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# DEVELOPMENT OF MATHEMATIC MODEL OF GAS-TURBINE UNIT COMBUSTION CHAMBER FUNCTIONING

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**Abstract.** The article presents the basic issues of ecological safety improvement of present-day steady-state gas turbine units. A mathematic model of diagnostics of combustion chamber considering the basic parameters of gas-turbine unit (GTU) operation under different technological modes was developed. These modes include the intake air temperature of the axial-flow compressor, the intake temperature of the actuating medium of the directing set of the high pressure turbine as well as the temperature of the environment.

**Key words:** combustion chamber, mathematic model, gas turbine engine, ecological characteristics, actuating medium.

## 1. Introduction

The problem of lowering the toxicity of combustion products of gas dispensing units (GDU) being operated at the compressor stations of the main gas pipelines is the constituent of the scientific and technical progress of the natural gas industry [1]. The key standards, that the low emissive combustion chambers are up to, have already been settled, based on the main ways of the working process arrangement and the tendencies of the improvement of the fuel scorching devices efficiency [2–7]. For their manufacturing application, it is indispensable to increase the design efficiency and improvement of the low emissive combustion chambers.

## 2. Analysis of present-day foreign and domestic studies and published works

The main ways to lower the toxicity of combustion products in the burning systems of various purposes are: the application of increased air surpluses (V. A. Khrystych, G. N. Liubchyk, N. K. Risk, D. Wels); the application of phasic combustion (I. Ya. Sigal, F. G. Tumanovskii, A. Lefer); the homogenization of the combustion area (G. Leonard, S. M. Korea); the application of micro torch combustion (S. D. Serdiuk, N. P. Danylenko, G. B. Varlamov, A. M. Markushin, A. M. Postnenkov, V. P. Savchenko); the application of catalytic combustion technologies (G. N. Liubchyk, G. S. Marchenko, B. I. Shelkovskii, C. Wilkes, S. V. Santanam).

The testing results of the Solar company device (in the combustion chamber GDU GTC – 10I) [8] demonstrated that when the air surplus increases over the above-mentioned level the CO emission increases roughly. Considering the air surplus ratio $\alpha \rightarrow 2{,}0$ the CO in the combustion products leads to $O_2$ = 15 %, and results in 100 мg/м$^3$, that can cause increased chemical deficiency of fuel combustion. Consequently, when decreasing the air surplus below the above-mentioned level not only the $NO_x$ and CO are essentially increased but also there is a high-frequency combustion, breakdown and a sudden flame [8]. The companies that use the given technology (such as: Solar, Rolls-Royce, Siemens etc.) eliminate such a failure using complex systems of regulating the mix proportion in the combustion area, and variable geometry of the combustion chambers and flow-type part of a compressor. The simulation of the working processes in the combustion chambers is considered to be one of the perspective methods that enables to reduce the cost of the research. In particular, the mathematic model of three-dimensional chemically reacted flows, engineered by G. B. Mostipanenko, enables to predict the output temperature and ecological features of the combustion chambers that use the gaseous fuel.

## 3. Emphasizing unresolved issues of the general problem

Neither the air consumption nor the actuating medium is measured at any section of gas route of the gas-turbine unit. Moreover, the records done by the existing methods are impossible, as it leads to the hydraulic losses that reduce the capacity and the economic efficiency of the cycle. The only kind of consumption that can be measured practically is the consumption of fuel gas. Evidentially, its component analysis can be considered to be known.

The objective of the research is to develop the mathematic model of diagnostics of combustion chamber considering the basic parameters of a gas-turbine unit operation under different technological modes including the intake air temperature of the axial-flow compressor, the intake temperature of the actuating medium of the directing set of the high pressure turbine as well as the temperature of the environment.

