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T. M. SERDYUK – cand. tech. sc., associate prof., Dnepropetrovsk National University of Railway Transport named after akad. V. Lazaryan, serducheck-t@rambler.ru
A. V. SHVETS – dr.phys.-math.sc., snr. staff sc., O. Ya. Usikov Institute for Radiophysics and Electronics of the National Academy of Sciences of Ukraine, lxndrshvts9@gmail.com.ru

NUMERICAL MODELING OF MULTIMODE TWEEK-ATMOSPHERICS IN THE EARTH-IONOSPHERE WAVEGUIDE

The paper is presented by dr.phys. math.sc., prof. V. I. Gavrilyuk (Ukraine), dr.phys. math. sc., s. r. sci. V. K. Ivanov (Ukraine)

Introduction

Tweek-atmospherics, electromagnetic pulses of 10 – 100 milliseconds duration, propagating in the natural Earth-ionosphere waveguide (EIWG), represent a response of the natural cavity to the excitation by lightning discharges. Tweeks are formed due to small losses in the ionosphere at altitudes from about 85 to 95 km, where ionosphere effectively reflects VLF electromagnetic waves during nighttime. A study of this altitude range in the ionosphere by traditional methods meets essential problems due to relatively small electron densities (10 – 1000 cm^{-3}). In this sense tweeks represent a useful natural means for radio sounding the lower ionosphere in a wide frequency band.

Approximation of the flat infinite waveguide with perfectly conducting walls quite well describes effects of the waveguide dispersion observed in experiments, especially near the cutoff frequencies [1]. Based on the waveguide propagation theory different methods of analysis of tweeks were elaborated to estimate the lower ionosphere effective height along the propagation path between a lightning discharge and the observation place and source-to-observer distance (SOD). Analysis of instantaneous frequency variations in tweek signals [1-5], phase spectrum of the longitudinal magnetic field component [6] interference between the zeroth and the first order modes in the tweek amplitude spectrum [7] are employed in the frequency region around the first waveguide cutoff frequency to find the effective waveguide height and a source-to-observer distance. But still lack of attention was devoted to using multimode tweeks that represented higher order EIWG modes to extend frequency range of analysis and to obtain information on the ionospheric vertical conductivity profile.

Tweek waveform in the range of the waveguide cutoff frequencies at ELF-VLF is resulted mainly from the dispersion properties of the EIWG and distance to a source. While the amplitude spectrum of tweek has a complicate structure arising as a result of interferences between different waveguide modes, the dynamic spectrum enhances time evolutions of instantaneous frequencies in the signal and allows us to separate from one to up to ten dispersion branches related to the first and higher order waveguide modes.

Two approaches are considered for estimating the EIWG cutoff frequencies and SOD. The first technique is based on traditional, “sonogram” method. The second, new one is based on linearizing transformation of dispersion branches for higher order waveguide modes enhanced from dynamic spectrum of tweek in the time domain by seeking an optimal value of the source range. These techniques were realized programmatically and tested by the numerical simulation.

Tweek model

To evaluate effectiveness of methods of estimation of the lower ionosphere parameters and lightning location, tweek waveforms have been synthesized based on the model of ELF/VLF propagation in the EIWG proposed in [4, 8-10]. The lower boundary of the flat infinite EIWG model, Earth is adopted as a perfect conductor, the upper boundary, ionosphere, is described by an exponential vertical conductivity profile with a single height scale [11]:

$$\sigma(z) = 2.5\times10^5 \varepsilon_0 e^{(z-H)/\zeta_0} = \\ = 2.5\times10^5 \varepsilon_0 e^{\beta(z-H)}, \qquad (1)$$

where ε_0 – dielectric permittivity of vacuum; ζ_0 – local scale height; β – inverse scale height; z – height above the ground; H – characteristic height. With this EIWG model the horizontal magnetic and vertical electric filed component on the Earth's surface are expressed as a sum of zeroth and higher waveguide modes [10]:

$$H_\phi = \frac{-Ids}{2(\rho\lambda)^{1/2}} e^{\frac{i\pi}{4}} \times \left[\frac{\delta_0 S_0^{1/2}}{h_0} e^{-(ikS_0+\alpha_0)\rho} + \right. \\ \left. + \sum_{m=1}^{\infty} \frac{\delta_m S_m^{1/2}}{h_1} e^{-(ikS_m+\alpha_m)\rho} \right] \qquad (2)$$

$$E_z = \frac{\eta Ids}{2(\rho\lambda)^{1/2}} e^{\frac{i\pi}{4}} \times \left[\frac{\delta_0 S_0^{3/2}}{h_0} e^{-(ikS_0+\alpha_0)\rho} + \right. \\ \left. + \sum_{m=1}^{\infty} \frac{\delta_m S_m^{3/2}}{h_1} e^{-(ikS_m+\alpha_m)\rho} \right] \qquad (3)$$

where Ids is a dipole current moment, ρ is the distance to a source, λ, η = 120π, and k are wavelength, wave impedance, and wavenumber in free space respectively, $C_m = m\pi / kh_1$ and $S_m = \sqrt{1-C_m^2}$ are modal cosine and sine. The altitudes h_0 and h_1 are characteristic parameters of the exponential conductivity profile (1). The lower altitude h_0 is the height at which the conduction current parallel to the magnetic field becomes equal to the displacement current, $\sigma(h_0) = \omega\varepsilon_0$. The upper altitude h_1 is the height at which the local wave number becomes equal to the reciprocal of the local scale height of the refractive index, $4\omega\mu_0\sigma(h_1)\zeta_0^2 = 1$. The altitudes h_0 and h_1 are calculated from equation (1) [10]:

$$h_0 = H - \zeta_0 \ln(2.5\times10^5 / 2\pi f)$$

$$h_1 = h_0 + 2\zeta_0 \ln(2.39\times10^4 / f\zeta_0), \qquad (4)$$

where f [Hz] is the frequency. Attenuation and excitation coefficients for the zeroth α_0, δ_0 and the higher α_m, δ_m modes are approximated in the model of ELF-VLF propagation under nighttime conditions in the lower ionosphere [8].

Waveforms of tweeks were obtained by applying the inverse FFT to the calculated model spectra Eq.(2).

Dynamic spectrum of a tweek retrieves dispersion branches for different modes. Waveforms resulted from modeling demonstrate main properties of tweeks observed experimentally, excluding effects of spherical geometry of the EIWG and an elliptical polarization of magnetic field connected with gyrotropic properties of the lower ionosphere [3,12-14]. Amplitude modulation, frequency dispersion, multi-mode content, decreasing the signal amplitude with increasing the mode order and source range, all these features implemented in the applied model will allow us to study effectiveness of tweek analysis techniques. Also, this model supposes different cutoff frequencies for higher modes, resulted from the frequency dependence of h_1 that monotonically decreases with increasing frequency (Eq.(4)).

Examples of synthesized amplitude spectrum, waveform, and dynamic spectrum of a tweek are shown in Fig.1. Calculations were made for the following parameters in Eqs. (2), (3), (4): H = 88 km, β = 0.6 km^{-1}, ρ = 1000 km.

Waveguide dispersion in tweek signals

Tweek-atmospheric represents a response of the EIWG on a pulse excitation by a lighting discharge. Waveform of a tweek-atmospheric in the range of the waveguide cutoff frequencies at ELF-VLF is resulted mainly from the dispersion properties of the EIWG.

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While the amplitude spectrum of tweek is characterized by intermodal interference, the dynamic spectrum allows us to reveal from one to up ten dispersion branches related to the first and higher waveguide modes. The model of flat infinite waveguide with perfectly reflecting walls quite describes effects of the waveguide dispersion observed in tweeks, especially near the cutoff frequencies [1]. Instantaneous frequency $F_m(\tau)$ of the m^{th} order mode (or branch) in the tweek, received at distance ρ from a source, is determined by the waveguide group velocity dispersion and can be expressed as follows (see e.g. [15]):

$$F_m(\tau) = \frac{f_m}{\sqrt{1 - \left(\frac{\rho}{\rho + c\tau} \right)^2}}, \qquad (5)$$

