$). This means that: $q_i^h = q_i^{h*} + \Delta\ddot{q}_i^h$. The consumer may want to change the stock by consuming from it, purchasing it for storage, or both. The net change in stock during a period depends on the size of the stock in the previous period ($\ddot{q}_{i,-1}^h$), prices (p^h) and income (m^h): $\Delta\ddot{q}_i^h = \Delta\ddot{q}_i^h\left(p^h, m^h, \ddot{q}_{i,-1}^h\right)$. Changes in the value of stock changes ($\Delta\ddot{y}_i^h = p_i^h \Delta\ddot{q}_i^h$) are assumed to consist of a deterministic component ($\Delta\underline{\ddot{y}_i^h}$) and a random component (υ_i^h): $\Delta\ddot{y}_i^h = \Delta\underline{\ddot{y}_i^h} + \upsilon_i^h$. The expenditure on storable goods is thus given by:

$$y_i^h = y_i^{h*} + \Delta\ddot{y}_i^h = \mu_i^h + \varepsilon_i^h + \Delta\underline{\ddot{y}_i^h} + \upsilon_i^h = \underline{y_i^h} + \vartheta_i^h, \quad (7)$$

where $\underline{y_i^h} = \mu_i^h + \Delta\underline{\ddot{y}_i^h}$ and $\vartheta_i^h = \varepsilon_i^h + \upsilon_i^h$. Since both random components (υ_i^h and ε_i^h) are assumed to be independent and identically normally distributed with zero expectations, the joint random term (ϑ_i^h) will be independent and identically normally distributed with a zero expectation and a variance given by: $\mathrm{var}\left(\vartheta_i^h\right) = \sigma_{h\vartheta_i^h}^2 = \sigma_{h\varepsilon_i^h}^2 + \sigma_{h\upsilon_i^h}^2$.

Here we allow the variance to vary between households. In particular, we suspect that the variance of

[1] These expenditure share functions are deduced from the expenditure functions in the cost minimization problem, but will equal the utility maximizing solution in optimum (discussed in section 1.1).

[2] Most empirical studies use a Stone index to approximate the price index within the AIDS model. Here, we use the full nonlinear price index directly in the estimation. The reason is that the Stone index may result in biased estimates, because it includes variables that are endogenous to the consumer (see Pashardes, 1993).

consumption may depend on the heating equipment. This is particularly true for households with a central heating system, who may have a larger variance than households with an oven burning firewood/ paraffin. This is because many households with central heaters may switch between electricity and fuel oil/firewood. Thus, some will use fuel oil/firewood only; some will use electricity only; whereas others use a combination.

This gives four different combinations of consumption and expenditures of storable goods, given the opportunity to consume the good:

- Case 1: Both consumption and expenditures are positive $\left(q_i^{h*} > 0 \wedge y_i^h > 0\right)$. In this case, the consumer may have a positive, negative or zero change in stock.
- Case 2: Consumption is positive and expenditures are zero, that is, the consumer is consuming from stock only ($q_i^{h*} > 0 \wedge y_i^h = 0$).
- Case 3: Consumption is zero while expenditures are positive, that is, all purchases are stored $\left(q_i^{h*} = 0 \wedge y_i^h > 0\right)$.
- Case 4: Both consumption and expenditures are zero $\left(q_i^{h*} = 0 \wedge y_i^h = 0\right)$.

As we can see, only in case (4) zero expenditure represent a corner solution in consumption. We may, however, also observe positive expenditures for consumers choosing a corner solution, as illustrated in case (3). The shares of households in our data in the different groups, given their consumption opportunities, are described in Appendix A.

2. The likelihood function

The stochastic properties described in section 2 are used to build a multivariate likelihood function accounting for differences in consumption opportunities, corner solutions and changes in stock.

2.1. The double hurdle model. The likelihood function is built around a DH model, to distinguish between consumers with different consumption opportunities. In a DH model, the probability density is a discrete-continuous mixture of consumers with positive expenditure and consumers with zero expenditure on a particular good:

$$f\left(y_i^h\right) = \begin{bmatrix} f_+\left(y_i^h\right) & if\ y_i^h > 0 \\ f_0 & if\ y_i^h = 0 \end{bmatrix}, \tag{8}$$

where the discrete component, f_0, is the probability mass measured at zero expenditure, and the continuous component, $f_+\left(y_i^h\right)$, is the density for consumers with a positive expenditure (see e.g. Smith, 2002; or Garcia and Labeaga, 1996; for a description of the DH model).

Since we assume that the stock of equipment is given in the short run (see the discussion in section 1.1), the consumer is assumed to have positive expenditure only if he has the opportunity to consume a good. The probability of positive expenditure on good *i* may thus be written as: $P\left(y_i^h > 0, OE_i^h = 1\right) = P\left(OE_i^h = 1\right) P\left(y_i^h > 0 \mid OE_i^h = 1\right)$. The probability of zero expenditure (f_0) is then $1 - P\left(OE_i^h = 1\right) P\left(y_i^h > 0 \mid OE_i^h = 1\right)$.