## 4. Reporting the principal subject matter

The combustion chamber of gas-turbine unit is specified for the actuating medium preparation that will perform the useful work when expanding at the turbine trowel. For this purpose, the gas fuel burning takes place in the primary air medium with the creation of pure combustion products in the combustion chamber. Then, the process of mixing with the reiterative air for obtaining the admissible temperature of the actuating medium follows. Hence, the composition of the actuating medium can be given like the mixture of the pure combustion products with the reiterative air. To describe the actuating medium the concept of the air surplus coefficient $\alpha$ is given. This is the ratio of total air consumption (primary $L$ and reiterative $L_0$) to the primary air consumption, theoretically necessary for combustion of 1kg of fuel gas [1].

$$\alpha = \frac{L+L_0}{L_0}. \quad (1)$$

The required quantity of air for combustion of 1 kg of fuel gas of the known component composition with the density $\rho'$ can theoretically be computed in the following way:

$$L_{\text{o}} = \frac{1,293}{21\rho}\left[0,5(\text{CO})+0,5(\text{H}_2)+2(\text{CH}_4)+ +\sum(m+n/4)(C_mH_n)+1,5(\text{H}_2\text{S})-(\text{O}_2).\right. \quad (2)$$

Here the symbols in the brackets mean the volumetric bits of every component in the mixture of gases. The air surplus in the combustion chamber in comparison with its theoretically indispensable quantity leads to the cooling of the actuating liquid. The air surplus coefficient is related to the maximum temperature of the ratio cycle:

$$\alpha = \frac{Q_H^P\eta_k + C_{\Pi}t_{\Pi} + C_P^{"}t_3}{L_0\left(C_P^{"}t_3 - C_P^{"}t_2\right)}, \quad (3)$$

where $C_{\Pi}$, $t_n$ – is the heat capacity and temperature of the fuel gas.

Without determining the heat value of the fuel $Q_H^p$ and its heat capacity $C_m$ it is pointed out that the air surplus coefficient can be found from the equation (3) on the basis of the fuel gas temperature dimension $t_т$ the air after the axial-flow compressor $t_2$ and the actuating liquid before the gas turbine $t_3$. Then the mass consumption of the actuating liquid and air can be shown through the mass consumption of the fuel gas.

$$M'' = \alpha(L_0+1)M_n, \quad (4)$$

$$M' = \alpha(L_0-1)M_n, \quad (5)$$

The volumetric air consumption is set to the input conditions of the first grade of the axial-flow compressor:

$$Q_1 = \frac{L_o(\alpha-1)M_{\Pi}}{P_1}287T_1 \quad (6)$$

Thus, using the equations (3) and (6) the productivity of the axial-flow compressor can be determined which will enable us to define the diagnostic features for every mode of the gas turbine unit. However, for the application of the equation (3) it is indispensable to determine the mean specific volume of the air heat capacity $C'_{m'}$ in the temperature span ($t_1$, $t_2$) and the actuating medium $C''_{p'm'}$ in the temperature span ($t_4$, $t_3$). If there is no difficulty in determining the air heat capacity that is mean in the known temperature span there is some difficulty in determining the mean mass heat capacity of the actuating medium $C''_{p'm'}$ as the composition of the actuating medium is unknown. Consequently for the computation of the air surplus coefficient the method of successive approximation is worth applying. The following computation algorithm is applied:

1. In the first approximation the values of actuating medium heat capacity $C''_{p'm'}$ are specified. This value is equal to the air heat capacity with the same data:

$$C''_{pm} = C'_{pm}. \quad (7)$$

2. When using equation (3) the first approximation of the air surplus coefficient is computed.

3. The mass consumption of the actuating medium on the assumption that of 1kg of fuel gas consumption is computed.

$$M'' = M_{\text{CO}_2} + M_{\text{H}_2\text{O}} + M_{\text{N}_2} + M_{\text{O}_2}, \quad (8)$$

where

$$\begin{aligned} &M_{\text{CO}_2} = \frac{0,44}{22,4}\left[(\text{CO}_2)+(\text{CO})+\sum m(C_mH_n)\right], \\ &M_{\text{H}_2\text{O}} = \frac{0,18}{22,4}\left[(\text{H}_2)+2(\text{CH}_4)+\sum\frac{n}{2}(C_mH_n)\right], \\ &M_{\text{N}_2} = 0,769L_0\alpha^{(1)}, \\ &M_{\text{O}_2} = 0,231L_0\left(\alpha^{(1)}-1\right). \end{aligned} \quad (9)$$

