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MODELING AND IDENTIFICATION OF SYSTEMS WITH FRACTIONAL ORDER INTEGRAL AND DIFFERENTIAL

Abstract. *A universal model of fractional-order differential equation is proposed. It is derived in form hyper neuron, based on a representation of the solution of the equation by finite increments and a modified form of the Riemann-Liouville. Implemented method for identifying parameters of objects by fractional differential equations is described on the base hyper neuron and modified genetic algorithms.*
Keywords*: fractional differential, fractional integral, hyper neuron, genetic algorithms, multipoint crossing, parameter identification*

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МОДЕЛИРОВАНИЕ И ИДЕНТИФИКАЦИЯ ПАРАМЕТРОВ СИСТЕМ С ДРОБНЫМИ ИНТЕГРАЛЬНО-ДИФФЕРЕНЦИРУЮЩИМИ ЭЛЕМЕНТАМИ

Аннотация. *Предложена универсальная модель дробно-дифференциального уравнения произвольного порядка в виде гипернейрона, основанная на представлении решения уравнения методом конечных приращений и модифицированной формы Римана-Лиувилля. На базе гипернейрона и модифицированных генетических алгоритмов реализован метод идентификации параметров объектов, описываемых дробно-дифференциальными уравнениями.*
Ключевые слова*: дробное дифференцирование, дробное интегрирование, гипернейрон, генетические алгоритмы, многоточечное скрещивание, идентификация параметров*

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МОДЕЛЮВАННЯ ТА ІДЕНТИФІКАЦІЯ ПАРАМЕТРІВ СИСТЕМ З ДРОБОВИМИ ІНТЕГРАЛЬНО-ДИФЕРЕНЦІЙНИМИ ЕЛЕМЕНТАМИ

Анотація. *Запропоновано універсальну модель дробово-диференційних рівнянь довільного порядку у вигляді гіпернейрона, яку засновано на поданні рішень рівняння методом кінцевих приростів і модифікованої форми Рімана-Ліувіля. На базі гіпернейрона і модифікованих генетичних алгоритмів реалізовано метод ідентифікації параметрів об'єктів з дробово-диференційними моделями.*
Ключові слова*: дробове диференціювання, дробове інтегрування, гіпернейрон, генетичні алгоритми, багато точкове схрещування, ідентифікація параметрів*

Introduction. Actual physical processes which occurring in nature and in man–made technological systems and environments are often dynamic systems, mathematical models which include differential equations of fractional order.

So, when controlling the transfer of heat processes at the expressed of diffusion, convection, it is advisable to describe the management and regulators of fractional differential equations of order 0,5. When the source is of heat (or cold) or performs condensing or boiling freon, the liquid drops, gas bubbles and the heat exchange medium converted into a structure which similar to fractal. In such an environment are observed effects of abnormal sub-and super-diffusion [6], which in describing order of the differential equations in the general case, different from 0,5. In the electrochemical capacitors (EchC), which currently get spread because of the possibility of rapid charge and discharge in systems for the recovery of the kinetic energy electric cars (KERS), due to the porous structures of the electrodes takes place the nonstationary diffusion current strength depends of voltage of fractional derivative [7]. As a result of EChC, as in the accumulators, accumulation and recoil energy is described by a combination of integrating and fractional integrating equations [8]. Described for a class of objects use of traditional methods of investigation can lead to significant errors in the identification of parameter discrepancy of dynamic and static of indicators systems, and as a result, errors in the synthesis control system [8].

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Purpose of work – develop a universal model of fractional differential equation of arbitrary order as for solving the equation method finite increments, as for the analysis and identification parameters of such systems with application genetic algorithms, implementing models of such objects at discrete control systems.

Materials of research. Fractional integration is a particular case of fractional differential equation of the following form:

$$\begin{aligned}&\dots a_2 D^{2+\mu} y + b_2 \frac{d^2 y}{dt^2} + a_1 D^{1+\mu} y + b_1 \frac{dy}{dt} + \\ &+ a_0 D^{\mu} y + b_0 y = kx,\end{aligned} \tag{1}$$

where $D^{\mu} y$ – the fractional derivative y of order μ.

