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MODELING OF FUNCTIONING OF CONVEYOR TRANSPORT SYSTEMS WITH DENTRITIC SELF-SIMILAR STRUCTURE

Abstract. *Based on dynamics of average method for Markov processes we developed a mathematical model of functioning of conveyer transport systems with serial and parallel connection of conveyer and hoppers, and also with dendritic and self-similar structures. We obtained recursive algorithm which determines capacity of these systems.*
Keywords: *conveyor transport system, hopper, Markov model, functioning, capacity, self-similar structure*

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МОДЕЛИРОВАНИЕ ФУНКЦИОНИРОВАНИЯ СИСТЕМ КОНВЕЙЕРНОГО ТРАНСПОРТА С ДРЕВОВИДНОЙ САМОПОДОБНОЙ СТРУКТУРОЙ

Аннотация. *На основании метода динамики средних для марковских процессов разработана математическая модель функционирования систем конвейерного транспорта с последовательным и параллельным соединением конвейеров и бункеров, а также с древовидной и самоподобной структурами. Получен рекуррентный алгоритм определения пропускной способности этих систем.*
Ключевые слова: *системы конвейерного транспорта, бункер, марковская модель, функционирование, пропускная способность, самоподобная структура*

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МОДЕЛЮВАННЯ ФУНКЦІОНУВАННЯ СИСТЕМ КОНВЕЄРНОГО ТРАНСПОРТУ З ДЕРЕВОВИДНОЮ САМОПОДІБНОЮ СТРУКТУРОЮ

Анотація. *На підставі методу динаміки середніх для марківських процесів розроблена математична модель функціонування систем конвеєрного транспорту з послідовним і паралельним з'єднанням конвеєрів і бункерів, а також з деревовидною і самоподібною структурами. Отримано рекурентний алгоритм визначення пропускної здатності цих систем.*
Ключові слова: *системи конвеєрного транспорту, бункер, марківська модель, функціонування, пропускна здатність, самоподібна структура*

Introduction. Conveyer transport systems of coal mines have difficult branched structure consisting of conveyers and hoppers which are connected together using batcher, loaders and unsolders. Failures of conveyers often lead to downtime in lavas and as result to poor productivity of conveyer transport systems.

Using method of structural reservation in coal mines i.e. using reserved conveyer lines because of high cost of capital works is almost. To increase carrying capacity conveyer transport systems of coal mines because of limited space accumulative hoppers (temporal redundancy) have received wide application. However the effective use of accumulative hoppers is limited by lack of mathematical ensuring and software that allows.

To optimize the process of conveyer transport system functioning.

Currently we developed mathematical models of conveyer transport system functioning without hoppers, which are used mainly for open mining [1].

Many researchers studied questions of functioning of conveyer transport system with hoppers [2 – 4].

In this case obtained mathematical models of conveyer transport system functioning mainly related to the system with a simple structure “conveyer – hopper – conveyer”.

We obtained simulation models for more complex structure of conveyer transport system [5 – 6].

In paper we developed mathematical models of conveyer transport system functioning with serial and parallel connection of hoppers and also with self-similar dendritic structure.

Mathematical model is based on method of dynamics of medium for markov process [7], where simple system “conveyer– hopper – conveyer” is replaced by element (conveyer) with equivalent parameters of failure and recovery.

Main part. Analysis of conveyer transport system of coal mines showed that in general they have a self-similar dendritic structure. In other words a block diagram of conveyer transport system can be divided into hierarchical levels. At each level the same graph is repeated. Such geometric structures are called dendritic fractals. [8].

Fig. 1 shows typical schemes of conveyer transport system of coal mines with serial and parallel connection of hoppers and also conveyer transport system with self-similar dendritic structure.

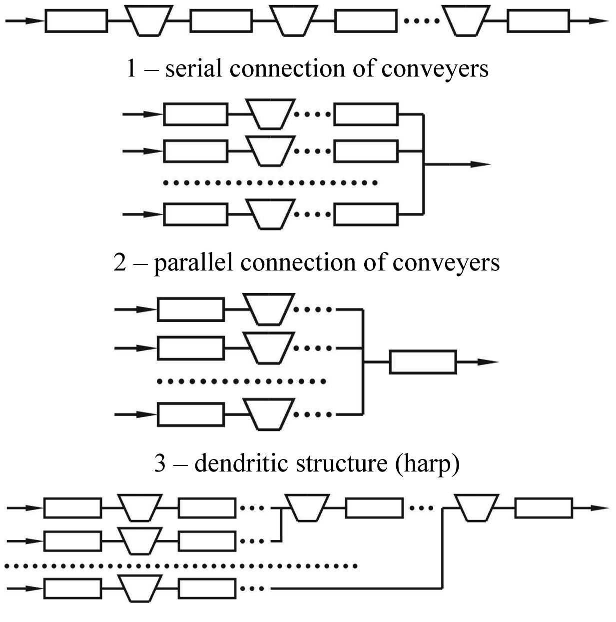

Fig. 1. Structural schemes of conveyer transport system of coal mines with hoppers