4. The mass bits of every given component in the actuating medium are computed:

$$\begin{aligned} &m_{\text{CO}_2} = M_{\text{CO}_2}/M''; \quad m_{\text{H}_2\text{O}} = M_{\text{H}_2\text{O}}/M''; \\ &m_{\text{O}_2} = M_{\text{O}_2}/M''; \quad m_{\text{N}_2} = M_{\text{N}_2}/M''. \end{aligned} \quad (10)$$

5. The heat capacity of the actuating liquid is computed:

$$C''_{pm} = m_{CO_2} C_{pm}^{CO_2} + m_{H_2O} C_{pm}^{H_2O} + m_{O_2} C_{pm}^{O_2} + m_{N_2} C_{pm}^{N_2}. \quad (11)$$

6. When using equation (3) the air surplus coefficient is defined more accurately and the calculations for achieving the given accuracy are computed:

$$\left|\alpha^{i-1} - \alpha^{i}\right| < \varepsilon_0 . \quad (12)$$

The complex processes happening in the combustion chamber demand the hypothesis admission while formalizing. Thus, for the narrow range of the air surplus coefficient change in the burning part of the combustion chamber, it can be admitted that the combustion temperature does not depend on the air surplus and is constant.

If to admit that the pressure and temperature after the axial-flow compressor as well as air and fuel gas consumption are known, then for the motion in the combustion chamber the following system of differential equations can be given:

$$\begin{cases} \dfrac{dP}{dt} + \dfrac{c^2}{F}\left(\dfrac{dM'}{dx} + ML_0(\alpha+1)\delta(x-x_0)\right) = 0, \\ \dfrac{dP}{dx} + \left(\xi_k / 2pF^2\right)M^2 + \dfrac{1}{F}\dfrac{dM}{dt} = 0, \\ \dfrac{dM}{dt} + \dfrac{M'}{\rho' F}\dfrac{dT}{dx} + \dfrac{ML_0(\alpha+1)C'_p}{p'' C''_p F} T_2 \delta(x-x_0) - \\ -\alpha \dfrac{d^2T}{dx^2} - \dfrac{\pi D(T_0 - T)}{p'' C''_p F} = 0. \end{cases} \quad (13)$$

The link between the basic parameters of the actuating medium (pressure $P$, temperature $T$ and mass consumption $M$) for different periods of time and various parts of the combustion chamber that are characterized with the linear coordinate $x$ can be given with the above-mentioned equations. At the point of combustion and mixing ($x = x_0$) the step consumption increase can be observed. It is caused by the reiterative air supply with the consumption $M_1$. Thus, the actuating medium consumption as a function of the linear coordinate can be given in the following equation:

$$M = M_r(L_0+1) + M' L_0(\alpha-1)\sigma(x-x_0), \quad (14)$$

where $L_0$ – is theoretically indispensable air for combustion of 1kg of fuel; $\alpha$ – air surplus coefficient; $\sigma(x-x_0)$ – Heaviside's unit function;

$$\sigma(x-x_0) = \begin{cases} 1, & if \quad x > x_0 \\ 0, & if \quad x < x_0 \end{cases} . \quad (15)$$

