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# MODELING OF SEWER NETWORKS BY MEANS OF DIRECTED GRAPHS

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**Abstract.** The paper presents an analysis of the known theoretical models of drainage network of the city as directed graphs of different types. It is proved that ringing sewer networks increases reliability of their work and improves maneuverability and regulation of drainage system as a whole. A theory of solving the optimization problem "capacity – economic efficiency" was proposed to design optimization schemes for sewage system, using the straight-dual algorithm proposed by Saharovych.

**Key words:** drainage network, mathematical model, structural scheme of drainage, directed graph.

## 1. Abstract

The proper functioning of urban drainage (sewer) system is one of the main factors that determine the quality of environmental and sanitary conditions of the settlement. For reliable operation of sewer systems it is necessary to implement effective monitoring and management tools. The problems of monitoring and managing drainage systems of major cities in real time are complex and interrelated problems.

Today in Ukrainian engineering practice the most common method of operation of sewer systems is manual control. With manual control corrective actions for each controlled network structure depend on individual experience and the operator's level of training. Thus, management is heuristic rather than systematic, theoretically grounded. In large cities due to the high complexity of urban sewer systems it is virtually impossible to ensure effective control and management regimes of their work manually. Perspective is the development and implementation of specialized computer control systems based on mathematical modeling of the processes in the elements of drainage systems taking into account changes in the time of their dynamic relationship.

The aim of the work is to analyze the known theoretical models of drainage networks in big cities in terms of management of working modes for appropriate sewer systems.

## 2. Materials and methods

### 2.1. Structured schemes of drainage networks

Sewer network are design according to directed-gravity concept, which provides that waste water mostly pass through the sewage network under the influence of gravity. When gravity flow of sewage is impossible, sewage pumping stations (SPS) are set, which pump liquid in the above located areas.

Usually scheme sewer networks are represented in the form of directed, coherent graph where the wells are presented as vertices (nodes) and drainage pipes and collectors - in the form of edges (areas). Depending on the particular planning and altitude schemes of the city, sewer network structure can be presented in the following graphs:

- "**reverse directed tree" type** – this model is the most common and can be used for a city that has one centralized sewage treatment plant (STP); so all wastewater enter these STP (an example of this model for the city of Nowy Sacz (Poland) is analyzed in detail in the work [1]);
- **directed, coherent and acyclic graph** – this model is used for big cities with two or more centralized STP network with presence of some closed-ring circuits (model of this type is analyzed in [2] on the example of Moscow city);
- **directed, network graph** – this model describes the sewer network, with ringing of all its areas (example of such model is given in [3]).

In Ukrainian sanitation practices, in accordance with applicable industry normative document DBN V.2.5-75 2013 [4] (p.8.1.4), "gravity and vacuum sewerage network is designed mostly in line." However, "in order to improve the reliability of gravity networks, the design of gravity overflow between different networks" is allowed if necessary. Placing on the sewer network ring-closed circuits increases its capital cost, but at the same time increases the reliability of the overall system and increases the possibilities of flow distribution and regulation of of wastewater costs in the network.

In the drainage networks of “reverse directed tree” (Fig. 1) [1] sewage enter the network nodes starting from the upper units 1-8 and finishing with the lower level nodes 14–15 and flow by gravity to the root of the tree where the STP is located (p.16 in Fig.1).

The capital cost of such networks is the lowest among the three above types. However, the reliability and consequently, safety of operation of such networks is minimal. Clog of any pipe causes flooding of upstream sections of the network and consequently, the release of waste water to the surface, causing considerable material and ecological damage. This is not always technically possible to pump the effluent to another working node located downstream. In these cases, flooding of the area with wastewater will continue until the complete elimination of the emergency.

Much more opportunities to regulate drainage network operating modes are provided by network diagrams as directed, coherent and non-cyclic graphs [2]. Fig. 2 shows an example of such scheme with three STP (components 20–22). The main feature and advantage of such schemes is the availability of alternative areas of transportation and wastewater treatment. These schemes involve extensive use of locking-regulating devices (regulating valves, gate, check valves, etc.). These design features allow to choose between alternatives of distribution of sewage streams, that is provide drainage system with manageability that makes it possible to avoid the above accidents or minimize damage from them.