Dendrite structure of conveyer transport system of coal mines can be explained by cyclical technology: penetration – mining – excavation. As a result each new section of conveyer transport system is connected to existing system, which was formed as a result of many cycles of coal mines. This process can be compared to the growth of tree, where the cycle is spring – summer – autumn.

Therefore mathematical modeling will be made for these types of conveyer transport system.

As was shown in [9], one of the main parameter of conveyer transport system functioning is average value of carrying capacity m_c, determined by formula

$$m_c = \sum_{i=0}^{s} P_i Q_i , \qquad (1)$$

where P_i – probability that conveyer transport system is in i-th state; Q_i – productivity of conveyer transport system i-th state; s – number of possible states of conveyer transport system while stops and failures of conveyers

It is clear from (1), to determine value m_c it is necessary to know the structure of conveyer transport system from which we can determine number of possible states of system s and probabilities P_i of transport system being in each i-th state ($i = 1, s$).

Consider first serial connection of hoppers (Fig. 2).

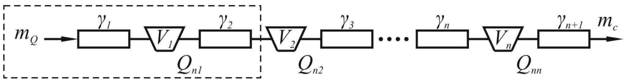

Fig. 2. Calculation scheme of serial connection hoppers

For getting mathematical models of conveyer transport system functioning with hoppers let's use property of self-similarity of structure with serial connection of hoppers.

Let's distinguish in this scheme from the left edge simple system "conveyer – hopper – conveyer" encircled by a dotted line (Fig. 2).

According to [10], average carrying capacity of this simple scheme can be determined from formula:

when $\overline{m}_{Q_1} > \overline{Q}_{n_1}$

$$m_{c_1} = \left[\frac{\dfrac{e^{A_{11}\rho V_1}}{\gamma_1} + \dfrac{\overline{m}_{Q_1}}{(\overline{m}_{Q_1} - \overline{Q}_{n_1})}\left(e^{A_{11}\rho V_1} - 1\right)}{1 + \dfrac{e^{A_{11}\rho V_1}}{\gamma_1} + \dfrac{\overline{m}_{Q_1}}{(\overline{m}_{Q_1} - \overline{Q}_{n_1})}\left(e^{A_{11}\rho V_1} - 1\right)} \right] \overline{Q}_{n1} , \qquad (2)$$

where m_Q – productivity of above-hopper conveyer; Q_{n1} – productivity of batcher; V_1 – volume of hopper; ρ – specific cargo weight, t/m^3;

$$A_{11} = \frac{\mu_1 \left[m_Q - (1+\gamma_1)\overline{Q}_{n_1} \right]}{\left(m_Q - \overline{Q}_{n_1} \right) \overline{Q}_{n_1}} ; \ \overline{m}_{Q_1} = \frac{m_Q}{1+\gamma_1} ;$$

$\overline{Q}_{n_1} = \dfrac{Q_{n_1}}{1+\gamma_2}$; $\gamma_1 = \dfrac{\lambda_1}{\mu_1}$; $\gamma_2 = \dfrac{\lambda_2}{\mu_2}$; γ_1, γ_2 – coefficients of accidents of above-hopper and under-hopper conveyers; λ_1, μ_1 and λ_2, μ_2 – parameters of failures and recoveries of above-hopper and under-hopper conveyers accordingly;

when $\overline{m}_{Q_1} \le \overline{Q}_{n_1}$

$$m_{c_1} = \left[\frac{1 + \dfrac{(Q_{n_1} - \overline{Q}_{n_1})}{(\overline{Q}_{n_1} - \overline{m}_{Q_1})}\left(1 - e^{A_{21}\rho V_1}\right)}{1 + \gamma_2 e^{A_{21}\rho V_1} + \dfrac{(Q_{n_1} - \overline{Q}n_1)}{(\overline{Q}_{n_1} - \overline{m}_{Q_1})}\left(1 - e^{A_{21}\rho V_1}\right)} \right] \overline{m}_{Q_1} , \qquad (3)$$

where $A_{21} = \dfrac{\mu_2 \left[\overline{m}_{Q_1}(1+\gamma_2) - Q_{n_1} \right]}{\overline{m}_{Q_1}\left(Q_{n_1} - \overline{m}_{Q_1} \right)}$.