Because

$$\frac{d^2 y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right),\ D^{1+\mu} y = \frac{d}{dt}\left(D^{\mu} y\right),$$

$$D^{2+\mu} y = \frac{d^2}{dt^2}\left(D^{\mu} y\right) = \frac{d}{dt}\left(D^{1+\mu} y\right),$$

then passing to finite increments and considering that

$$\begin{aligned}&dy \to \Delta y_i = y_i - y_{i-1};\\ &d^2 y \to \Delta\left(\Delta y_i\right) = \Delta y_i - \Delta y_{i-1};\dots\\ &D^{1+\mu} y \to \left(\Delta D^{\mu} y\right)_i = \left(D^{\mu} y\right)_i - \left(D^{\mu} y\right)_{i-1};\\ &D^{2+\mu} y \to \Delta\left(\Delta D^{\mu} y_i\right) = \left(\Delta D^{\mu} y\right)_i - \left(\Delta D^{\mu} y\right)_{i-1};\dots\end{aligned}$$

obtain

$$\frac{\Delta}{\Delta t}\left(\frac{\Delta y_i}{\Delta t}\right) = \frac{y_i - 2y_{i-1} + y_{i-2}}{\Delta t^2},$$

$$\frac{\Delta}{\Delta t}\left(\frac{\Delta D^{\mu} y_i}{\Delta t}\right) = \frac{D^{\mu} y_i - 2D^{\mu} y_{i-1} + D^{\mu} y_{i-2}}{\Delta t^2}.$$

As a result, transformation of the original equation of replacing "of fractional difference":

$$\begin{aligned}D_i^{\mu} y &= Kx_i - A_2 D_{i-2}^{\mu} y - B_2 y_{i-2} + \\ &+ A_1 D_{i-1}^{\mu} y + B_1 y_{i-1} - B_0 y_i,\end{aligned} \tag{2}$$

where

$$A_1 = \frac{2a_2 + a_1 \Delta t}{a_2 + a_1 \Delta t + a_0 \Delta t^2};\ A_2 = \frac{a_2}{a_2 + a_1 \Delta t + a_0 \Delta t^2};$$

$$B_0 = \frac{b_2 + b_1 \Delta t + b_0 \Delta t^2}{a_2 + a_1 \Delta t + a_0 \Delta t^2};\ B_1 = \frac{2b_2 + b_1 \Delta t}{a_2 + a_1 \Delta t + a_0 \Delta t^2};$$

$$B_2 = \frac{b_2}{a_2 + a_1 \Delta t + a_0 \Delta t^2};\ K = \frac{k \Delta t^2}{a_2 + a_1 \Delta t + a_0 \Delta t^2}.$$

The desired value of output coordinates can be found by means of fractional integration found value of fractional derivative $y_i = I^{\mu} D_i^{\mu} y$. In discrete form the equation (2) may be represented by structural diagram shown in Fig. 1. Obvious that changing the amount of feedbacks and recalculating coefficients A_n, B_n, can solve an equation of order $n + \mu$.

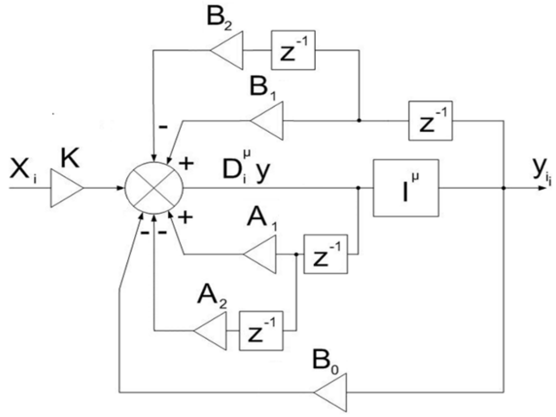

Fig. 1. Structural diagram of hyper neuron

Named it totality links of hyper neuron whose properties on contemporary representations of the close to the properties of biological neurons [1]. However, when computing solution of equation (2) on $i-$th step there is a problem with the lack of value of the output coordinates y_i in the moment of calculation by the right side.

We rewrite (2) as follows:

$$\begin{cases} D_i^{\mu} y' = Kx_i - A_2 D_{i-2}^{\mu} y - B_2 y_{i-2} + A_1 D_{i-1}^{\mu} y + B_1 y_{i-1}, \\ D_i^{\mu} y = D_i^{\mu} y' - B_0 y_i. \end{cases}$$

Fractional integral of order $0 < \mu \le 1$ can be calculated from the approximate formula, based on the modified discrete form of the Riemann-Liouville [2]:

$$y_i = I^{\mu} D_i^{\mu} y \approx \sum_{j=1}^{i} k_{i-j+1}^{\mu} D_j^{\mu} y, \tag{3}$$

where

$$k_n^{\mu} = \frac{\Delta t^{\mu}}{\Gamma(1+\mu)}\left(n^{\mu} - (n-1)^{\mu}\right).$$