Utilities that have the structure of directed, network graph (eg. a circuit shown in Fig. 3), are characterized by the highest capital cost and degree of reliability. Full ringing is necessarily used for the main water distribution networks of medium and large cities. On the other hand, ringing of drainage networks today is rarely used in practice, because of their high capital cost, especially for rainwater and general sewage networks. Technical problems in the implementation of circular drainage schemes have the potential of backwater or even upstream processes in certain areas of the network that is absolutely unacceptable from a technological point of view.

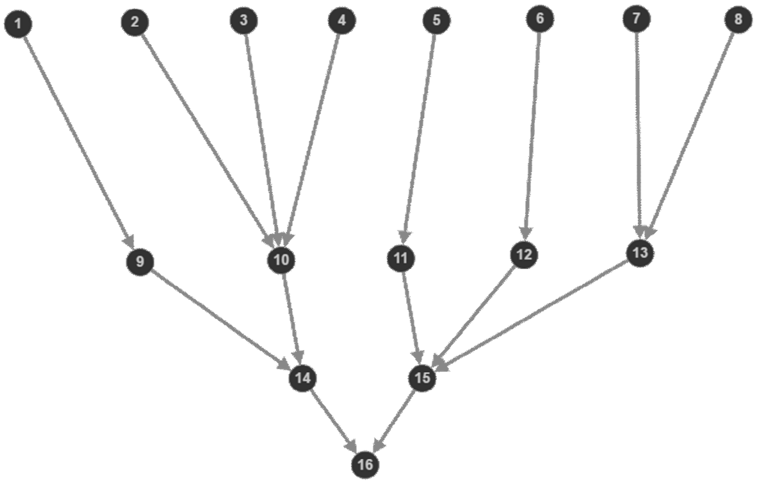

**Fig. 1.** Example of traditional block diagram of the sewerage network as a “reverse-directed tree” graph

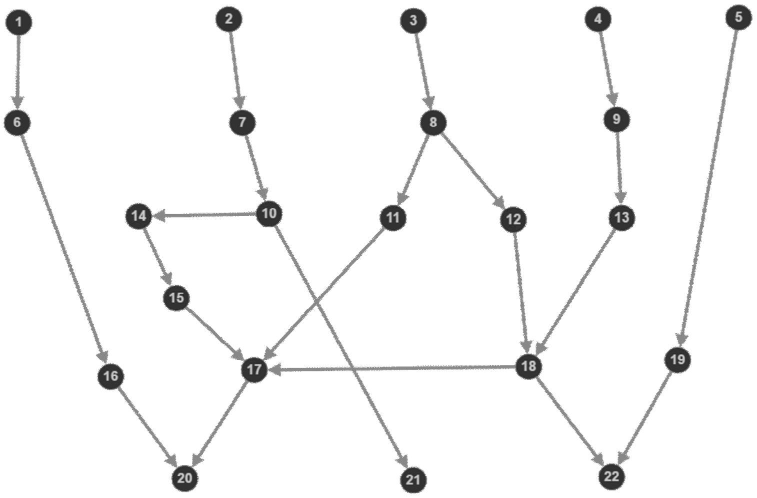

**Fig. 2.** Example of sewerage scheme as directed, coherent and acyclic graph

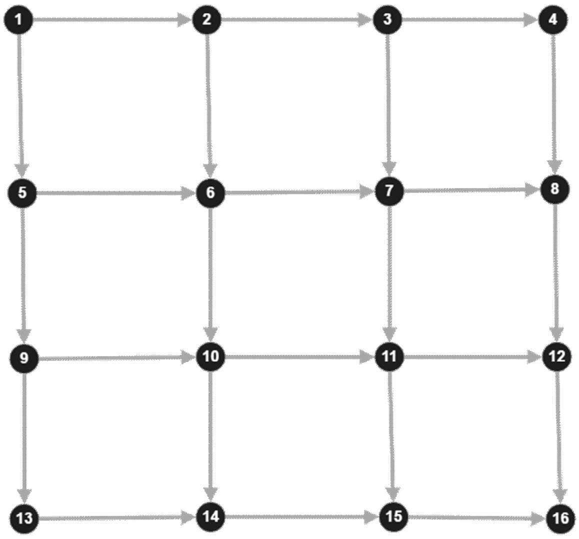

**Fig. 3.** Example of the sewerage scheme as directed, network graph

However, ring sewerage scheme provides maximum flexibility and maneuverability for drainage system [3]. Failure on any of the sites of the network leads to disconnection from the network only the subscribers connected directly to this site. At the same time all other areas of the network can function properly.

## 2.2. The General Formulation of Optimization Problem of Sewerage Schemes

To improve the reliability and efficiency of the drainage system, the theory of solving one of the basic optimization problems can be applied, providing maximum utility for minimum system operation costs [5]. In terms of graph theory, the problem is to find such set of volumetric flow rate $Q_i$, which satisfies the conditions:

$$F = \sum_{i \in G} p_i Q_i \to \min , \qquad (1)$$

for a given node costs of wastewater inflow $Q_u$ (which, in general, are variable in time) for compliance with the conditions of non-overflow:: $0 \le Q_i \le Q_{max.i}$ ;

where $G[U,N]$ − coherent, directed graph, where each region $i \in N$ is characterized by the range of eligible costs $[0;Q_{\max.i}]$; $Q_{\max.i}$ − maximum capacity of the *i*-th section of the graph; $p_i$ − specific cost of the unit flow rate passing through the *i*-th section of the graph; $Q_i$ − flow rate through the *i*-th section of the graph to be optimized.