Since we focus on the utilization of already existing heating equipment, the choice of equipment ownership is predetermined. The choice of zero expenditure and equipment ownership is, thus, stochastically independent, because the stock of equipment is exogenous in this decision. Thus, the probability of a positive expenditure conditional on the possibility of consuming a particular good equals the marginal probability: $P\left(y_i^h > 0 \mid OE_i^h = 1\right) = P\left(y_i^h > 0\right)$.

This means that we apply a DH model with independence (see Garcia and Labeaga (1996) for a discussion). Given a short-run analysis where the stock of heating equipment is predetermined, the continuous part of the distribution is given by: $f_+\left(y_i^h\right) = f\left(y_i^h \mid y_i^h > 0\right) P\left(y_i^h > 0\right) P\left(OE_i^h = 1\right)$, where $f\left(y_i^h \mid y_i^h > 0\right)$ is the truncated density function of y_i^h.

Assuming expenditures to be independently and identically distributed, the likelihood function in the DH model is the product of all densities for all households, that is:

$$L = \prod_{h_+} f_+\left(y_i^h\right) \prod_{h_0} f_0 = \prod_{h_+} f\left(y_i^h \mid y_i^h > 0\right) P\left(y_i^h > 0\right) P\left(OE_i^h = 1\right) \prod_{h_0} \left[1 - P\left(OE_i^h = 1\right)\right] P\left(y_i^h = 0\right). \tag{9}$$

where h_+ is the set of consumers with a positive expenditure and h_0 is the set of consumers with zero expenditures on the good. This equals the Cragg specification of the DH model if the distributions are assumed to be normal (Cragg, 1971; Smith, 2002). Equation (9) represents the likelihood function for a single-equation DH model with independence.

2.2. Modifications of the double hurdle model. As noted by Smith (2002), it is assumed in the DH model that it is not possible to separate different sources of zero observations in the data. In our data, however, we are able to identify whether the consumer has zero expenditure because of limited consumption opportunities, corner solutions or consuming from stock.

We are also able to distinguish between consumers with both positive expenditure and consumption, and consumers with a corner solution and positive expenditure. Thus, we want to decompose both the discrete and the continuous parts of the DH model to take this information into account in the estimation. Applying the property that the discrete part of the density equals the probability of not having the opportunity to consume good *i*, $P\left(OE_i^h=0\right)$, and the probability of having the opportunity to consume but choosing a zero expenditure, $P\left(y_i^h=0\right)\left[1-P\left(OE_i^h=0\right)\right]$, the likelyhood function can be written as:

$$L=\prod_{\substack{OE_i^h=1\\ y_i^h>0}} f\left(y_i^h \mid y_i^h>0\right)P\left(y_i^h>0\right)P\left(OE_i^h=1\right)\prod_{\substack{OE_i^h=1\\ y_i^h=0}} P\left(y_i^h=0\right)P\left(OE_i^h=1\right)\prod_{OE_i^h=0} P\left(OE_i^h=0\right). \tag{10}$$

When incorporating changes in stock, we need to correct both the density and probability function of having a positive expenditure. For the continuous component, $f_+\left(y_i^h\right)$, we are either in case (1) where both consumption and expenditures are positive $\left(\left(q_i^{h*}>0 \wedge y_i^h>0\right)\right)$, or in case (3) where consumption is zero while expenditures are positive, that is, all purchases are stored ($q_i^{h*}=0 \wedge y_i^h>0$).

First, we look at the expressions for the standardized normal *density functions* given positive expenditure. In case i), the conditional density of observing positive expenditure and positive consumption of a storable good is

$$f\left(y_i^h \mid y_i^h>0 \wedge q_i^{h*}>0\right)=\frac{1}{\sigma_{h\vartheta_i^h}}\varphi\left(\frac{y_i^h-\underline{y_i^h}}{\sqrt{\sigma_{h\varepsilon_i^h}^2+\sigma_{h\upsilon_i^h}^2}}\right),$$

where $\sigma_{h\vartheta_i^h}=\sqrt{\sigma_{h\varepsilon_i^h}^2+\sigma_{h\upsilon_i^h}^2}$ is the standard variation of consumer *h*'s expenditures on good *i* in case (1), and $\frac{y_i^h-\underline{y_i^h}}{\sqrt{\sigma_{h\varepsilon_i^h}^2+\sigma_{h\upsilon_i^h}^2}}=\frac{\vartheta_i^h}{\sigma_{h\vartheta_i^h}}$ is the standardized error terms in this case. In case (3), where the consumer changes his stock without consuming the good, the conditional probability of a positive expenditure and zero consumption is

$$f\left(y_i^h \mid y_i^h>0 \wedge q_i^{h*}=0\right)=\frac{1}{\sigma_{h\upsilon_i^h}}\phi\left(\frac{y_i^h-\underline{\Delta\ddot{y}_i^h}}{\sigma_{h\upsilon_i^h}}\right).$$