Then

$$y_i = \sum_{j=1}^{i-1} k_{i-j+1}^{\mu} D_j^{\mu} y + k_1\left(D_i^{\mu} y' - B_0 y_i\right). \tag{4}$$

After transformations, obtain the formula to the calculations verify y_i:

$$y_i = \frac{\sum_{j=1}^{i-1} k_{i-j+1}^{\mu} D_j^{\mu} y + k_1 D_i^{\mu} y'}{1 + k_1 B_0}. \tag{5}$$

And for the next calculation step, we find

$$D_i^{\mu} y = D_i^{\mu} y' - B_0 y_i. \tag{6}$$

The expressions obtained allow excluding a circular reference in calculating of the output signal hyper neuron. Using expressions (5) and (6) and the calculation of the output signals of a closed system with a given fractional order astatism μ, the open loop which describes transfer function

$$H^{\mu}{}_{opt}(p)=\frac{1}{\alpha T_{\nu}^{\mu}p^{\mu}}\cdot\frac{1}{T_{\nu}p+1}, 0<\mu\le 1, \quad (7)$$

where α – parameter settings, T_{ν} – uncompensated short time constant of the control object. Solution of the corresponding of fractional differential equation in the operator form:

$$\left(\alpha T_{\nu}^{\mu+1}p^{\mu+1}+\alpha T_{\nu}^{\mu}p^{\mu}+1\right)y=x. \quad (8)$$

Comparing (1) and (8), we obtain

$$a_1=\alpha T_{\nu}^{\mu+1}, a_0=\alpha T_{\nu 1}^{\mu}.$$

The results of the calculation of output of signals for μ=0,4; 0,5; 0,8 shown in Fig. 2 and coincide with the solutions those obtained by other methods, for example by the formula Robotnova-Hartley.

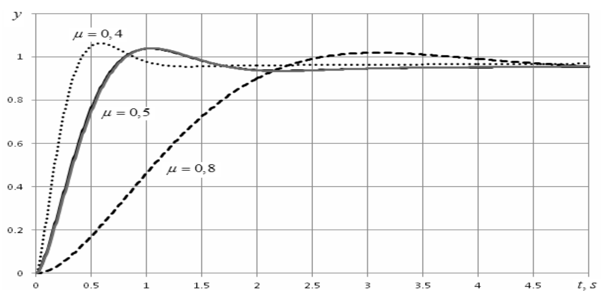

Fig. 2. Transition functions $y=f(t)$ for equation (8)

For equations of higher order

$$\left(b_2p^2+a_1p^{\mu+1}+b_1p+a_0p^{\mu}+1\right)y=x \quad (9)$$

for some combinations of coefficients graphics transition functions are shown in Fig. 3.

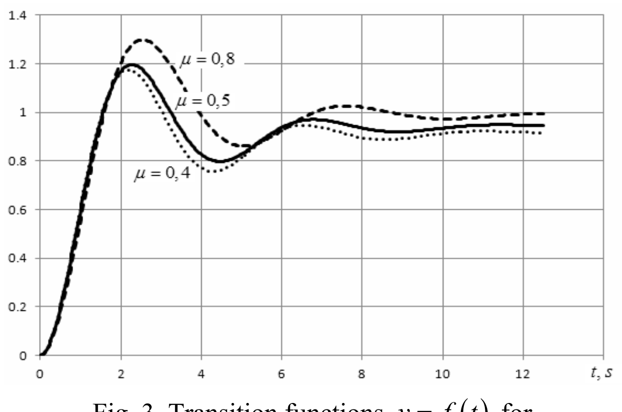

Fig. 3. Transition functions $y=f(t)$ for equation (9)

If the transition functions received as a result of experimental studies, the for identification parameters of the objects described possibly of fractional differential equations (1) can be used genetic algorithms [3].

From all their variety selected multipoint method crossbreeding 40 % of the best individuals with replacement of 60 % culled individuals as follows. Multi-point crossing is the core of a genetic algorithm, which is to prevent the degeneration of the population supplemented by random variations in the parameters of the descendants ±5 % [1]:

$$Mask=\left[\left(2^b-1\right)Rnd\right],$$
$$Child_1=\left(\left(Parent_1\ AND\ Mask\right)OR\left(Parent_2\ AND\ \overline{Mask}\right)\right)\times$$
$$\times\left(0{,}95+0{,}1Rnd\right), \quad (10)$$
$$Child_2=\left(\left(Parent_1\ AND\ \overline{Mask}\right)OR\left(Parent_2\ AND\ Mask\right)\right)\times$$
$$\times\left(0{,}95+0{,}1Rnd\right),$$

where *Mask* – the random $b-$ bit binary mask, $Child_i$, $Parent_i$ – parameters descendants and parents that are normed to $b-$ the bit binary number, Rnd – a random number between 0 and 1. Received two children replace 40 % culled individuals.