The solution of the equation (1) is equivalent to overlaying on the graph $\mathrm{G}[\mathrm{X},\mathrm{N}]$ of maximum flow with minimal costs. To solve this problem, for example, a straight-dual algorithm proposed by Saharovych can be used [6].

According to [2] this method is used for simulation using graphs.

All nodes of the graph are numbered. The total number of nodes is equal to r. It is more comfortable to number as follows: the sequence starts with k input nodes, all other nodes are numbered in the sequence from the (k + 1) to r, for Fig. 1 total number of units r, is 22, and the number of input node k is 5. For Figure 2 r = 8 and k = 16 respectively.

The matrix form is best suited for analytical description of the network. In this description the conception of a route is the main for the development of a matrix description of the network. We define a route as a series of network structures (graph nodes) that starts from the inlet and ends with the outlet, along which no structure will be covered more than once. Suppose the inlet 19 (see. Fig. 2) receives the stream as a result of wastewater discharges $q_1$. This stream can be transferred through the network of collectors in two possible ways: some sewage will be transported through the structure in order 2 ± 7 ± 10 ± 14, ± 15, ± 17, ± 20; and the rest through the structures 2 ± 7 ± 10 ± 21. All possible routes from all ports to the outlets are defined similarly. The number of routes ***p*** for the real network is finite. For the graph in Fig. 1 the number of routes p = 8, for Fig. 2 the number of routes p = 9 for Fig. 3 p = 14. In the order of numbering adopted for the nodes and routes the corresponding graph of sewer system can be described as incidence matrix *A* structure/route, which is $(r \times p)$ matrix consisting of elements $a_{ij}$. Elements $a_{ij}$ ($i$ = 1, 2, ..., r; $j$ = 1, 2, …,$p$.) Matrix *A* is defined as: $a_{ij}$ =1 if $j$ the route includes the structure $i$ or $a_{ij}$ = 0 otherwise.

For further use it is convenient to introduce matrix *A* $(r \times p)$ as a unit,

$$A = \begin{bmatrix} A_1 \\ \dots \\ A_2 \end{bmatrix}, \qquad (2)$$

where $A_1$ $(r \times p)$ corresponds to the input nodes (intake vent), and $A_2$ $((r-k) \times p)$ for all other nodes of the graph.

