UDC 621.313.3

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CRITICAL LOAD OF SELF-EXCITED INDUCTION GENERATORS

The maximum load of a self-excited autonomous induction generator is derived as a function of velocity and capacitance. The maximum value of the capacitance and the minimum frequency are determined as well. Three-dimensional plots of self-excitation boundaries are presented for a practical example.

Отримано аналітичну залежність критичного навантаження автономного асинхронного генератора з самозбудженням від швидкості та ємності конденсаторів. Визначено максимально допустиме значення ємності та мінімально можливу частоту. Представлено тривимірну межу самозбудження асинхронного генератора.

Получена аналитическая зависимость критической нагрузки автономного асинхронного генератора с самовозбуждением от скорости и емкости конденсаторов. Определена максимально допустимая величина емкости и минимально возможная частота. Представлено трехмерную границу самовозбуждения асинхронного генератора.

1. Introduction

Induction generators have found applications in renewable energy (wind and hydro), due to their ability to generate electric power at frequencies that are not exactly tied to their frequency of rotation. The focus of the paper is on self-excited induction generators (SEIG), which generate power off-grid with capacitors providing the necessary reactive power. A mathematical model of SEIG with resistive load includes six differential equations and a static non-linearity due to saturation of the magnetizing inductance [2,4]. To determine possible self-excitation conditions, it is necessary to find parameters of the model that guarantee singularity of a 6x6 matrix [1,3].

The objective of the paper is to extend the results of [1,3] to determine the critical resistive load of SEIG, the maximum possible capacitance for self-excitation, the minimum sufficient frequency and the maximum load for different frequencies. The self-excitation boundaries of the SEIG will be presented as three-dimensional plots suitable for the development of future control strategies.

2. Self-excitation boundaries of induction generator

Consider a two-phase induction generator with resistive loads connected in parallel with capacitors to the stator windings. The self-excitation boundary of the generator is described by the quartic equation [1,3]

$$f_1\omega_e^4 + f_2\omega_e^2 + f_3 = 0\,, \qquad (1)$$

where

$$f_1 = C^2 L_S (L_S L_R - L_M^2) > 0,\ \ f_3 = L_R (Y_L R_S + 1)^2 > 0,$$

$$f_2 = Y_L^2 L_S (L_S L_R - L_M^2) + C^2 R_S^2 L_R - C(2L_S L_R - L_M^2),$$

with $L_S = L_{\sigma S} + L_M$, $L_R = L_{\sigma R} + L_M$.

Fig. 1 shows the general shape of the magnetization inductance L_M as a function of the magnitude of the magnetizing current i_M [2,4]. The curve includes an ascending part rising from L_{M0} to L_{MAX}, a (more or less) horizontal part at L_{MAX} corresponding to a linear magnetic regime, and a descending part corresponding to magnetic saturation.

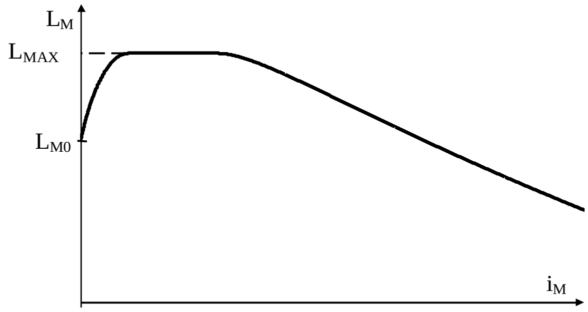

Fig.1. The magnetization inductance as a function of magnetizing current

The following notation is used: ω_e is the angular electrical frequency, C is the capacitance, Y_L is the admittance of the resistive load, $L_{\sigma S}$ and $L_{\sigma R}$ denote the stator and rotor leakage inductances, L_S and L_R denote the stator and rotor inductances.