For simulating the step change of temperature consumption in the point of supply of the reiterative air ($x = x_0$) the Dirac's function of source in the equations $\delta(x = x_0)$ is given. It is worth considering that for some combustion chambers the diameter $D$ and the sectional area $F$ can be changed on the linear coordinate $x$. The air density $\rho'$, actuating medium $\rho''$, and pure combustion products $\rho$ (when there are the specified conditions in the combustion chamber) can be shown through pressure and temperature:

$$\rho' = \frac{P}{R'T}, \quad \rho'' = \frac{P}{R''T}, \quad \rho = \frac{P}{RT}. \quad (16)$$

Thus, the system (13) is a closed system of the differential equations that combine the parameters of the combustion chamber functioning with its typical dimensions and physical properties of the actuating medium. The system includes also the coefficient of the hydraulic resistance of the combustion chamber $\xi_k$ and the coefficient of the heat exchange with the environment $K$. The gas-turbine units operation proved that exactly these coefficients demonstrate the real condition of the combustion chamber. So, the coefficient of the hydraulic resistance of the combustion chamber and the absolute coefficient of the heat exchange with the environment $K$ can be used as the diagnostic features. The simplification of the mathematic model given in the equation (13) for the standard mode has been performed due to the division of the gas dynamic and thermodynamic processes. For the permanent gas dynamic process in the combustion chamber the equation of motion can be presented in the following way:

$$\frac{dP}{dx} = -\frac{\varepsilon_\kappa M''^2}{2D\rho F^2}. \quad (17)$$

The actuating medium consumption can be determined through the measured consumption of the fuel gas $M_г$ and the coefficient of the air surplus in the combustion chamber computed by the approach given above:

$$M'' = \alpha(L_0+1)M_{\Pi} . \quad (18)$$

Through the computed composition of the actuating medium its gas constant from the equation can be determined:

$$\frac{1}{R''} = \sum_{i=1}^{4} \frac{m_i}{R_i}, \quad (19)$$

where $m_i$, $R_i$ – are mass fractions of the $i$ component in the actuating medium and its gas constant. The mean pressure and temperature in the mixing zone are computed as the arithmetic mean:

$$P_c = \frac{1}{2}(P_2 + P_3), \quad (20)$$

where $P_2, P_3$ – are the input and output pressures of the combustion chamber, that are determined via measurements; $T_{2L}$ – is the intake air temperature of the mixing zone of the combustion chamber, the approach of determining it will be given later; $T_3$ – the measured temperature at the output of the combustion chamber.

Then the density of the actuating medium under the average conditions in the combustion chamber can be determined from the equation of the state:

$$P_c = (P_2 + P_3)/2; \; T_c = (T_{2L} + T_3)/2,$$

$$\rho = \frac{P_c}{R'' T_c}. \tag{21}$$

The answer from the equation (17) can be given in the following way:

$$\frac{P_2 - P_3}{L} = \frac{\xi_k M''^2}{2\rho F^2}, \tag{22}$$

where $L$, $F$ – are the length and the area of the section of the combustion chamber correspondingly.

From equation (22) the coefficient of the resistance of the combustion chamber as the following can be computed:

$$\xi_k = 2\rho F^2 \left(P_2 - P_3\right) / LM^2. \tag{23}$$

The permanent heat exchange of the actuating medium that moves in the flue tube after the mixing zone with the flow of the reiterative air in the conditions of counterflow can be given as:

$$K_1 \pi D(T_3(x) - T_2(x))dx = M'' c_p'' dT_3(x), \tag{24}$$

where $K_1$ – is the coefficient of the heat exchange from the actuating medium to the air; $D$ – is the diameter of the flue tube (given); $M'' = M_{\Pi}\left(L_0 + 1\right)\alpha$ – is the mass consumption of the actuating medium; $C''$ – heat capacity of the actuating medium under the temperature $T_3$.