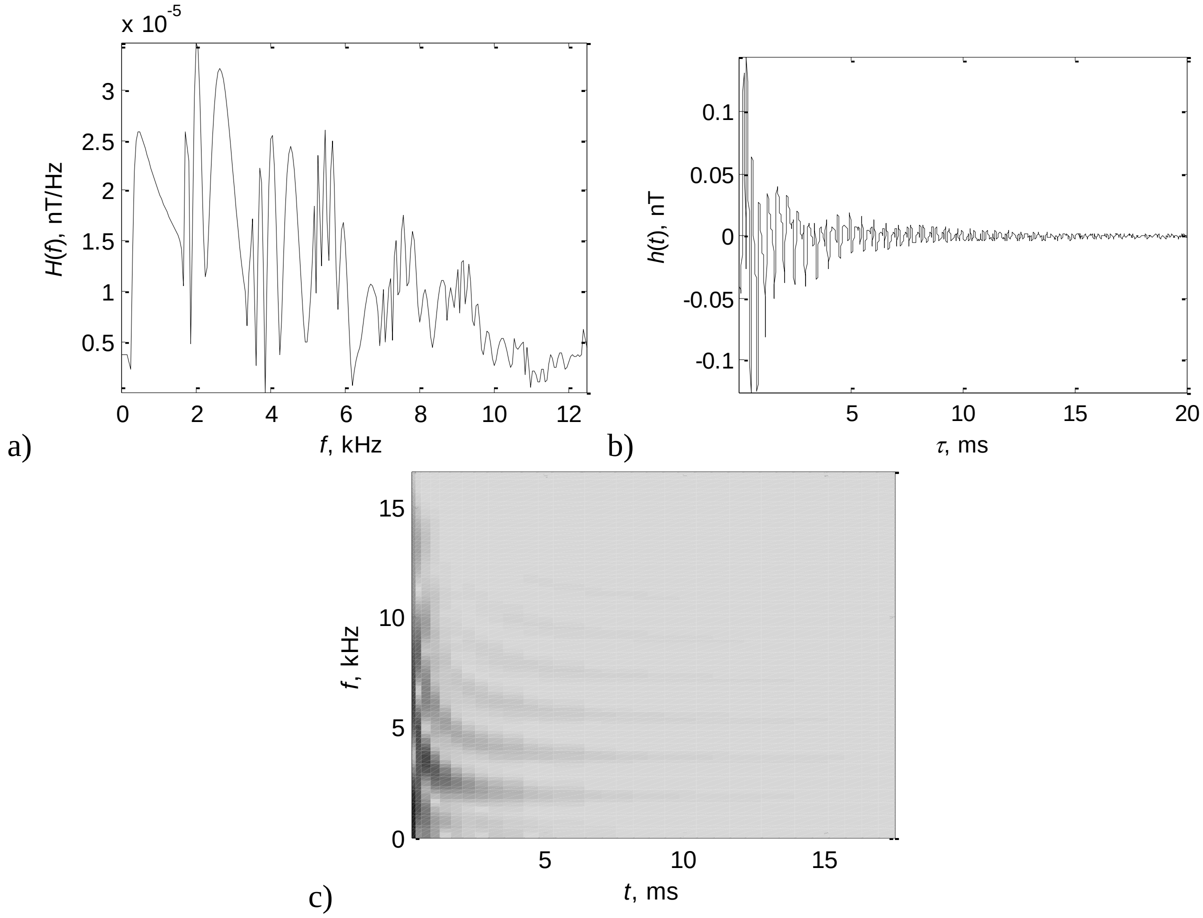

Fig. 1. Amplitude spectrum (a), waveform (b), and dynamic spectrum (c) of a model tweek-atmospheric calculated for the following parameters: H = 88 km, β = 0.6 km^{-1}, ρ = 1600 km

where τ is the delay from the moment of arrival of the ground wave to an observation point, cutoff frequency is determined by the reflection height h from the lower ionosphere: $f_m = cm/2h$, and c is the speed of light in free space.