The quartic equation is a quadratic equation in ω_e^2, which has a real positive solution if and only if

$$f_2 < -2\sqrt{f_1 f_3}\,. \qquad (2)$$

If (2) is satisfied, there are two real positive solutions to the quadratic equation, and we denote the two square roots of these solutions $\omega_{e,\min}$ and $\omega_{e,\max}$. These are the solutions of the quartic equation. The velocity ω can be determined from ω_e using [1,3]

$$\omega = \frac{1}{n_P}\left(\omega_e - \frac{Y_L R_S R_R - \omega_e^2 C R_R L_S + R_R}{\omega_e\left(Y_L\left(L_S L_R - L_M^2\right) + R_S L_R C\right)}\right), \qquad (3)$$

where n_p denotes a number of pole pairs.

From $\omega_{e,\min}$ and $\omega_{e,\max}$, velocities $\omega_{\min}$ and $\omega_{\max}$ can be determined in this manner. The velocities constitute the boundaries where self-excitation is possible. If the procedure is applied with $L_M = L_{MAX}$, the boundaries of self-excitation (which may require triggering to be reached) are obtained. If $L_M = L_{M0}$ is used, the boundaries of spontaneous self-excitation are obtained [2,3].

Alternatively, equation (1) can be presented as a quadratic equation

$$g_1 C^2 - g_2 C + g_3 = 0\,, \qquad (4)$$

where

$g_1 = L_S(L_S L_R - L_M^2)\omega_e^4 + R_S^2 L_R \omega_e^2 > 0\,,$

$g_2 = (2L_S L_R - L_M^2)\omega_e^2 > 0\,,$

$g_3 = Y_L^2(L_S(L_S L_R - L_M^2)\omega_e^2 + L_R R_S^2) + 2L_R R_S Y_L + L_R > 0\,.$

The quadratic equation has two real positive solutions if and only if

$$g_2{}^2 - 4g_1 g_3 > 0\,. \qquad (5)$$

The solutions of the quadratic equation are $C_{\min}$ and $C_{\max}$. Substitution them into equation (3) gives $\omega_{\min}$ and $\omega_{\max}$.

3. Critical load of induction generator for different frequencies

Rewrite equation (1) in the following manner

$$a_1 Y_L^2 + a_2 Y_L + a_3 = 0\,, \qquad (6)$$

where

$a_1 = L_S(L_S L_R - L_M^2)\omega_e^2 + L_R R_S^2 > 0\,,\; a_2 = 2L_R R_S > 0\,,$

$a_3 = C^2(L_S(L_S L_R - L_M^2)\omega_e^4 + R_S^2 L_R \omega_e^2) -$

$- C(2L_S L_R - L_M^2)\omega_e^2 + L_R.$

Equation (6) has a single solution

$$Y_L = \left(-a_2 + \sqrt{a_2^2 - 4a_1 a_3}\right)/(2a_1) \qquad (7)$$

that is positive or zero if and only if $a_3 \le 0$.

The last inequality can be put in a form

$$a_3(C) = b_1 C^2 - b_2 C + b_3 \le 0\,, \qquad (8)$$

where $b_1 = L_S(L_S L_R - L_M^2)\omega_e^4 + R_S^2 L_R \omega_e^2 \ge 0$,

$b_2 = (2L_S L_R - L_M^2)\omega_e^2 \ge 0\,,\; b_3 = L_R > 0\,.$

The function $a_3(C)$ is negative or zero for all capacitance values between the two solutions of equation (8)

$$C_{1,2} = \left(b_2 \pm \sqrt{b_2^2 - 4b_1 b_3}\right)/(2b_1)\,, \qquad (9)$$

if and only if $b_2^2 - 4b_1 b_3 \ge 0$. Note, that the values $C = C_{1,2}$ correspond the case with $Y_L = 0$.

Substituting b_1, b_2 and b_3 into the last inequality leads to

$$\omega_e \ge 2L_R R_S / L_M^2\,. \qquad (10)$$

Since equations (1) and (6) describe the self-excitation boundary, the minimum possible operating frequency of the generated voltage of the SEIG cannot be lower than $f_{\min op} = L_R R_S / \left(\pi L_{MAX}^2\right)$ and the minimum value is for the unloaded generator. The minimum spontaneous self-excitation frequency of the unloaded SEIG cannot be lower than $f_{\min se} = L_R R_S / \left(\pi L_{M0}^2\right)$

The maximum value of the load Y_L in (7) corresponds to a minimum value of $a_3(C)$. Equating $\partial a_3(C)/\partial C$ to zero gives the value of capacitance

$$C_{ex} = \frac{b_2}{2b_1} = \frac{2L_S L_R - L_M^2}{2(L_S(L_S L_R - L_M^2)\omega_e^2 + R_S^2 L_R)} > 0\,. \qquad (11)$$

Since $\partial^2 a_3(C)/\partial^2 C = 2b_1 > 0$, the value C_{ex} corresponds to the minimum of $a_3(C)$ and $a_3(C_{ex}) < 0$.

