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# WIDEBAND WATTMETER OF TRANSFER POWER WITHOUT SELF CONSUMPTION ERROR

*The article considers the method of transfer power measurement without self consumption error using sum-difference method.*

***Keywords:*** *transfer power, self consumption error, sum–difference method.*

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*Описан метод измерения проходной мощности без погрешности от собственного потребления с использованием суммо–разностного метода.*

***Ключевые слова:*** *проходная мощность, погрешность от собственного потребления, суммо–разностный метод.*

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# ШИРОКОПОЛОСНИЙ ВАТМЕТР ПРОХІДНОЇ ПОТУЖНОСТІ БЕЗ ПОХИБКИ ВІД ВЛАСНОГО СПОЖИВАННЯ

*Описано метод вимірювання прохідної потужності без похибки від власного споживання з використанням сумо–різницевого методу.*

***Ключові слова:*** *прохідна потужність, похибка від власного споживання, сумо–різницевий метод*

Active power is defined as integral of multiplied instantaneous values of load voltage and load current.

$$P_a = \frac{1}{T}\int_0^T u_i(t) i_i(t)\cdot dt, \qquad (1)$$

where T is time of observing, that are divisible on period of signal; $u_i(t)$ is load voltage; $i_i(t)$ is load current.

For getting product in expression (1) two methods are used.

*First method* is based on direct multiplying of instantaneous values of voltage $u_i(t)$ and current $i_i(t)$ signals with using of analog and digital multipliers [1, 2].

Error of conversation has amplitude and phase (time) components. Phase error is important under reactive load.

*Second method* is based on getting product of voltage $u_i(t)$ and current $i_i(t)$ by taking square of sum of voltage and current signals

$$\left[u(t) + Z\cdot i(t)\right]^2 = u^2(t) + Z^2\cdot i^2(t) + 2Z\cdot u(t)\cdot i(t) \;. \qquad (2)$$

There are few algorithms for deliverance $u^2(t)$, and $Z^2\cdot i^2(t)$, for example, method of three voltmeters [3], sum–difference method [4,5].

In voltmeters of transfer power exist specified errors of self consumption. They usually present in circuits of voltage or of current conversation, or both of them.

Because of presence of such errors high accuracy is possible to achieve only with certain ratios of voltage and current. One of the methods to deliver errors, that are caused by consumption in wattmeters circuit, is shown in [5]. Its disadvantage is presence of additional devices of error compensation and necessity of consideration this additional devices.

Purpose of this research is the development of wideband wattmeter's with absent (minimum) error of self consumption that gives additional possibilities of optimization by wideband criteria of wattmeter's input link.

That's why we will consider equivalent circuit of wattmeter's input links that is described in [5]. The scheme is shown on Fig.1.

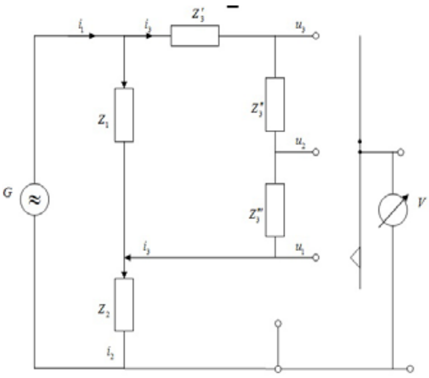

Fig. 1. Equivalent schematic of wattmeter's input links

On Fig. 1 are: G – signal generator; V – voltmeter; $Z_1$ – load resistance; $Z_3' + Z_3'' + Z_3''' = Z_3$ – resistance of two–stage voltage divider; $Z_2$ – shunt resistance.

We will create 3 equations

$$u_1 = i_2 Z_2, \quad (3)$$

$$u_2 = u_1 + i_3 Z_3''', \quad (4)$$

$$u_3 = u_1 + i_3 (Z_3''' + Z_3''). \quad (5)$$

Let's write obvious equations

$$i_1 + i_3 - i_2 = 0, \ (6)$$

$$i_3 = \frac{u_i}{Z_3}, \ (7)$$

$$i_2 = \frac{U_1}{Z_2}, \quad (8)$$

$$i_1 = i_i. \quad (9)$$

After substitution (6) in (7) – (9) will get:

$$i_i + \frac{u_i}{Z_3} - \frac{u_1}{Z_2} = 0, \quad (10)$$

or

$$u_1 = u_i \frac{Z_2}{Z_3} + i_i Z_2. \quad (11)$$

Use of (11) and (7) in (4) gives:

$$u_2 = u_i \frac{Z_2 + Z_3'''}{Z_3} + i_i Z_2. \quad (12)$$

Substituting (11) and (7) into (5) gives

$$u_3 = u_i \frac{Z_2 + Z_3''' + Z_3''}{Z_3} + i_i Z_2. \quad (13)$$

Let's define:

$$A = Z_2 / Z_3 \quad (14)$$

$$B = (Z_2 + Z_3''') / Z_3 \quad (15)$$

$$C = (Z_2 + Z_3''' + Z_3'') / Z_3. \quad (16)$$

Then equation (11, 12, 13) will have such look

$$u_1 = u_i A + i_i Z_2 \quad (17)$$

$$u_2 = u_i B + i_i Z_2 \quad (18)$$

$$u_3 = u_i C + i_i Z_2. \quad (19)$$

We will take square of each equation and multiply with coefficients ($K_1$, $K_2$, $K_3$), and result of measurements will note like

$$N = N_2 - N_1 - N_3, \quad (20)$$

where

$$N_2 = K_2 u_2^2 \quad (21)$$

$$N_1 = K_1 u_1^2 \quad (22)$$

$$N_3 = K_3 u_3^2. \quad (23)$$

Then

$$N = \left[u_н^2 B^2 + (i_н Z_2)^2 + 2u_н i_н Z_2 B\right] K_2 - \\ -\left[u_н^2 A^2 + (i_н Z_2)^2 + 2u_н i_н Z_2 A\right] K_1 - \\ -\left[u_н^2 C^2 + (i_н Z_2)^2 + 2u_н i_н Z_2 C\right] K_3. \quad (24)$$

In (24) components, that include $u_i i_i$ are proportional to power, components, that include, $u_i{}^2$ and $i_i{}^2$ are redundant. It is necessary to find invariance conditions concerning components, that include $u_i{}^2$ and $i_i{}^2$.To do this let's collect coefficients at defined components and equate them with zero. Then invariance conditions for $i_i{}^2$ will be

$$K_2 - K_1 - K_3 = 0. \quad (25)$$

Invariance condition for $u_i{}^2$ will be

$$K_2 B^2 - K_1 A^2 - K_3 C^2 = 0. \quad (26)$$

From (25) and (26) it's obvious, that invariance conditions for $u_i{}^2$ and $i_i{}^2$ aren't equal and they can act at the same time only with certain values of $B$, $A$, $C$, $K_2$, $K_1$, $K_3$.

Invariance conditions for square of current (25).

As we have only two equations (25) and (26) and six variables it's impossible to solve these equations relatively to each variable. In such cases some of variables should be defined and also some ratios between other variables should be found. From (25) we can define few cases

$$K_2 = K_1 + K_3 \quad (27)$$

$$K_1 = K_2 - K_3 \quad (28)$$

$$K_3 = K_2 - K_1. \quad (29)$$

To consider condition (26) we will make few changes and substitutions.

Let's define:

$$Z_3'''/Z_3=\alpha \qquad (30)$$

$$Z_3''/Z_3=\beta . \qquad (31)$$

From equations (14), (15), (16) and (30), (31) will get

$$B=A+\alpha \qquad (32)$$

$$C=A+\alpha+\beta . \qquad (33)$$

Then

$$B^2=A^2+2A\alpha+\alpha^2 \qquad (34)$$

$$C^2=A^2+\alpha^2+\beta^2+2A\alpha+2A\beta+2\alpha\beta . \qquad (35)$$

Substituting (34), (35) into (26) gives

$$K_2(A^2+2A\alpha+\alpha^2)-K_1A^2-$$
$$-K_3(A^2+\alpha^2+\beta^2+2A\alpha$$
$$+2A\beta+2\alpha\beta)=0 \ . \qquad (36)$$

Grouping coefficients at $A^2$, $A\alpha$, $\alpha^2\beta^2$, $A\beta$, $\alpha\beta$ and substituting invariance conditions (25), which appears in the formula gives:

$$(K_2-K_3)\alpha(2A-\alpha)-\beta K_3(\beta+2A+2\alpha)=0 . \qquad (37)$$

In [5] is shown, that maximum value of coefficient at product $u_i\cdot i_i$ will take place when

$$\alpha=\beta . \qquad (38)$$

Taking this into account we will transform (37) and get

$$\alpha\left[(K_2-K_3)(2A+\alpha)-K_3(2A+3\alpha)\right]=0 . \qquad (39)$$

We can find K3 from (39)

$$K_3=\frac{K_2(2A+\alpha)}{4(A+\alpha)} . \qquad (40)$$

Substitution (14), (30) into (40) gives

$$K_3=\frac{K_2}{4}\left(1+\frac{Z_2}{Z_2+Z_3'''}\right) . \qquad (41)$$

Taking (25) and (41) into account

$$K_1=\frac{K_2}{4}\left(3-\frac{Z_2}{Z_2+Z_3'''}\right) . \qquad (42)$$

Let's assume, that $K_2=1$,

$$K_3=\frac{1}{4}\left(1+\frac{Z_2}{Z_2+Z_3'''}\right) \qquad (43)$$

$$K_1=\frac{1}{4}\left(3-\frac{Z_2}{Z_2+Z_3'''}\right) . \qquad (44)$$

Invariance conditions for squares of voltage and current (25) and (26) are true with such values of $K_2$, $K_1$, $K_3$ and also error of self consumption is considered. If invariance conditions (25), (26) are implemented from equation (24) will get