Continuing this process n times (n – number of hoppers in system), we come to recurrence formulas:

when $\overline{m}_{Q_i} > \overline{Q}_{n_i}$

$$m_{c_i} = \left[\frac{\dfrac{e^{A_{1i}\rho V_i}}{\gamma_{э_i}} + \dfrac{\overline{m}_{Q_i}}{(\overline{m}_{Q_i} - \overline{Q}_{n_i})}\left(e^{A_{1i}\rho V_i} - 1\right)}{1 + \dfrac{e^{A_{1i}\rho V_i}}{\gamma_{э_i}} + \dfrac{\overline{m}_{Q_i}}{(\overline{m}_{Q_i} - \overline{Q}_{n_i})}\left(e^{A_{1i}\rho V_i} - 1\right)} \right] \overline{Q}_{n_i} , \qquad (4)$$

where $A_{1i} = \dfrac{\mu_c \left[m_Q - (1+\gamma_{э_i})\overline{Q}_{n_i} \right]}{\left(m_Q - \overline{Q}_{n_i} \right) \overline{Q}_{n_i}}$;

$$\overline{m}_{Q_i} = \frac{m_Q}{1+\gamma_{э_i}} = m_{c_{i-1}} ; \ \overline{Q}_{n_i} = \frac{Q_{n_i}}{1+\gamma_{i+1}} ;$$

$$\gamma_{э_i} = \frac{m_Q}{m_{c_{i-1}}} - 1 ; \ \gamma_i = \frac{\lambda_i}{\mu_i} ;$$

$$(i = 1,n;\ m_{c_0} = \frac{m_Q}{1+\gamma_1};\ \mu_c = \mu_i);$$

when $\overline{m}_{Q_i} \le \overline{Q}_{n_i}$

$$m_{c_i} = \left[\frac{1+\dfrac{(Q_{n_i}-\overline{Q}_{n_i})}{(\overline{Q}_{n_i}-\overline{m}_{Q_i})}\left(1-e^{A_{2i}\rho V_i}\right)}{1+\gamma_{i+1}e^{A_{2i}\rho V_i}+\dfrac{(Q_{n_i}-\overline{Q}_{n_i})}{(\overline{Q}_{n_i}-\overline{m}_{Q_i})}\left(1-e^{A_{2i}\rho V_i}\right)}\right]\overline{m}_{Q_i}, \quad (5)$$

where $A_{2i} = \dfrac{\mu_c\left[\overline{m}_{Q_i}(1+\gamma_{i+1})-Q_{n_i}\right]}{\overline{m}_{Q_i}\left(Q_{n_i}-\overline{m}_{Q_i}\right)}$; $\overline{m}_{Q_i} = m_{c_{i-1}}$;

$\overline{Q}_{n_i} = \dfrac{Q_{n_i}}{1+\gamma_{i+1}}$; $(i = 1,n;\ m_{c_0} = \dfrac{m_Q}{1+\gamma_1};\ \mu_c = \mu_i)$.

In this case carrying capacity of entire conveyer transport system with serial connection of hoppers is determined on n-th iteration by formula:

$$m_c = m_{c_n}, \quad (6)$$

where n – number of hoppers in system.

Consider now system conveyer transport system with parallel connection of hoppers (Fig. 3).

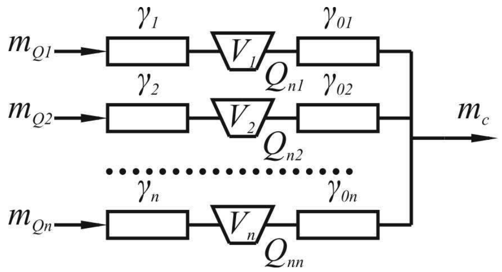

Fig. 3. Calculation scheme of parallel connection of hoppers

For this system as in previous case using self-similarity of its structure we will obtain recursive ratios when $\overline{m}_{Q_i} > \overline{Q}_{n_i}$