Substituting equation (11) into equations (8) and (7) gives the dependency of the maximum possible loads for different angular frequencies.

$$Y_L = \left(-a_2 + \sqrt{a_2{}^2 + \frac{b_2^2}{\omega_e^2} - 4a_1 b_3}\right)/(2a_1)\,. \qquad (12)$$

Combining the final expression with equation (3) gives the dependency on velocities.

4. Maximum possible value of the capacitance

Rewrite inequality (8) as follows

$$a_3(\omega_e) = d_1\omega_e^4 + d_2\omega_e^2 + d_3 \le 0\,, \qquad (13)$$

where $d_1 = C^2 L_S(L_S L_R - L_M^2) > 0$, $d_3 = L_R > 0$,

$d_2 = R_S^2 L_R C^2 - C(2L_S L_R - L_M^2)\,.$

The quartic equation is a quadratic equation in ω_e^2, which has a real positive solution if and only if

$$d_2 < -2\sqrt{d_1 d_3}\,. \qquad (14)$$

If (14) is satisfied, the function $a_3(\omega_e)$ is negative or zero for all angular frequencies values between the two solutions of equation (13)

$$\omega_{e1,2} = \sqrt{\left(-d_2 \pm \sqrt{d_2^2 - 4d_1 d_3}\right)/(2d_1)}\,. \qquad (15)$$

Note, that the values $\omega_e = \omega_{e1,2}$ correspond to the no load case $Y_L = 0$.

Substituting d_1, d_2 and d_3 into inequality (14) gives

$$C < \frac{2L_S L_R - L_M^2 - 2\sqrt{L_S L_R(L_S L_R - L_M^2)}}{R_S^2 L_R}\,. \qquad (16)$$

Therefore, the maximum capacitance that cannot be exceeded for successful triggered self-excitation of SEIG is

$$C_{\max op} = \frac{2L_S L_R - L_{MAX}^2 - 2\sqrt{L_S L_R(L_S L_R - L_{MAX}^2)}}{R_S^2 L_R}\,. \qquad (17)$$

In this case $\omega_{e1} = \omega_{e2} = \sqrt{-d_2/(2d_1)}$ and $Y_L = 0$. Substituting L_{M0} instead L_{MAX} into (17) gives the maximum possible value $C_{\max se}$ for spontaneous self-excitation of the unloaded SEIG.

It would be tempting to assume that the minimum value of the negative $a_3(\omega_e)$ would correspond the maximum Y_L. In this case, in equation (7), not only a_3

but also a_1 are functions of ω_e. Unfortunately, differentiation of Y_L with respect to ω_e and equating the result to zero does not give an implicit solution for extreme angular frequency. It is necessary to find a real solution of a polynomial of 12-th order.

5. Maximum possible value of the load

Substituting a_1, a_2 and b_2, b_3 into equation (12) and dividing both the nominator and denominator by $L_S L_R$ results in

$$Y_L = \frac{-2R_S + L_S\sqrt{(\sigma-1)^2\omega_e^2}}{2L_S^2\sigma\omega_e^2 + 2R_S^2}, \tag{18}$$

where $\sigma = \left(L_S L_R - L_M^2\right)/(L_S L_R) > 0$.

Since $\sigma - 1 = -L_M^2/(L_S L_R) < 0$ and $Y_L \geq 0$ then

$$Y_L = \frac{-2R_S - L_S(\sigma-1)\omega_e}{2L_S^2\sigma\omega_e^2 + 2R_S^2}, \tag{19}$$

and $\omega_e \geq 2R_S/(L_S(1-\sigma)) = 2L_R R_S / L_M^2$.