## 2.3. Wastewater Scheme of Lviv

General-purpose sewage system is functioning in most parts of Lviv city. A mix of residential, industrial and rainwater sewage enters the Lviv sewer treatment plant and after purification is dumping into the river Poltva basin of the Western Bug.

City drainage system consists of 765 km of sewerage networks, 15 sewage pumping stations and two separate lines of treatment facilities - STP-I with a design capacity of 140 ths. $m^3$/day and STP-II – 350 ths. $m^3$/day.

The characteristic features of the city sewage system is that the area of the city is divided by Major European Watershed into two drainage basins – the Baltic and the Black Sea, and the main sewer collector of the city is the Poltva river, which is closed in an underground reinforced concrete channel.

The central part of Lviv city has a general-purpose sewer system that transports mix of domestic, industrial and surface waste water to STP. Economic and industrial and domestic waste water from the Black Sea drainage basin of the city are transported to the main sewer collector and then to the main STP of the city by means of pump stations, pressure reservoirs and deep-laying mine collector.

Due to the presence of two lines of STP, wastewater which is transported by different collectors and the availability of an interceptor between the main collector and collector № 4, the current circuit city sewers can most reasonably be presented as directed, coherent and acyclic graph. Problems associated with overloading of the existing Lviv CSTP loss during periods of intense rainfall, determine the relevance of search for the ways of reconstruction of drainage network of the city with the implementation of an automated control system of runoff. To find the best wastewater management schemes of Lviv territory it is necessary to create appropriate mathematical model of urban sewage networks.

## 3. Results and Discussion

The purpose of next studies is to develop theoretically grounded mathematical models of drainage networks of the considered types and getting standard algorithms of optimization of the structure of drainage network at the stages of planning and reconstruction and development of optimal schemes for flow control in drainage networks during operation.

## 4. Conclusions

Block diagrams of sewage system of cities can be described using graph theory in the form of graphs of three main types: 1) reverse – directed tree; 2) directed, coherent and non-cyclic graph; 3) directed, network graph.

Creation of circular urban sewage networks significantly increases the reliability of their work, improves maneuverability and regulation of drainage system as a whole.

Design and construction of circular or partially circular drainage networks are constrained by their high capital cost and insufficient study of hydraulic, technical and economic characteristics for gravity and gravity-pressure circular piping systems.

To develop the schemes of reconstruction of sewage systems, a theory of solving the optimization problem “capacity – economic efficiency” can be applied using the straight-dual algorithm proposed by Saharovych.

The existing scheme of Lviv sewers can most reasonably be presented as directed, coherent and acyclic graph.

## References

[1] Królikowska J. & Królikowski A. Applying the treedendrical scheme failure method to evaluate the reliability of sewage collection draining reliability evaluation subsystems // Environmental Engineering III, Taylor & Francis Group, London. − 2010. − P. 191−195.

[2] Ermolin Y. A. Mathematical modelling for optimized control of Moscow's sewer network // Applied Mathematical Modelling, N23. − 1999. − P. 543−556.

[3] Chupina R. V., Zelenin A. M. Potokorozpodil v systemakh vodovidvedennya kil’tsevoyi struktury // Inzhenerne zabezpechennya budynkiv i sporud. Resursozberezhennya ta ekolohiya. – S. 124–126.

[4] DBN V.2.5.75 diye do: 2013. Kanalizatsiya. Zovnishni merezhi ta sporudy. Osnovni polozhennya proektuvannya. – Do.: Minrehionbud Ukrayiny, 2013. – 128 s.

[5] Vazquez J., Bellefleur D., Gilbert D., Grandjean B. Real time control of a combined sewer network using graph theory // Water Sci. Tech., Vol. 36, No. 5. − 1997. − P. 301−308.

[6] Sakharovitch M. Optimisation combinatoire: graphs of programmation lineaire // Hermann, Paris. − 1984. − 249 p.