Assuming expenditures on all available energy goods to be independent and identically normally distributed, using the expenditure system discussed in section 2 and the probability of choosing a corner solution discussed in section 1.1, the decomposition of the conditional density function in cases (1) and (3) is given by equation (11):

$$\begin{aligned}f\left(y_i^h \mid y_i^h>0\right)&=f\left(y_i^h \mid y_i^h>0 \wedge q_i^{h*}>0\right)P\left(q_i^{h*}>0\right)+f\left(y_i^h \mid y_i^h>0 \wedge q_i^{h*}=0\right)P\left(q_i^{h*}=0\right)=\\&=\frac{1}{\sigma_{h\vartheta_i^h}}\varphi\left(\frac{y_i^h-\underline{y_i^h}}{\sigma_{h\vartheta_i^h}}\right)\left(1-\Phi\left(\underline{\psi_i^h}\right)\right)+\frac{1}{\sigma_{h\upsilon_i^h}}\varphi\left(\frac{y_i^h-\underline{\Delta\ddot{y}_i^h}}{\sigma_{h\upsilon_i^h}}\right)\Phi\left(\underline{\psi_i^h}\right).\end{aligned} \tag{11}$$

We also need an expression for the probability of observing a positive expenditure. We assume that the utility of the expenditures on the storable good, $\ddot{V}_i^{h*}=V\left(y_i^h\right)$, consists of a common deterministic ($\underline{\ddot{V}_i^{h*}}$) and a random ($\ddot{v}_i^{h*}$) component: $\ddot{V}_i^{h*}=\underline{\ddot{V}_i^{h*}}+\ddot{v}_i^{h*}$. The stochastic component is assumed to be independent and identically normally distributed with zero expectation and a heteroscedastic variance $\left(\ddot{v}_i^{h*}\sim IIN\left(0,\sigma_{h\ddot{v}_i^{h*}}^2\right), \forall h=1,....,H\right)$. We assume that the consumer will choose case (1) if the indirect utility of both positive expenditure and positive consumption is positive. In this case, we may express the probability of choosing case (1) by

$$P\left(y_i^h>0 \mid q_i^{h*}>0\right)=P\left(\underline{\ddot{V}_i^{h*}}+\ddot{v}_i^{h*}\geq 0\right)=P\left(-\ddot{v}_i^{h*}\leq\underline{\ddot{V}_i^{h*}}\right)=P\left(\underline{\ddot{V}_i^{h*}}\right).$$

We furthermore assume that the deterministic component equals the value of the consumption and changes in stock multiplied by a welfare weight on consumption and stock changes, respectively, $\left(\gamma_i^{h*},\ddot{\gamma}_i^h\right)$: $\underline{\ddot{V}_i^{h*}}=\gamma_i^{h*}\mu_i^h+\ddot{\gamma}_i^h\Delta\underline{\ddot{y}_i^h}$. From this, we may write the conditional probability of observing case (1) as $P\left(\underline{\ddot{V}_i^{h*}}\right)=\Phi\left(\gamma_i^{h*}\mu_i^h+\ddot{\gamma}_i^h\Delta\underline{\ddot{y}_i^h}\right)$. In case (3), when the consumer changes his stock without consuming the good, the indirect utility, $\ddot{V}_i^h=V\left(\Delta\ddot{y}_i^h\right)$, is assumed to be given by $\ddot{V}_i^h=\underline{\ddot{V}_i^h}+\ddot{v}_i^h$. The stochastic component is assumed to be distributed as $\ddot{v}_i^h\sim IIN\left(0,\sigma_{\ddot{v}_i^h}^2\right)$, $\forall h=1,....,H$. We may express the conditional probability of observing case (3) as $P\left(y_i^h>0 \mid q_i^{h*}=0\right)=P\left(-\ddot{v}_i^h\leq\underline{\ddot{V}_i^h}\right)=\Phi\left(\ddot{\gamma}_i^h\Delta\underline{\ddot{y}_i^h}\right)$. Using this and the probability of choosing a corner solution from the Kuhn-Tucker condition, discussed in section 1.1, the probability of observing a positive expenditure may be written as:

$$\begin{aligned} P\left(y_i^h>0\right) &= P\left(y_i^h>0 \mid q_i^{h*}=0\right)P\left(q=0\right)+P\left(y_i^h>0 \mid q_i^{h*}>0\right)P\left(q_i^{h*}>0\right)= \\ &= \Phi\left(\dddot{\gamma}_i^h \Delta\underline{\dddot{y}_i^h}\right)\Phi\left(\underline{\psi_i^h}\right)+\Phi\left(\gamma_i^{h*}\mu_i^h+\dddot{\gamma}_i^h\Delta\underline{\dddot{y}_i^h}\right)\left(1-\Phi\left(\underline{\psi_i^h}\right)\right) \end{aligned} \quad (12)$$