Equating $\partial Y_L(\omega_e)/\partial\omega_e$ to zero gives a quadratic equation from which the value of the angular frequency corresponding to the maximum load can be deduced to be

$$\omega_{eYLMAX} = 2R_S\left(1 \pm \frac{\sigma+1}{2\sqrt{\sigma}}\right)/(L_S(1-\sigma)). \tag{20}$$

The condition of inequality from (19) is satisfied if the "+" sign is chosen. After simple transformations, the formula becomes

$$\omega_{eYLMAX} = \frac{R_S(1+\sqrt{\sigma})}{L_S\sqrt{\sigma}(1-\sqrt{\sigma})}. \tag{21}$$

Substituting ω_{eYLMAX} into equation (19) gives the maximum load that cannot be exceeded

$$Y_{LMAX} = \frac{(\sqrt{\sigma}-1)^2}{4R_S\sqrt{\sigma}}. \tag{22}$$

Substituting ω_{eYLMAX} into equation (11) gives a corresponding value of the capacitance

$$C_{YLMAX} = \frac{L_S(\sqrt{\sigma}-1)^2}{4R_S^2}. \tag{23}$$

Therefore, the SEIG can sustain a largest load if the angular frequency equals ω_{eYLMAX} and the capacitance is C_{YLMAX}. The largest load cannot exceed Y_{LMAX}. Substituting L_{MAX} into (21)-(23) gives the maximum possible load for the operating mode of SEIG (or for triggered self-excitation) while substituting of L_{M0} gives the maximum possible load during spontaneous self-excitation.

6. Computation and experimental results

A small two-phase induction motor (Bodine KCI-22A1, with rated values 7.5W, 24V, 60 Hz, and 3350 rpm) was used for experiments. The following parameters of the generator were determined in [4] R_S =49.5 Ω, R_R =24 Ω, $L_{\sigma S} = L_{\sigma R}$ =0.027H, n_p =1, L_{MAX} =0.305 H, L_{M0} =0.24 H. Computed self-excitation boundaries for the generator with no load, 700Ω, and 500Ω loads based on equations (1) and (3) with $L_M = L_{MAX}$ are shown in Fig. 2. The experimental validation of these boundaries was performed by determining *collapse* regions. Specifically, the generator was spun at a high velocity and an initial voltage was applied to the capacitor in one of the phases, triggering self-excitation. Then, the velocity was reduced until the voltage collapsed. The result defined the experimental boundary for self-excitation.

The induction motor was tested as a generator by coupling it to a DC motor/tachometer under closed-loop velocity control. A DS1104 data acquisition and control board from dSPACE was used to implement the PID control law for the DC motor and to collect the data.

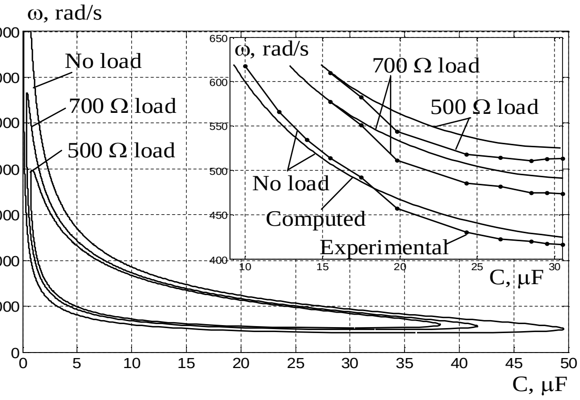

Fig.2. Self-excitation boundaries

Computation gives the following values of the critical capacitances $C_{\max op}$ =49.59 μF, $C_{\max se}$ =34,39 μF, frequencies $f_{\min op}$ =56.26 Hz, $f_{\min se}$ =73.07 Hz, loads Y_{LMAXop} =0.00468 Ω^{-1}, $C_{YLMAXop}$ =12.398 μF, $\omega_{eYLMAXop}$ =870.338 rad/s, Y_{LMAXse} =0.003638 Ω^{-1}, $C_{YLMAXse}$ =8.598 μF, $\omega_{eYLMAXse}$ =1083.07 rad/s, where the additional index "op" denotes computation results with $L_M = L_{MAX}$ and the index "se" denotes computation results with $L_M = L_{M0}$. Note that the minimum operating frequency is lower than the rated frequency of the generator (this case corresponds to no load).