$$m_{c_i} = \left[\frac{\dfrac{e^{A_{1i}\rho V_i}}{\gamma_i}+\dfrac{\overline{m}_{Q_i}}{(\overline{m}_{Q_i}-\overline{Q}_n)}\left(e^{A_{1i}\rho V_i}-1\right)}{1+\dfrac{e^{A_{1i}\rho V_i}}{\gamma_i}+\dfrac{\overline{m}_{Q_i}}{(\overline{m}_{Q_i}-\overline{Q}_{n_i})}\left(e^{A_{1i}\rho V_i}-1\right)}\right]\overline{Q}_{ni}, \quad (7)$$

where $A_{1i} = \dfrac{\mu_c\left[m_{Q_i}-(1+\gamma_i)\overline{Q}_{n_i}\right]}{\left(m_{Q_i}-\overline{Q}_{n_i}\right)\overline{Q}_{n_i}}$; $\overline{m}_{Q_i} = \dfrac{m_{Q_i}}{1+\gamma_i}$;

$\overline{Q}_{n_i} = \dfrac{Q_{n_i}}{1+\gamma_{0_i}}$; $\gamma_i = \dfrac{\lambda_i}{\mu_i}$;

when $\overline{m}_{Q_i} \le \overline{Q}_{n_i}$

$$m_{c_i} = \left[\frac{1+\dfrac{(Q_{n_i}-\overline{Q}_{n_i})}{(\overline{Q}_{n_i}-\overline{m}_{Q_i})}\left(1-e^{A_{2i}\rho V_i}\right)}{1+\gamma_{0_i}e^{A_{2i}\rho V_i}+\dfrac{(Q_{n_i}-\overline{Q}_{n_i})}{(\overline{Q}_{n_i}-\overline{m}_{Q_i})}\left(1-e^{A_{2i}\rho V_i}\right)}\right]\overline{m}_{Q_i}, \quad (8)$$

where $A_{2i} = \dfrac{\mu_c\left[\overline{m}_{Q_i}(1+\gamma_{0_i})-Q_{n_i}\right]}{\overline{m}_{Q_i}\left(Q_{n_i}-\overline{m}_{Q_i}\right)}$; $\overline{m}_{Q_i} = \dfrac{m_{Q_i}}{1+\gamma_i}$;

$\overline{Q}_{n_i} = \dfrac{Q_{n_i}}{1+\gamma_{0_i}}$; $i = 1,n$; $\mu_c = \mu_i$.

In this case carrying capacity of entire conveyer transport system with parallel connection of hoppers is determined by formula:

$$m_c = \sum_{i=1}^{n} m_{c_i}, \quad (9)$$

where n – number of hoppers in system.

For dendritic harp structure of hoppers connection (Fig. 4) average value of carrying capacity of conveyer transport system also is determined by same formulas (7) and (8), where values of coefficient of accident γ_{0_i} of above-hopper conveyer is replaced by value of coefficient of accident γ_0 made-up conveyer ($\gamma_{0_i} = \gamma_0$, $i = 1,n$).

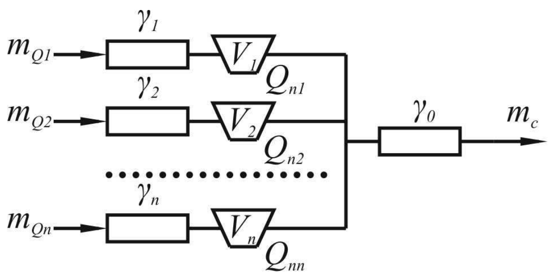

Fig. 4. Calculation scheme of harp structure of hoppers connection

Consider self-similar dendritic structure of conveyer transport system with hoppers (Fig. 5).

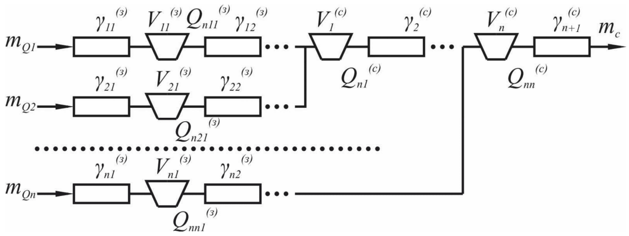

Fig. 5. Calculation scheme of self-similar dendritic structure of hoppers connection