Inserting Equations (11) and (12) into equation (10), denoting the share of a consumer with the opportunity to consume good i as κ_i, we obtain the following likelihood function for good i:

$$L_i=\begin{cases} \prod\limits_{\substack{OE_i^h=1\\ y_i^h>0}} \left[\begin{array}{l}\left[\dfrac{1}{\sigma_{h\vartheta_i^h}}\varphi\left(\dfrac{y_i^h-\underline{\underline{y_i^h}}}{\sigma_{h\vartheta_i^h}}\right)\left(1-\Phi\left(\underline{\psi_i^h}\right)\right)+\dfrac{1}{\sigma_{h\upsilon_i^h}}\varphi\left(\dfrac{y_i^h-\underline{\underline{\Delta\dddot{y}_i^h}}}{\sigma_{h\upsilon_i^h}}\right)\Phi\left(\underline{\psi_i^h}\right)\right]\cdot \\ \left[\Phi\left(\dddot{\gamma}_i^h\Delta\underline{\dddot{y}_i^h}\right)\Phi\left(\underline{\psi_i^h}\right)+\Phi\left(\gamma_i^{h*}\mu_i^h+\dddot{\gamma}_i^h\Delta\underline{\dddot{y}_i^h}\right)\left(1-\Phi\left(\underline{\psi_i^h}\right)\right)\right]\end{array}\right]\kappa_i \times \\ \\ \prod\limits_{\substack{OE_i^h=1\\ y_i^h=0}} 1-\left[\Phi\left(\dddot{\gamma}_i^h\Delta\underline{\dddot{y}_i^h}\right)\Phi\left(\underline{\psi_i^h}\right)+\Phi\left(\gamma_i^{h*}\mu_i^h+\dddot{\gamma}_i^h\Delta\underline{\dddot{y}_i^h}\right)\left(1-\Phi\left(\underline{\psi_i^h}\right)\right)\right]\kappa_i \times \\ \prod\limits_{OE_i^h=0}\left(1-\kappa_i\right) \end{cases} \quad (13)$$

Unfortunately, we are not able to decompose the probability of observing a zero expenditure given the opportunity to consume the good, by households who consume from stock only (case (2)) and households who choose a corner solution (case (4)). This is because we do not have any continuous information about expenditures, consumption or stock changes for this group of households. Thus, cases (2) and (4) is treated as one group in the likelihood function.

The likelihood function in equation (13) is specified for a single good. To find the likelihood function attached to all commodities (apart from the residual one, $i = 4$), we decompose the simultaneous multivariate density for the expenditure on all energy goods, $f\left(y_1^h, y_2^h, y_3^h\right)$, into its conditional counterparts assuming that the expenditures on different goods are uncorrelated across households and goods, that is, $E\left(\varepsilon_i^h \varepsilon_j^r\right)=0$ for all $h \neq r$ and $i \neq j$[1]. In this case, the simultaneous density is the product of the marginal densities: $f\left(y_1^h, y_2^h, y_3^h\right)=f_1\left(y_1^h\right) f_2\left(y_2^h\right) f_3\left(y_3^h\right)$. Using this, assuming expenditures on all available energy goods to be simultaneously normally distributed, we obtain an expression for the simultaneous multivariate likelihood function to be estimated. See Appendix B for a more detailed description of the simultaneous likelihood function.

3. Empirical illustration

To illustrate how the model may be used to obtain estimates of consumption and changes in stock, we estimate the annual household consumption of electricity, firewood and fuel oils based on Norwegian expenditure data.

3.1. The data. The main data source is the annual Norwegian SCE, with an additional questionnaire concerning energy consumption for the years 1993, 1994 and 1995. The data set contains information on 3,511 individual households. It includes information about the purchase of, and expenditure on, paraffin, fuel oil and firewood, as well as electricity expenditure for the 12 months prior to the interview. The data include information about household total consumer expenditure and household gross income during the past 12 months. An additional energy questionnaire to the SCE for these years contains information about the available heating technology, whether the household utilizes the different heating technologies that are available to them and capacity of the existing heating equipment to heat the residence on a cold winter day. The SCE also contains information about characteristics of the household and the residence such as the type of residence (apartment block, detached house, farmhouse, etc.).

Oil and firewood prices are obtained from the SCE, calculated as expenditure divided by the physical amount of purchases for households reporting both a positive expenditure and a positive purchased amount of the good. These prices are averaged by county and applied to households in that county that do not have both positive expenditures on, and a purchased amount of, firewood and/or fuel oils. Information on electricity prices is collected from the households' individual electricity suppliers and the Norwegian Water Resources and Energy Directorate. If price information for a household is missing, the mean price of all power suppliers distributing to the household's area of residence (municipality) is allocated to the household[2].

[1] We have tested the correlation between the residuals of the electricity, fuel oil and firewood functions in the estimation, and they are very close to zero. This indicates that this assumption is likely to hold.

[2] In this period, most households used their local power distributor.

In the model, the probability of choosing a corner solution is a function of the marginal rate of substitution in optimum and the relative price on the energy good (i = 1, 2, 3) and all other goods (i = 4). The marginal rate of substitution between the energy good and all other consumption is not observable. Thus, we need an instrument that is correlated with the marginal rate of substitution. It is reasonable to believe that the marginal rate of substitution will vary with the substitution possibilities between energy goods in residential space heating. If substitution possibilities are good, the marginal rate of substitution is likely to be lower, and the probability of a corner solution for a particular good is higher given the relative prices. We have information about the available stock of heating equipment and its capacity to heat the residence on cold winter days, which is used as a proxy for the marginal rate of substitution. We have information about the price level of all energy goods. We do not, however, have information about the price level of all other goods (good 4). Since we have data from three different years, we use the consumer price index to calculate all prices, income and expenditures in 1995 values.