To minimize influence of the EIWG curvature on the analysis with use Eq.(5) obtained for the flat waveguide we consider the "tail" part of a tweek waveform starting from some delay τ_0 from the beginning depending on the source range. From the expressions that encounter curvature of the EIWG [15] for practical use we can obtain the following empirical expression for τ_0 that provides accuracy of the instantaneous frequency in dispersion branches (Eq.(5)) for the first and higher order modes better than 10 Hz: τ_0 [ms] $\geq$ 0.75 + 1.02ρ [Mm].

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Instantaneous frequency can vary in a wide range at the beginning of a tweek waveform, depending on the distance to the source. So, consideration of the “tail” part of a tweek waveform allows us also to diminish a difference between dispersion properties of the idealized EIWG model with perfectly conducting boundaries and cases when the effective reflection height from the ionosphere depends on frequency.

Having measured from the dynamic spectrum of tweek instantaneous frequencies Fm for a particular mode at two values of τ, the two constant parameters fm and ρ can be directly resolved from Eq.(5).

Determination of the source range and the waveguide height is usually done by the least square fitting the theoretical dependence (Eq.(5)) to the experimental points F_m and we can find the cutoff frequencies f_{m} for the selected branches and the effective EIWG heights, corresponding to the m^{th} order mode, that are determined as follows:

$$h_m = cm / 2 f_m . \qquad (6)$$

In this paper we examine the effectiveness of application of the dispersion relations, deduced for the flat infinite waveguide with perfectly conducting boundaries, to the estimation of parameters of more complicate waveguide model, described in the previous chapter, with the use of multimode tweek atmospherics. Cutoff frequencies and corresponding effective heights of the EIWG for the higher modes are determined by parameters of the vertical conductivity profile of the lower ionosphere.

Dispersion dependences were constructed using dynamic spectra of tweek’s magnetic field component. To reduce influence of the border effects on the spectral estimates, elementary waveforms were multiplied by the Hamming window before calculation of the FFT. The peak frequencies of spectral peaks and their amplitudes are determined by interpolation with a parabola inscribed to the point of maximum and two adjacent points in the discrete amplitude spectrum.

A nonlinear scaling technique

There are some problems for accurate measurement of the instantaneous frequency in tweek-atmospherics. It deals with the fast changing of signal frequency in the beginning of a tweek signal. First, to compensate frequency dispersion, an approach was proposed in [2] and [4], where a tweek signal was multiplied by a complex analytical signal, pseudospheric, with known frequency dispersion. In the case of coincidence of parameters in the pseudo-spheric and experimental waveform a straight horizontal line appeared in the dynamic spectrum of the synthesized signal around zero frequency, corresponding to a chosen mode.

In this section we propose nonlinear transformation of a tweek waveform to suppress changes of frequencies simultaneously for all dispersion branches.

Tweek waveform for a distance ρ from a source in frame of the plate infinite waveguide with perfectly conducting boundaries can be represented as a succession of pulses from a set of virtual dipoles, as it is shown in Fig.3 by thick vertical arrows, simultaneously radiating short pulses (see, e.g., [15]). Only one virtual dipole above the earth, related to the first reflection from the ionosphere, is shown for the sake of simplicity. These virtual dipoles represent successive multiple reflections from the ionosphere.

Delay between successive pulses increases, asymptotically approaching the value of $2h/c$ at a receiver point. Our task is to expand delays between pulses to obtain equidistant pulse succession on a new time axis. The time of arrival of the first reflected pulse relative to the direct, ground wave, from Fig.2, is proportional to the path difference $\Delta\rho_1$ between them:

$$\tau = \Delta\rho_1 / c = \sqrt{(\rho / c)^2 + (2h / c)^2} - \rho / c .$$

On the new time axis this delay is transformed to the following value: $\tau \rightarrow$ □$t = 2h/c$. Substituting t instead $2h/c$ in the previous equation for τ, we can easily express t versus τ:

$$t(\tau, \rho) = \sqrt{\tau^2 + 2\tau\rho / c} . \qquad (7)$$

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The new, stretched waveform will be expressed as follows:

$$w_1\left[t(\tau,\rho)\right]=w(\tau). \qquad (8)$$

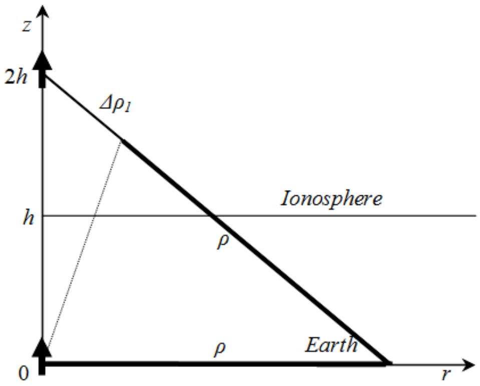

Fig. 2. Scheme of transformation to the new time axis. Path difference $\Delta\rho_1$ between the direct wave and the wave first reflected from the ionosphere is projected to the asymptotic value $2h$

To get equidistant discrete points in the rescaled discrete waveform of tweek we use linear interpolation to apply the FFT for further analysis. Now, for the stretched waveform, we can extend an elementary interval to construct a dynamic spectrum and in such a way to improve estimations of the instantaneous frequency and, respectively, estimations of source range and EIWG height. With the help of the proposed transformation it is possible to obtain instantaneous frequencies in the individual modes, which do not change during impulse when the parameter ρ equals to the true source range.

Synthesized tweek (ρ = 1000 km, H = 88 km, β = 0.6 km^{-1}), its stretched waveform and their amplitude spectra are shown in Fig.3. We can see that the intermode interference practically disappear and the spectrum of the transformed tweek waveform consists of the peaks representing separate modes.

The fitting procedure for a stretched tweek waveform to obtain the effective heights of the EIWG for higher order modes by varying the source range ρ is demonstrated in Fig.4.

Dispersion branches are plotted by different symbols marked at the legends on the graphs with corresponding mode numbers. Horizontal dashed lines show heights h_1 for the corresponding modes calculated from the model.

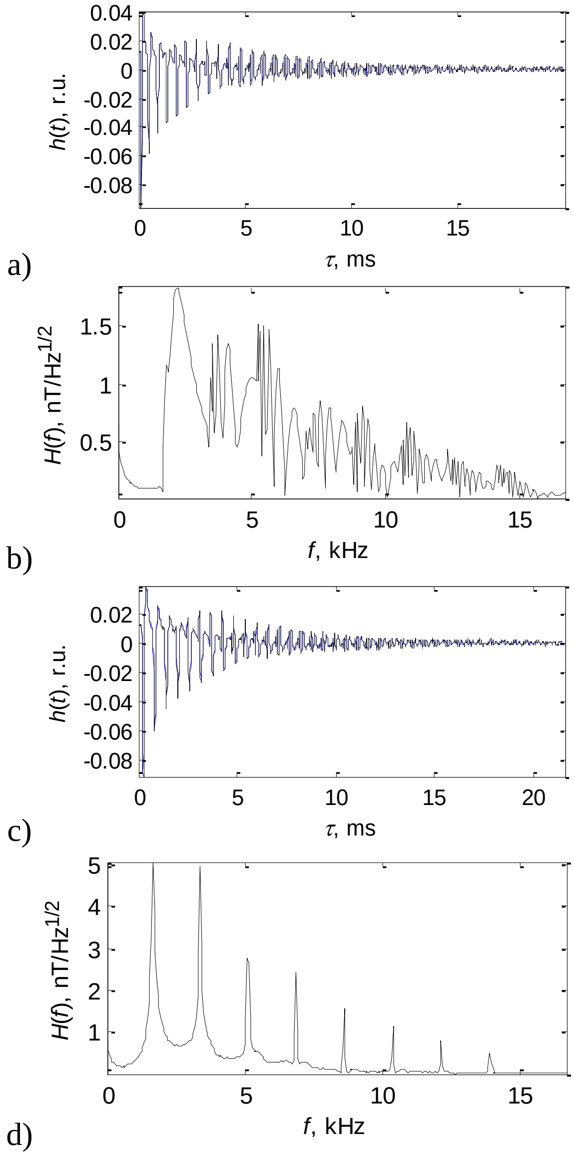

Fig. 3. Original (a) and stretched (b) waveforms, and their amplitude spectra (c) and (d), respectively, of a model tweek-atmospheric calculated for the following parameters: H = 88 km, β = 0.6 km^{-1}, ρ = 1000 km