The permanent heat exchange of the reiterative air with the actuating medium and environment can be presented with the following equation:

$$\begin{gathered} K_1 \pi D\left(T_3(x) - T_2(x)\right) dx - \\ -K_2 \pi D_0 T_2(x) - T_0(x) dx = M' C_p' dT_2, \end{gathered} \tag{25}$$

where $K_2$ – is the coefficient of heat transmission from the reiterative air into environment through the casing with the given diameter $D_0$ under the temperature of the environment $T_0$; $C_p$ – is the air heat capacity under the temperature $T_2$.

The equation for the heat balance of the combustion area is the following:

$$T_{30} C_p'' \left(\alpha L_0 + 1\right) = \alpha L_0 C_p' T_{2L} + \left(\alpha_1 L_0 + 1\right) C_p T_{03}, \tag{26}$$

where $T_{30}$ – is the intake temperature of the actuating medium into the mixing zone; $T_{03}$ – is the torch temperature in the combustion area; $T_{2L}$ – is the intake temperature of the primary air that comes into the combustion area with the surplus coefficient $\alpha_1$; $C_p$ – is the heat capacity of the combustion products under the temperature $T_{03}$.

The joint answer to the equations (23)–(25) under the specified intake temperature in the combustion chamber $T_2$ and the temperature of the actuating medium in the blade of the directing set of the high pressure turbine $T_3$ can be given as the following:

$$\begin{gathered} (((T_{2i} - T_{0i})/(1 - W_T(K_T/W_m + K_0/W_{\Pi} - \\ -\sqrt{(K_T/W_m + K_0/W_m) - 4K_0K_T/W_TW_{\Pi}}/2K_T + T_{0i}) \times \\ \times C_p''(\alpha L_0 + 1) - (\alpha L_0 + 1)C_pT_{03}/\alpha L_0 - T_{0i} = \\ = 1 - W_T(K_T/W_m + K_0/W_{\Pi}) - \\ -\sqrt{(K_T/W_m + K_0/W_m) - 4K_0K_T/WW_{\Pi}}/2K_T) \times \\ \times (T_{3i} - T_{0i} - (\alpha_1 L_0 + 1)C_PT_{03}/\alpha L_0 - \\ -T_{0i}(\exp(-((K_T/W_m + K_0/W_{\Pi}) + \\ +\sqrt{(K_T/W_m + K_0/W_m) - 4K_0K_T/W_TW_{\Pi}}))\frac{L}{2} - \\ -(\exp(-((K_T/W_m + K_0/W_{\Pi}) - \\ -\sqrt{(K_T/W_m + K_0/W_m) - 4K_0K_T/W_TW_{\Pi}}))\frac{L}{2})), \end{gathered} \tag{27}$$

where $W_{\Pi}$, $W_m$ – are the water equivalents of the air and actuating medium correspondingly;

$$W_{\Pi} = C_p' M' \cdot W_m = C_p'' M'', \tag{28}$$

$W_m$ – is the mixed water equivalent;

$$W_m = W_n'' \cdot W_m / (W_n + W_m), \tag{29}$$

$K$, $K_0$ – are the given coefficients of heat transmission from the actuating medium to the reiterative air and from the reiterative air into the environment.

## 5. Conclusions

The present mathematic model enables to determine the given heat transmission coefficients $K_n$ and $K_0$ as well as the intake of reiterative air in the mixing zone of the combustion chamber temperature $T_{2L}$, if the operation parameters of GTU in three modes are known. Whereupon, the technological modes due to which the combustion chamber is diagnosed differ by the intake air temperature of the axial-flow compressor, by the output actuating medium temperature into the directing set of the high pressure turbine as well as by the temperature of the environment. Besides, the steady-state conditions of GTU operation are adhered to in every mode.

Thus, the given mathematic model of the burning processes in the combustion chamber will enable not only to determine the key features of the combustion chamber but also to study the basic technological modes to decrease the emission of the polluted substances of the adjacent territories of compressor stations.

Such an approach will enable to carry out various computations and research as well as determine the indicated indices of exhaust combustion products and develop the ecological characteristics under the different operative modes of the GTU.

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