In the model, change in stock is assumed to be a function of last year's stock, prices and income. We would particularly expect the own price to be important, as it is reasonable to believe that consumers add to stock when the own price is relatively low (over time) and consume from stock in periods when the price is high, given the decision to consume. Unfortunately, we do not have *individual* price information about energy prices in *previous* periods in our data. The household's opportunity to store firewood and fuel oils is likely to depend on the type of residence, since storage opportunities are more limited in blocks than in detached houses. Furthermore, it is also reasonable to assume that households with central heating systems have larger storage facilities than households with wood- or oil-burning stoves. Some households with central heating systems have large fuel tanks (up to 4,000 liters) in their cellars or buried in their yards. Unfortunately, we do not have information about the storage capacity and level of the previous year's stock in our data. We thus use information about the type of residence and a dummy for a central heating system as a proxy for the size of the storage facilities.

3.2. Estimation results. These data are used to estimate parameters in the consumption functions and changes in the stock of firewood and fuel oils for the mean household. Maximization of the likelihood function is performed by applying the MINIMIZE procedure in LIMDEP (Greene, 1995) on the simultaneous log-likelihood function described in Appendix B, under the restriction that consumption of all the commodities must be nonnegative. The results from this estimation are presented in Table 1.

Table 1. Results from a simultaneous ML estimation of expenditures on, and consumption of electricity and changes in stock of firewood and fuel oils (1000 NOK)

	Electricity	Firewood	Fuel oils
A) Consumption ($\underline{y}_i^h$)			
Constant	0.544***	0.250***	0.544***
Electricity price (øre per kWh)	-0.018	0.031***	-0.067***
Firewood price (øre per kWh)	0.010***	0.000	-0.004**
Fuel oil price (øre per kWh)	-0.016***	-0.008***	0.012
Household gross income (10,000 NOK)	-0.145***	-0.029***	-0.001
B) Probability of positive consumption ($\underline{\psi}_i^h$)			
Constant		0.431**	0.164
Total heating capacity (1, 2, 3, ...)		0.079***	
Electric heating capacity (1, 2, 3, ...)			-0.057***
Own price (øre per kWh)		-0.004***	0.000
Electricity price (øre per kWh)		0.007*	0.0134**
Central heating system (0, 1)		-0.207*	0.2700***
C) Changes in stock ($\Delta\underline{\ddot{y}}_i^h$)			
Constant		1.979***	2.827***
Detached houses (0, 1)		-0.293***	0.568***
Apartment block (0, 1)		-0.911***	-0.649
Central heating system (0, 1)		1.366***	4.661***
Positive consumption		-0.693**	-0.876
Household gross income (10,000 NOK)		-0.008*	
D) Welfare weights ($\gamma_i^h, \ddot{\gamma}_i^h$)			

Table 1 (cont.). Results from a simultaneous ML estimation of expenditures on, and consumption of electricity and changes in stock of firewood and fuel oils (1000 NOK)

	Electricity	Firewood	Fuel oils
Consumption		1.128***	0.540
Changes in stock		2.504***	1.351***
E) Standard deviation (σ)			
Positive consumption			
Constant	4.101***	2.110***	1.716***
Central heating system (0, 1)		0.896***	2.088***
Corner solution		1.609***	2.551***

Note: *, **, or *** are significant at the 10, 5 and 1 percent levels, respectively.

The table shows that most estimated parameters are significant. Starting with the decision whether to consume B), we find the capacity of various heating equipment to be important, but the mechanism varied between firewood and fuel oils. For the decision to use firewood, it is the total capacity of the heating system that is of importance. This is because firewood is used in combination with electricity to heat the residence. For fuel oils, it is the capacity of the electric heating equipment that is of importance. This is because much of the fuel oil consumption is used in central heating systems (which is not so common for firewood), and thus used as a pure alternative to electricity. After the oil price shocks of the early 1970s and early 1980s, fuel oil was replaced by electric heating (either in the existing central heating systems or by separate electric heaters). Consumers with high capacities on their electric heating systems are less likely to use fuel oil to heat their residences than those with low electric capacities. We also see that an increase (decrease) in the own price reduces (increases) the probability of consumption, although the effect of the own price on fuel oil is not statistically significant. Finally, owning a central heating system has the opposite effect on firewood and fuel oil. The negative coefficient of households owning central heating systems with firewood indicates that these households are less inclined to use the wood burners on their central heaters than are households that own wood-burning stoves[1].

Given that the household decides to consume the good, we see that many coefficients in the consumption functions are significant (see Section A of Table 1). The negative signs of the income coefficients imply that energy goods are considered a necessity good[2]. The estimated coefficients are significant for electricity and firewood. The cross-price effects between electricity and firewood are significant and positive (positive coefficients) whereas the cross-price effects between fuel oils and all other goods are negative. From previous analyses of these data, we know that these negative signs are due to income effects and not because electricity and fuel oils are seen as complements in consumption (Halvorsen et al., 2010). Finally, we see that none of the own price effect on consumption are significant. Thus, the *consumption* decision seems to be independent of variations in the own price.