Fig. 3 shows a three-dimensional plot of self-excitation boundaries based on equation (7) with $L_M = L_{MAX}$. The parabolic function $Y_L = f(C)$ was computed for different angular frequencies over the critical value defined by (10) within the capacitance range defined by (9). The corresponding velocity values were taken from (3). All the operating points of the generator are inside the figure limited by the surfaces $Y_L = f(C,\omega)$ and $Y_L = 0$. Curve 1 shows maximum loads for different frequencies (velocities) according to (12). Curves 2, 3, and 4 are the self-excitation boundaries for 500 Ω, 700 Ω and no load cases respectively computed based on (4). Point 5 is the point with $Y_L = Y_{LMAXop}$. As can be seen from the figure, this point is unique. The value of the frequency is twice the rated value.

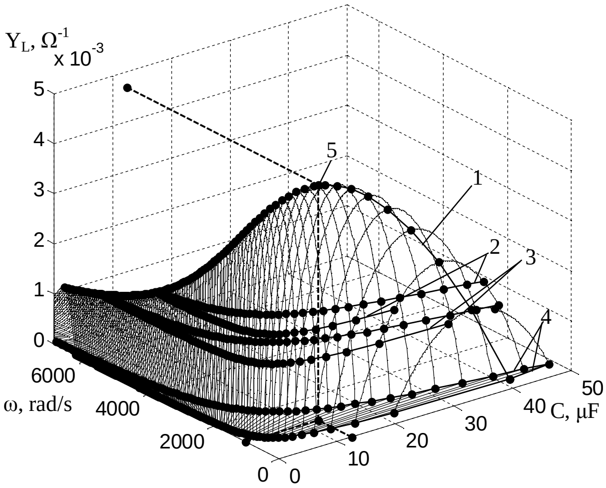

Fig.3. $Y_L = f(C, \omega)$ computed through the range of angular frequencies

Fig. 4 also shows a three-dimensional plot of the self-excitation boundaries based on equation (7) with $L_M = L_{MAX}$. This time, the parabolic functions $Y_L = f(\omega)$ were computed for capacitances lower than the critical value defined by (17) within the frequency range defined by (15). Curve 1 shows the maximum loads for different capacitances, which were computed by iterations. Curves 2, 3, and 4 are the self-excitation boundaries for the 500 Ω, 700 Ω and no load cases respectively, computed based on (1). Point 5 is the point with $Y_L = Y_{LMAXop}$. As may be expected, point 5 is the same point on the both figures.

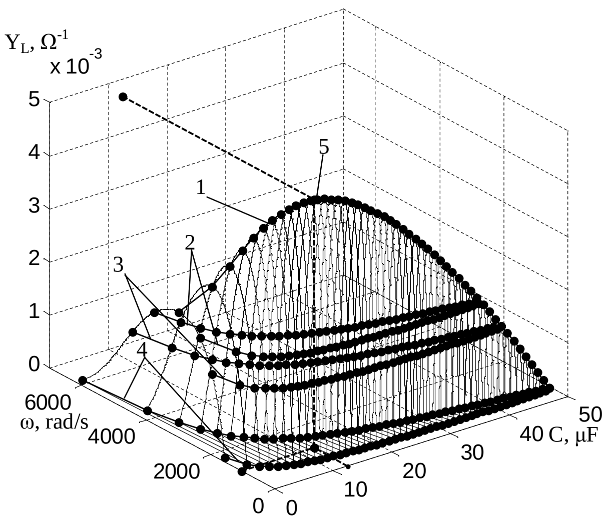

Fig. 4. $Y_L = f(C, \omega)$ computed through the range of capacitances

7. Conclusions

The paper develops simple formulas for the critical values of maximum capacitance and load, and minimum angular frequency of SEIG. This allows a quick choice of necessary equipment to implement a self-excitation system of SEIG. The three-dimensional plot of the self-excitation boundaries can be a basis for the development of a control strategy of self-excitation to provide the best loading of the generator.

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Received 05.07.2011

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