$$N=2u_ii_iZ_2(BK_2-AK_1-CK_3) . \qquad (45)$$

If (38) is implemented

$$B=A+\alpha \qquad (46)$$

$$C=A+2\alpha . \qquad (47)$$

Substituting (46), (47) into (45) gives:

$$N=2u_ni_nZ_2\left[(A+\alpha)K_2-AK_1-(A+2\alpha)K_3\right]=$$
$$=2u_ni_nZ_2\left[(A(K_2-K_1-K_3)+\alpha(K_2-2K_3)\right] . \qquad (48)$$

After taking (25) into account

$$N=2u_ii_iZ_2\alpha(K_2-2K_3) . \qquad (49)$$

After substitution value of $K_3$ from (43) into (49) and determined values $K_2=1 ; \alpha=\frac{Z_3'''}{Z_3} ; \varepsilon=\frac{Z_2}{Z_2+Z_3'''}$ will get:

$$N=u_ii_iZ_2\alpha(1-\varepsilon) . \qquad (50)$$

Instantaneous power is product of voltage and current

$$p=u_ii_i . \qquad (51)$$

Power on the load will be:

$$p=u_ii_i=\frac{N}{Z_2\alpha(1-\varepsilon)} . \qquad (52)$$

Substituting N from (20), (21), (22), (23) into (52) gives

$$p=u_ii_i=\frac{K_2u_2^2-K_1u_1^2-K_3u_3^2}{Z_2\alpha(1-\varepsilon)} . \qquad (53)$$

Let's recall, that: $Z_2$ – shunt resistance;

$\alpha=Z_3'''/Z_3$ – transfer coefficient of first voltage divider

$$K_1=0,25(3-\varepsilon) ; \qquad (54)$$

$$K_2=1 ; \qquad (55)$$

$$K_3=0,25(1+\varepsilon) ; \qquad (56)$$

$$\varepsilon=\frac{Z_2}{Z_2+Z_3'''} . \qquad (57)$$

For clarity of formula (53) we will make some conversions and substitutions and will get

$$p=\frac{Z_3}{Z_3'''}\cdot\frac{u_2^2-0,75u_1^2-0,25u_3^2}{Z_2}\times$$
$$\times\left\{1+\frac{\varepsilon}{1-\varepsilon}\left[1+\frac{0,25\left(u_1^2-u_3^2\right)}{u_2^2-0,75u_1^2-0,25u_3^2}\right]\right\} . \qquad (58)$$

Expression (58) is correct formula of power. It is structured in such way, that correction of self consumption error is separated like component that is in parenthesis after unit in formula (58)

$$\delta_p = \frac{\varepsilon}{1-\varepsilon}\left[1+\frac{0,25\left(u_1^2-u_3^2\right)}{u_2^2-0,75u_1^2-0,25u_3^2}\right]. \quad (59)$$

Also let’s estimate correction of self consumption error in the worst case, while scheme is working on dc current. In this case we will put $u_3 = 3u_1, u_2 = 2u_1$ and if $\varepsilon << 1$ then we can write

$$p \cong \frac{Z_3}{Z_3'''}\cdot\frac{u_2^2-0,75u_1^2-0,25u_3^2}{Z_2}\times$$

$$\times\left[1+\varepsilon\left(1+0,25\cdot\frac{u_1^2-u_3^2}{u_2^2-0,75u_1^2-0,25u_3^2}\right)\right]. \quad (60)$$

## Conclusions

After detailed analysis of scheme on fig. 1 we got accurate expression of power (58) including error of self consumption in the circuits in input links of wattmeter.

In expression (58) no conditions about values of resistors $Z_2$ and $Z_3'''$ are imposed, that gives possibility to chose, them due to criteria of maximum wideband.

Comparing with wattmeter that is described in [5], there are no additional devices for neglecting error of self consumption in scheme on fig. 1 that may also extend the frequency range and reduce measurement error.

During optimization of input links using criteria of frequency error consumption of the source voltage is increased, nevertheless it doesn’t cause an error of load power measurement.

For implementation of wattmeter, that is based on scheme (Fig. 1) it’s necessary to know exact values of resistors: $Z_2$, $Z_3'''$, $Z_3''$, $Z_3$.

Voltmeter for measuring $u_1$, $u_2$, $u_3$, should be wideband and have possibility to measure squares of RMS of voltages values.

Advantage of scheme on figure 1 is that voltages $u_1$, $u_2$, $u_3$ is measured relatively to ground point of signal source (generator).

The drawback of scheme on fig. 1 is that the conversion coefficient of scheme on fig. 1 is in 4 times smaller, then with classic sum-difference method.

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