The estimated parameters for changes in firewood stock are all significant, whereas some are not for fuel oils (see Section C of Table 1). We see that households that are consuming the good are purchasing less for storage than are other households, but this effect is significant only for firewood. Households living in detached houses and apartment blocks are purchasing less firewood for storage compared with other households (farmhouses are used as reference). This is because farmhouses use a lot of firewood for heating and, presumably, have the largest storage opportunities. The coefficient is largest for apartment blocks, as expected. Households with central heating systems running on the storable good in question purchase more of both firewood and fuel oils for storage compared with other households. Finally, there is a tendency that wealthier households do not buy as much for storage, as they are less vulnerable to price changes. For fuel oils, living in a detached house increases the purchases for stock compared to other types of houses. Also, owning a central heating system significantly increases the purchases for stock.

Looking at the estimation of the standard deviations (Section E of Table 1), we find that households with a central heating system, both for firewood and fuel oil, have a significantly larger variance in their consumption and changes in stock than other households.

4.3. Predictions of consumption and changes in stock. The empirical results presented in Table 1 are used to predict the value of consumption, changes in stock and purchases of firewood and fuel oils as well as the consumption of electricity for all households in the sample. The estimated value of consumption is predicted for each household by using the estimated parameters in Section A of Table 1 on the AIDS model in equation (6)

[1] Wood burners on central heating systems are rare in Norway, and are often installed in combination with either oil and/or electric burners.

[2] See, e.g., Deaton and Muellbauer (1980) for more information on how to interpret coefficients from an AIDS model.

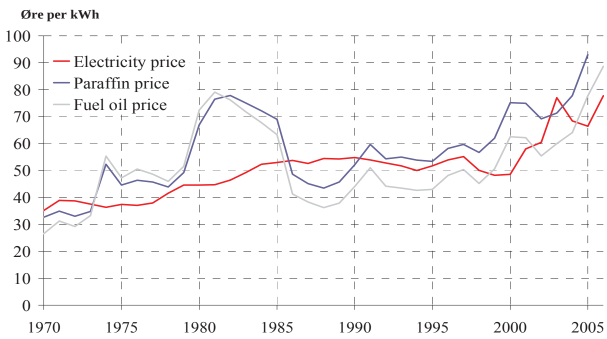
and the relationship between the value of consumption and changes in stock, and expenditures. Changes in the value of the stock for households in group (1) are found by using the estimates in Section C of Table 1, assuming changes in the value of the stock to be a linear function of the explanatory variables. For households in group (2), we assume that the reduction in the value of the stock equals the value of consumption (since they have zero expenditure). For households in group (4), both consumption and changes in stock are zero. Total expenditures are the sum of the value of consumption and changes in stock. The mean results (converted into kWh)[1] are presented in Table 2[2].

Table 2. Mean predicted expenditures on, consumption of, and changes in stock of energy goods in different samples (kWh)

	All	(1) Positive consumption and positive expenditures	(2) Positive consumption and zero expenditures	(3) Zero consumption and positive expenditures
Firewood				
Consumption	1 434	2 296	2 163	0
Changes in stock	623	1 561	-2 163	2 911
Purchases	2 057	3 857	0	2 911
Fuel oils				
Consumption	329	1 815	1 753	0
Changes in stock	915	6 099	-1 753	6 834
Purchases	1 238	7 895	0	6 834

We see that, on average, approximately 70 percent of the purchased quantity of firewood and 27 percent of the purchased quantities of fuel oil are consumed; the rest is stored. Looking at the different sub-samples, we find that households in group (1), who both has a positive consumption and expenditure, are increasing their stores of both firewood and fuel oils in this period. This increase is particularly large for the stores of fuel oils, where more than three quarters of all purchases are stored for late use. With respect to firewood, we see that forty percent of purchases are stored. Looking at the group (2), who are only consuming from stores, we see that the consumption of both firewood and fuel oils are lower, but still close to that of group (1). With respect to group (3), who store all their purchases, we see that changes in stores are higher but purchases are lower than for households who also consume (group (1)), both for firewood and fuel oils.

These results indicate that many households that purchased firewood and fuel oils in this period ended up using electricity to heat their residences, storing the fuel for later use. The large increases in stock, in particular for fuel oils, may sound surprising, because it is reasonable to believe that the purchases will equal consumption over time. This result must, however, be seen in the light of the time period it is estimated on. During the early 1990s, the use of electricity for space heating became common in Norway after the high oil prices in the early 1980s (see Figure 1), increasing the substitution opportunities. We also see from Figure 1 that there is a reduction in the price of fuel oils from 1992 to 1993, when our data begin, after a rather rapid price increase a couple of years earlier. It is thus reasonable that some households use this opportunity to fill their oil tanks for later consumption, even if electricity is relatively cheep in this period.



Source: Statistics Norway.