Another 20 % of descendants produced as the arithmetic mean of the genes of parents:

$$Child_3=\frac{Parent_1+Parent_2}{2},$$

and one of the descendants of the best specimens obtained in the previous generation, is the individual, whose parameters are determined by the gradient changes of parameters best individuals in the last few generations:

$$\forall Parent_i=best\Rightarrow Child_3=Parent_i+gradient.$$

Fig. 4 illustrates the results of identification parameters of the object described by of fractional differential equation with coefficients

$$a_0=0{,}35; a_1=0{,}175; b_1=2; b_2=0{,}006; \mu=0{,}5.$$

The obtained coefficients of the identified object

$$a_0=0{,}39; a_1=0{,}173; b_1=7{,}5; b_2=0{,}006; \mu=0{,}53$$

is ensured by obtaining the mean squared error $0{,}64\,\%$, the nature of the transition process is exactly the same with the original, so you can use to identify the parameters for subsequent analysis and synthesis of control systems.

Figures 5 and 6 illustrate the learning process – shows the variation of mean μ and square error values of the output signal the best individual in relation to the original signal F_{best} from the generation number *Gen* (Fig. 5) and the number of descendant better during this same process (Fig. 6).

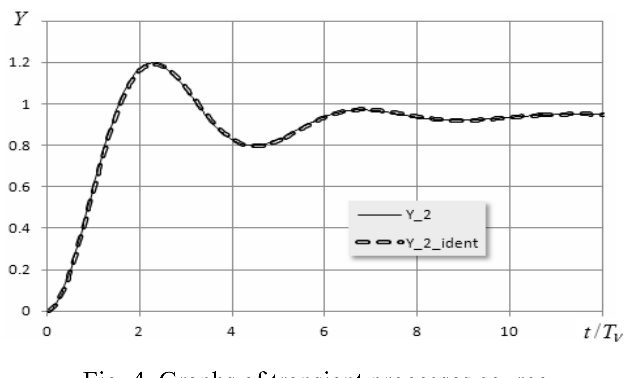

Fig. 4. Graphs of transient processes source and identify objects

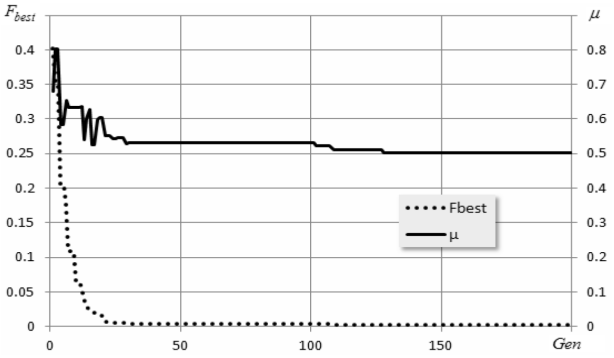

Fig. 5. Graphs changes μ and F_{best} in the learning process

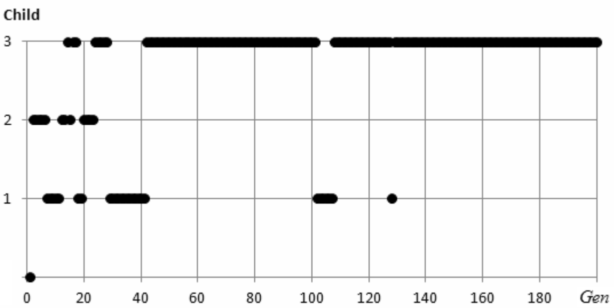

Fig. 6. Changing the number best child in the learning process

Obvious that at the initial stage basic role for the achievement of the best parameters plays a crossbreeding $(Child_{1/2})$, and in the refinement the parameters is often the best $Child_3$.

Conclusion. To describe the behavior of objects in the models that include fractional differential equations of arbitrary order, designed hyper neurona model based on the method of finite increments and modified discrete form of the Riemann-Liouville.Volume of computational operations in hyper neuron minimized by a single operation of fractional integration and computational accuracy improved by excluding of circular references. It allows to use hyper neuron as a built model of such facilities in digital control systems and when combined with the genetic algorithms - for identification of their parameters with high accuracy.

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