For this system using self-similarity of structure as in previous cases we will obtain recursive ratios which determine average carrying capacity of the system:

when $m_i^{(s)} > \overline{Q}_{n_i}^{(c)}$

$$m_{c_i}=\left[\frac{\dfrac{e^{A_{1i}\rho V_i^{(c)}}}{\gamma_{\text{э}_i}^{(c)}}+\dfrac{m_i^{(s)}}{(m_i^{(s)}-\overline{Q}_{n_i}^{(c)})}\left(e^{A_{1i}\rho V_i^{(c)}}-1\right)}{1+\dfrac{e^{A_{1i}\rho V_i^{(c)}}}{\gamma_{\text{э}_i}^{(c)}}+\dfrac{m_i^{(s)}}{(m_i^{(s)}-\overline{Q}_{n_i}^{(c)})}\left(e^{A_{1i}\rho V_i^{(c)}}-1\right)}\right]\overline{Q}_{n_i}^{(c)},\quad(10)$$

where $A_{1i}=\dfrac{\mu_c\left[m_i^{(s)}(1+\gamma_{\text{э}_i}^{(c)})-(1+\gamma_{\text{э}_i}^{(c)})\overline{Q}_{n_i}^{(c)}\right]}{\left[m_i^{(s)}(1+\gamma_{\text{э}_i}^{(c)})-\overline{Q}_{n_i}^{(c)}\right]\overline{Q}_{n_i}^{(c)}}$;

$$\gamma_{\text{э}_i}^{(c)}=\frac{\sum_{k=1}^{i}m_{Q_k}}{m_i^{(s)}}-1;\quad m_i^{(s)}=m_{c_{i-1}}+\frac{m_{Q_i}}{1+\gamma_{\text{э}_i}^{(\text{з})}};$$

$$\overline{Q}_{n_i}^{(c)}=\frac{Q_{n_i}^{(c)}}{1+\gamma_{i+1}^{(c)}};\quad m_{c0}=0;\quad \mu_c=\mu_i;\quad i=1,n;$$

$\gamma_i^{(c)}$ – coefficients of accidents of shafts paths with hoppers; $\gamma_{\text{э}_i}^{(c)}$ – equivalent coefficients of accidents of shafts paths with hoppers; $\gamma_{\text{э}_i}^{(\text{з})}$ – equivalent coefficients of accidents of faces paths with hoppers;

when $m_i^{(s)}\le\overline{Q}_{n_i}^{(c)}$

$$m_{c_i}=\left[\frac{1+\dfrac{(Q_{n_i}^{(c)}-\overline{Q}_{n_i}^{(c)})}{(\overline{Q}_{n_i}^{(c)}-m_i^{(s)})}\left(1-e^{A_{2i}\rho V_i^{(c)}}\right)}{1+\gamma_{i+1}^{(c)}e^{A_{2i}\rho V_i^{(c)}}+\dfrac{(Q_{n_i}^{(c)}-\overline{Q}_{n_i}^{(c)})}{(\overline{Q}_{n_i}^{(c)}-m_i^{(s)})}\left(1-e^{A_{1i}\rho V_i^{(c)}}\right)}\right]m_i^{(s)},\quad(11)$$

where $A_{2i}=\dfrac{\mu_c\left[m_i^{(s)}(1+\gamma_{i+1}^{(c)})-Q_{n_i}^{(c)}\right]}{m_i^{(s)}\left(Q_{n_i}^{(c)}-m_i^{(s)}\right)}$

($\mu_c=\mu_i$; $i=1,n$).

Here efficient coefficients of accidents of faces paths with hoppers are deter-mined by formulas:

$$\gamma_{\text{э}_i}^{(\text{з})}=\frac{m_{Q_i}}{m_{c_i}^{(\text{з})}}-1,\ (\gamma_{\text{э}_1}^{(c)}=\gamma_{\text{э}_1}^{(\text{з})},\ i=1,n),\quad(12)$$

where $m_{c_i}^{(\text{з})}$ – average carrying capacity *i*-th face path of conveyer transport system with hoppers which are determined as in (4) and (5).

Average carrying capacities of faces paths $m_{c_i}^{(\text{з})}$ are determined by formulas (4) – (6) for serial connection of faces conveyers with hoppers.

To determine minimum and maximum values of carrying capacities mentioned above conveyer transport systems with hoppers set up in above recursive formulas values $V_i=0$ and $V_i\to\infty$ accordingly to minimum and maximum values.