Cases of underestimated range, ρ = 900 km, exact range, ρ = 1000 km, and overestimated range, ρ = 1100 km are shown in Fig.4(a,b,c) respectively.

We can observe small deviations in the dispersion branches due to large elementary interval, about 10 ms, used for constructing the dynamic spectrum.

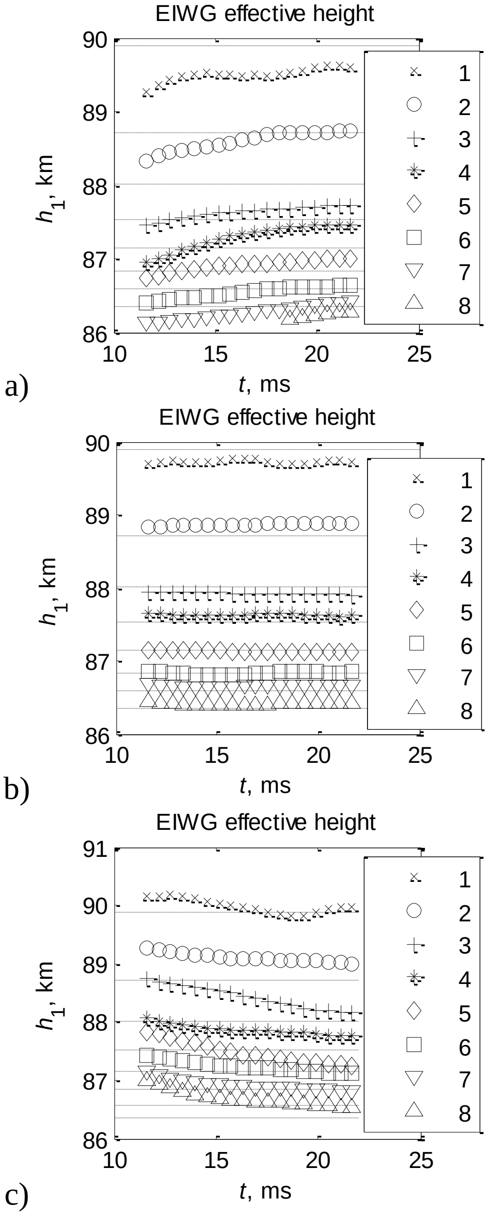

Fig. 4. Example of selecting the source range ρ to get effective heights of the EIWG by the stretched tweek waveform. Horizontal dashed lines show heights h_1 for the corresponding modes calculated from the model.
a) underestimated range, ρ = 900 km, b) exact range, ρ = 1000 km, c) overestimated range, ρ = 1100 km

Accuracy estimations

To estimate accuracy of the proposed methods we synthesized tweek waveforms of 20 ms length for source ranges from 250 to 3500 km. For these distances we expect to observe two or more dispersion branches in the experimental records of tweeks [16] that can be used for estimation of the ionosphere conductivity profile. Together with the first and second methods described in the previous sections for comparison we apply also the third, known method (see e.g. [1,5]), consisting in the two parameters optimization to fit theoretical dependence of the instantaneous frequency Eq.(4) to the synthesized tweeks. In this case we determine the pair of parameters, the source range and the waveguide height, separately for each branch.

Results of numerical simulation for the three methods are presented in Tables 1, 2 where the absolute errors of determination of the source range and effective ionospheric height by the first four modes are shown. In the simulation an additive white noise is applied formed by random values drawn from a normal distribution with mean zero and standard deviation 0.5 of the standard deviation σ of the analyzed part of a tweek signal.