Fig. 1. Prices on electricity, fuel oil and paraffin (1960-2006)

Despite this, the consumption of fuel oils seems small compared with changes in stock, and we may have a problem identifying the consumption function for fuel oils in the estimation for several reasons. First, only 18 percent of the sample actually consumed fuel oils. Another problem is that we look at fuel oils combined, which is used both in central heating systems and in separate paraffin ovens. It is reasonable to believe that there is large heterogeneity in both the consumption and the storage decision between these groups. We have allowed the variance to differ across the groups, but there may be additional differences that we have not accounted for. Unfortunately, our estimation program (LIMDEP) does not allow us to treat fuel oil for central heating systems

[1] The value predictions (in NOK) are converted into kWh by dividing the mean price.

[2] Since the restriction, that purchases equal consumption plus changes in stock, is laid on the individual households (and not on the mean), this relation is not necessarily fulfilled for the mean of all households.

and paraffin consumption separately because of capacity restrictions. Finally, we are not able to decompose the probability of observing a zero expenditure given the opportunity to consume the good, by households who consume from stock only (case (2) and households who choose a corner solution (case (4) in the estimation. This is because we have no continuous information to compare these groups. Thus, important information about the storage and consumption decision may be lost. However, if we may assume that households in group (1) and (2) may be represented by the same the consumption function, this will not bias the results. This implies that the consumption decision is assumed to be independent of changes in stock. This will be fulfilled if the household consume continuously from stock, and fill the stores when stocks are running low and need to be filled before the heating season (given relative energy prices).

Concluding remarks

The main aim of this paper has been to implement a method for estimating consumption and changes in stock of storable goods based on expenditure data. This is done by utilizing discrete information about consumption opportunities, whether the consumers are buying and/or consuming the good, in addition to information about actual expenditures. This method is illustrated by applying data from the Norwegian Survey of Consumer Expenditure for the years 1993, 1994 and 1995, which includes an additional questionnaire about energy use.

The empirical results indicate that this is a period when much of the storable energy goods that were purchased were stored for later use, as we estimate a net increase in stock. We find that the probability of consuming these goods increases with the substitution possibilities embedded in the capacity of the heating system installed. We also saw that the probability of consuming the good decreases with the relative cost of the good. Both results are as expected from the Kuhn-Tucker conditions of choosing a corner solution.

In this study, the method is applied to household energy consumption. This method will, however, also be relevant in other cases where households have the possibility to store the good in question, and where the stock may last longer than the observation period. We only need additional information about who is choosing a corner solution. The approach may also be applied if some households may be identified to belong to a group with a particular consumption pattern, e.g. households with small children. In this case, we may assume that all households within the group consume the good (e.g. diapers), whereas all other households do not consume the good (choose a corner solution). Furthermore, in the case where corner solutions do not exist and all households consume the good, we may rewrite the likelihood function by including the information that the probability of consuming the good equals one. In these cases, a modified version of this approach may be used as an alternative to infrequency of purchases methods for example when we only have registered information for two weeks of purchases, but where the stock may last longer than that (consumption of frozen food, flour, canned food, etc.). To do this, we reduce the period to the observation period (e.g. two weeks), and predict consumption and changes in stock for this period. Based on these results, we may calculate predictions for annual consumption. Thus, with some simple modifications, this approach may be applied to an array of problems where the value of consumption does not equal expenditures during the observation period.

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Appendix A. Variation in expenditures and utilization of available equipment

In the estimation, the variation in the discrete information about the opportunity to consume, actual consumption and positive expenditures are used to decompose the expenditures on consumption and changes in stock. Table B1 shows the distribution of these characteristics across households in the sample. The table also shows the distribution of these attributes in different subsamples, in addition to the full sample. The four subsamples are: (a) households with positive firewood expenditures; (b) households with positive firewood consumption; (c) households with positive fuel-oil expenditures; and d) households with positive fuel-oil consumption.

We see from the table that 80 percent of the households in the sample had the opportunity to consume firewood, and 28 percent had the opportunity to consume fuel oils. In the sample, 55 percent had expenditures on firewood, whereas 63 percent had positive consumption. This means that a considerable number of households with positive consumption are consuming from the stock only. If we look at the subsamples, we see that only 75 percent of households that consumed firewood did actually buy firewood during the previous 12 months. On the other side, several households with positive expenditures on firewood did not actually consume the good (14 percent), but purchased for storage only. Only 16 percent of the households had expenditures on fuel oils, whereas 18 percent had a positive consumption. Of the households that did consume fuel oils, 72 percent of households had a positive expenditure, whereas 19 percent of households with a positive expenditure did not consume the good.

Table A1. Share with consumption opportunities, expenditure and consumption of firewood and fuel oils in different subsamples (percentages)

	All	(a) Positive firewood expenditure	(b) Positive firewood consumption	(c) Positive fuel-oil expenditure	(d) Positive fuel-oil consumption
Opportunity to consume firewood	80	100	100	81	86
Expenditures on firewood	55	100	75	44	47
Consumption of firewood	63	86	100	56	68
Opportunity to consume fuel oils	28	24	27	100	100
Expenditures on fuel oils	16	13	14	100	72
Consumption of fuel oils	18	16	20	81	100

A large group of households is also able to use both firewood and fuel oils to heat their residences, and several of these households do. We see that approximately half of the households that use or purchase fuel oils also use or purchase firewood. The percentage of households that purchase or use fuel oils is also approximately the same as for the overall mean.