Conclusions. On the basis on method of dynamics of medium for markov process we obtained mathematical models of conveyer transport system functioning with serial and parallel connection of hoppers and also with self-similar dendritic structure that allow to determine carrying capacity of coal mines conveyer transport system.

It was found that if average carrying capacity from lavas of cargo m_{Q_i} is higher than batcher productivity Q_{n_i} ($m_{Q_i}\ge Q_{n_i}$), then carrying capacity of conveyer transport system is much lower than carrying capacity in case when $m_{Q_i}<Q_{n_i}$.

Moreover, with increasing of accumulative hoppers volume to certain value carrying capacity of conveyer transport system increases, but when values of volume are higher from that value – almost doesn't change.

References

1. Kiriya R.V. Primenenie fraktalov k opredeleniyu propusknoy sposobnosti sistem konveyernogo transporta gornykh predpriyatiy [Application of Fractals to Determination of Carrying Capacity of Conveyer Transport Systems of Mining Enterprises], (2010), *Sistemnye Tekhnologii: Regionalnyu Mezhvuzovskiy Sbornik Nauchnykh Trudov*, Dnepropetrovsk, Ukraine, Issue 2(67), pp. 167–174 (In Russian).

2. Ponomarenko V.A., Kreymer E. L. and Dunaev G. A. Sistemy podzemnogo transporta na ugolnykh shakhtakh [Underground Transport Systems on Coal Mines], (1975), *Nedra Publ.*, Moscow, Russian Federation, 309 p. (In Russian).

3. Stepanov P.B. and Alotin L.M. Nadezhnost mnogobunkernykh konveyernykh liniy [Reliability of Multihopper Conveyer Lines], (1978), *Izvestiya Vuzov. Gornyu Zhurnal Publ.*, Yekaterinburg, Russian Federation, No. 1, pp. 94–99 (In Russian).

4. Cherkesov G.N. Nadezhnost tekhnicheskikh sistem s vremennoy izbytochnostyu [Reliability of the Technical Systems with Temporal Redundancy], (1974), *Soviet Radio Publ.*, Moscow, Russian Federation, 296 p. (In Russian).

5. Alotin L.M. and Stepanov P.B. Modelirovanie i raschet transportnykh sistem gornykh predpriyatiy [Design and Calculation of Transport Systems of Mining Enterprises], (1979), *Nauka Publ.*, Alma-Ata, KazSSR, 214 p. (In Russian).

6. Gorlik A.M. Raschet parametrov nadezhnosti shakhtnykh konveyernykh liniy s primeneniem EVM [Calculation of Parameters of Reliability of Mine Conveyer Lines Using Computers], (1971), *Izvestiya Vuzov. Gornyu Zhurnal Publ.*, Yekaterinburg, Russian Federation, No. 11, pp. 113 – 119 (In Russian).

7. Venttsel E.S., and Ovcharov L.A. Teoriya sluchaynykh protsesov i yee inzhenernye prilozheniya: uchebnoe posobie. 5-e izd. [Theory of Random Processes and its Engineering Applications: Train Aid. 5th Publ.], (2011), *KNORUS*, Moscow, Russian Federation, 448 p. (In Russian).

8. Shreder N. Fraktaly, khaos, stepennye zakony. Miniatiury iz beskonechnogo raya [Fractals, Chaos, Power Laws. Miniatures from Endless Paradise], (2005), *NITS*, Izhevsk, Russian Federation, 528 p. (In Russian).

9. Kiriya R.V., Monastyrsky V.F., and Maksuytenko V.Yu. Adaptivnoe upravlenie konveyernymi liniyami gornykh predpriyatiy [Adaptive Control of Conveyer Lines of Mining Enterprises], (2011), *Forum Gornyakov–2011: Materialy Mezhdunar. Konf., 12-15 Oktyabrya 2011 g. National Mining University*, Dnepropetrovsk, Ukraine, pp. 12 – 15 (In Russian).

10. Kiriya R.V., Mishenko T.F., and Babenko Yu.V. Razrabotka bystrogo algoritma opredeleniya propusknoy sposobnosty sistemy "konveyer – bunker – konveyer" [Development of Fast Algorithm of Determination of Carrying Capacity of the System "Conveyer – Hopper – Conveyer"], (2013), *Sistemnye Tekhnologii: Regionalnyu Mezhvuzovskiy Sbornik Nauchnykh Trudov*, Dnepropetrovsk, Ukraine, Issue 1, pp. 146 – 158 (In Russian).

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