Mean errors for the distance and height estimations and their standard deviations are presented in Table I (noise = 0.5σ) and Table II (noise = 0). We can see that the errors decrease with increase of the mode number for all methods that can be connected with increase of relative accuracy of frequency estimation for the higher order modes.

Table 1

Errors of estimations of the SOD and the effective EIWG height. Model parameters: H = 88 km, β = 0.6 km^{-1}, noise = 0.5σ.

Source-to-observer distance (SOD) error, km		
SOD range, km	Method 1	Method 2
250 - 3500	61±88	36±34
EIWG height error, km		
Mode# (SOD range, km)	Method 1	Method 2
1 (250 – 3500)	0.54±0.31	0.2±0.14
2 (250 – 3500)	0.37±0.41	0.16±0.12
3 (250 – 3000)	0.28±0.46	0.08±0.05
4 (250 – 2250)	0.11±0.15	0.05±0.05

Table 2

Errors of estimations of the SOD and the effective EIWG height. Model parameters: H = 88 km, β = 0.6 km^{-1}, noise = 0.

SOD error, km		
SOD range, km	Method 1	Method 2
250 - 3500	40±37	9±12
EIWG height error, km		
Mode # (SOD range, km)	Method 1	Method 2
1 (250 – 3500)	0.36±0.20	0.16±0.06
2 (250 – 3500)	0.24±0.25	0.12±0.07
3 (250 – 3000)	0.20±0.24	0.04±0.03
4 (250 – 2250)	0.09±0.07	0.03±0.02

As it is follows from the Tables, while the first and the third method show concurring accuracy, the second method shows about two times better results in estimation of the source range and EIWG height.

Summary and conclusion

The proposed technique for estimating the EIWG cutoff frequencies is based on linearizing transformation of dispersion branches for higher order waveguide modes enhanced from dynamic spectrum of tweek in the time domain by seeking an optimal value of source range ρ.

Algorithms of the EIWG parameters estimation for the traditional "sonogram" and the proposed techniques are formulated as follows:

First technique:

1) calculation of dynamic spectrum;

2) selection of dispersion branches;

3) minimization deviation between measured and calculated (Eq.(5)) dispersion branches by seeking optimal values of the SOD and cutoff frequencies.

Second technique:

- minimization the sum of the slope coefficients for selected branches in the tweek by seeking an optimal value of the SOD that includes:

- rescaling tweek waveform with a new value of ρ by transformations Eqs.(7,8);

- calculation of dynamic spectrum;

- selection of dispersion branches;

- calculation of the slope coefficients for the selected dispersion branches.

As we can see, the proposed algorithm is much more computer resource consuming in comparison to the first one. It includes rescaling tweek, calculating the dynamic spectrum and selecting dispersion branches at each cycle of seeking the optimal value of ρ, while in the first algorithm calculation of the dynamic spectrum and selection of the dispersion branches are performed only once. But the second algorithm has essential advantage in the accuracy of parameters estimation as it was demonstrated in the previous section.

Concluding this study we note the following.

Based on the numerical modeling of multimode tweek-atmospherics in the EIWG with exponential vertical conductivity profile of the lower ionosphere, it was shown that the dispersion relations, deduced for the simplest infinite plate waveguide model with perfectly conducting boundaries, can be applied to evaluate parameters of the vertical conductivity profile of the lower ionosphere with high accuracy for a wide range of distances to sources, from about 100 to a few thousand kilometers, as long as two or more dispersion branches in tweek can be detected.

A new technique has been proposed consisting in nonlinear scaling tweek waveform along the time axis that compensates the frequency dispersion in the signal and allows us to improve accuracy of estimation of the EIWG cutoff frequencies, especially in the presence of noises.

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Ключові слова: хвилевод Земля-іоносфера, твік-атмосферик, локація блискавок, діагностика нижньої іоносфери.

Ключевые слова: волновод Земля-ионосфера, твик-атмосферик, локация молний, диагностика нижней ионосферы.

Keywords: Earth-ionosphere waveguide; tweek-atmospheric; lightning location; lower ionosphere diagnostics.

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