Appendix B. The simultaneous likelihood function

The simultaneous likelihood function in the estimation is given by:

$$
L = \left\{\prod_h \frac{1}{\sigma^h_{\varepsilon 1}}\varphi\left(\frac{y_1^h-\mu_1^h}{\sigma_1^h}\right)\right\}\times
$$

$$
\left\{\begin{array}{l}
\prod_h \left\{\begin{array}{l}\left[\frac{1}{\sigma^h_{\vartheta 2}}\varphi\left(\frac{y_2^h-\underline{\underline{y_2^h}}}{\sigma^h_{\vartheta 2}}\right)\left(1-\Phi\left(\underline{\psi_2^h}\right)\right)B_2^h+\frac{1}{\sigma^h_{\upsilon 2}}\varphi\left(\frac{y_2^h-\Delta\underline{\underline{\dddot{y}_2^h}}}{\sigma^h_{\upsilon 2}}\right)\Phi\left(\underline{\psi_2^h}\right)\left(1-B_2^h\right)\right] \\ \left[\Phi\left(\dddot{\gamma}_2^h\Delta\underline{\dddot{y}_2^h}\right)\Phi\left(\underline{\psi_2^h}\right)\left(1-B_2^h\right)+\Phi\left(\gamma_2^{h*}\mu_2^h+\dddot{\gamma}_2^h\Delta\underline{\dddot{y}_2^h}\right)\left(1-\Phi\left(\underline{\psi_2^h}\right)\right)B_2^h\right]\kappa_2\end{array}\right\}^{D_2^hOE_2^h}\cdot \\
\prod_h \left\{\left[1-\left[\Phi\left(\dddot{\gamma}_2^h\Delta\underline{\dddot{y}_2^h}\right)\Phi\left(\underline{\psi_2^h}\right)\left(1-B_2^h\right)+\Phi\left(\gamma_2^{h*}\mu_2^h+\dddot{\gamma}_2^h\Delta\underline{\dddot{y}_2^h}\right)\left(1-\Phi\left(\underline{\psi_2^h}\right)\right)B_2^h\right]\right]\kappa_2\right\}^{\left(1-D_2^h\right)OE_2^h}\cdot \\
\prod_h \left(1-\kappa_2\right)^{\left(1-OE_2^h\right)}
\end{array}\right\}\times
$$

$$
\times\left\{\begin{array}{l}
\prod_h \left\{\begin{array}{l}\left[\frac{1}{\sigma^h_{\vartheta 3}}\varphi\left(\frac{y_3^h-\underline{\underline{y_3^h}}}{\sigma^h_{\vartheta 3}}\right)\left(1-\Phi\left(\underline{\psi_3^h}\right)\right)B_3^h+\frac{1}{\sigma^h_{\upsilon 3}}\varphi\left(\frac{y_3^h-\Delta\underline{\underline{\dddot{y}_3^h}}}{\sigma^h_{\upsilon 3}}\right)\Phi\left(\underline{\psi_3^h}\right)\left(1-B_3^h\right)\right] \\ \left[\Phi\left(\dddot{\gamma}_3^h\Delta\underline{\dddot{y}_3^h}\right)\Phi\left(\underline{\psi_3^h}\right)\left(1-B_3^h\right)+\Phi\left(\gamma_3^{h*}\mu_3^h+\dddot{\gamma}_3^h\Delta\underline{\dddot{y}_3^h}\right)\left(1-\Phi\left(\underline{\psi_3^h}\right)\right)B_3^h\right]\kappa_3\end{array}\right\}^{D_3^hOE_3^h}\cdot \\
\prod_h \left\{\left[1-\left[\Phi\left(\dddot{\gamma}_3^h\Delta\underline{\dddot{y}_3^h}\right)\Phi\left(\underline{\psi_3^h}\right)\left(1-B_3^h\right)+\Phi\left(\gamma_3^{h*}\mu_3^h+\dddot{\gamma}_3^h\Delta\underline{\dddot{y}_3^h}\right)\left(1-\Phi\left(\underline{\psi_3^h}\right)\right)B_3^h\right]\right]\kappa_3\right\}^{\left(1-D_3^h\right)OE_3^h}\cdot \\
\prod_h \left(1-\kappa_3\right)^{\left(1-OE_3^h\right)}
\end{array}\right\},
$$

where κ_i denotes the share of households having the opportunity to consume good *i*, indicating the probability of belonging to this household group; ϕ denotes the normal density function; and Φ denotes the normal probability function. Thus, $\Phi\left(\underline{\psi_i^h}\right)$ denotes the probability of choosing a corner solution for energy good *i*, given the opportunity to consume good *i*.

The properties of the density functions depend on the individual household's consumption opportunities. We use the dummy variable, OE_i^h, indicating equipment ownership, to separate households with different consumption opportunities, the dummy variable, D_i^h, indicating whether the household has positive expenditure on energy good *i*, and the dummy variable, B_i^h, indicating whether the household has positive consumption of energy good *i* (to identify corner solutions).