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COMPUTER MODELING OF DYNAMIC PROCESSES IN ANALYTIC NUMBER THEORY

G. Vostrov, R. Opiata

Odessa national polytechnic university

Abstract. *In the work of computer simulation methods, the properties of prime numbers are investigated, as a dynamic evolving system. Methods for the classification of prime numbers based on Fermat's small theorem are constructed. It is proved that, based on the results of computer simulation, the bases for the development of analytic and algebraic number theory as dynamical systems were created.*

Key words: *Prime numbers, Fermat's little theorem, dynamical systems, the dynamics of the development of prime numbers.*

Introduction

Statements K.F. Gauss "Mathematics – the queen of sciences, the theory of numbers – the queen of mathematics" remains absolutely correct at the present time. In the theory of numbers, such areas of research have been formed: analytical, algebraic, computational, combinatorial, geometric, and probabilistic. Despite their diversity and a huge number of publications, the number of unresolved problems is quite impressive and growing continuously. In studies on number theory, considerable effort is associated with prime and natural numbers. In [1, 2] the authors very thoroughly presented a complete picture of scientific research and its problems. Despite the impressive results, many problems remain unsettled, both from the theoretical and practical points of view. In the noted books, the state of research towards deepening the achieved results is presented quite fully.

The theory of numbers, beginning with prime numbers and ending with complex numbers, is actively used in practically all subject areas of modern pure and applied mathematics. One of such domains is the mathematical theory of dynamical systems [3]. On the one hand, the properties of dynamical systems defined over certain subsets of numbers from the point of view of the distribution of fixed points of the functions defining a dynamical system are investigated, depending on their properties. On the other hand, the functions of dynamical systems allow one to investigate the properties of numbers from the domain of definition of a dynamical system. The system is considered from the point of view of the influence of their properties on the laws of dynamic behavior, regardless of the choice of the function that determines the dynamic system.

The mathematical theory of dynamical systems in the form of topological dynamics began to develop actively since the work of the Ukrainian mathematician A. N. Sharkovsky [4]. Let $f(x)$ be a continuous function defined on the set S, which is a compact. In connection with the fact that we consider rational numbers as the set S, and among them numbers of the form $1/p$ where p is a prime number, we will assume that this set is any interval including the interval of real numbers $(0,1)$. Consider the class of difference equations $x_{n+1} = f(x_n)$. According to [4], the point x_0 is fixed if there exists a natural number m such that condition $x_0 = f^{(m)}(x_0)$ is satisfied. This means that at point x_0 the function generates a difference iterative process:

$$x_0,\ f^{(1)}(x_0) = x_1,$$
$$f^{(2)}(x_2) = f(f(x_0)) = x_2, \ldots, \quad (1)$$
$$f^{(0)}(x_0) = f(f(..f(x_0)..)), \ldots$$

The sequence $f^{(m)}(x_0) = x_0$ $(x_0, x_1, x_2, \ldots, x_{l-1})$ in dynamic systems is usually called the trajectory of iterations of a dynamical system or an orbit. According to the theory of topological dynamics, the existence of x_0, the sequence of iterations of a certain length, is determined by the properties of the function $f(x)$. It is not assumed that the values of x as numbers of a compact that can have properties that affect the value of $f^{(l)}(x)$. Let us consider the case when this set is a set of real numbers $(0,1) \subset R$.

It is proved [4] that different values of x iteration of the function $f(x)$ can generate cycles of different length and structure. Moreover, the existence of an iterative cycle of length m can cause the existence of cycles of other long lengths, say k. It is accepted in this case to write $m > k$, although it can be much larger than m.

It was shown in [4] that in the general case the lengths of the iteration cycles are related by the order relation of the following form:

$$3 > 5 > 7 > 9 > ..., 2 \cdot 3 > 2 \cdot 5 > 2 \cdot 7 > ... \\ 2^2 \cdot 3 > 2^2 \cdot 5 > 2^2 \cdot 7 > ... > 2^3 > 2^2 > 2 > 1 \quad (2)$$

This means that N for this order relation consists of an infinite set of ordinal type ω of equivalent ordered odd natural numbers by the principle $3 > 5 > 7 > 9 > ... > 2n+1 > ... > 1$.

In [4] and other authors, many surprising results were obtained during more than half a century, but none of the studies investigated the effect of the properties of the numbers x on the value of m in $f^{(m)}(x) = x$ and also on the structure of the iteration cycle at this point. The study of the mathematical regularities of such influence is advisable, since if we consider the set of all prime numbers $P < N$, then it is possible to establish by virtue of which properties of numbers they have properties that for any function $f(x)$ or a certain class of functions will determine the iteration cycle length and its structure is not depending on the choice of function $f(x)$. It is obvious that $1/p$ for $p \in P$ belong to the interval $(0,1)$.

Such studies are necessary because the iterative processes of the diffuse structure cause "chaotic processes" that cause the appearance of structural attractors. In this case, "chaos" may be due to the non-linear properties of function $f(x)$, but it is possible that the properties of the values of variable x can also affect the structure of "chaos". In addition, then the set of fixed points can be divided into classes according to their properties, which make it possible to determine the structure of the entire set of fixed points, and consequently, of strange attractors.

The study of fixed-point structures and attractors in the general case is a complex problem even for sufficiently simple functions $f(x)$. Sometimes such functions are called mappings. An interesting example of such an investigation can be work [5] in which the set of fixed points of the map "tent" considered only on the interval $(0,1)$ was investigated. Consider two types of mapping "tent": symmetric and asymmetric. The display " tent " is selected for the reason that the aim of the work is to study the properties of prime numbers p which in the given mappings are represented as 1 / p. Symmetric " tent " is given to the integer form in order to avoid the use of rounding processes in computer modeling processes. The display of an asymmetric " tent " cannot always be given to an integer form. In this paper we need an exclusively symmetric " tent " for the reason that in this form it is congruent to a mapping that is constructed on the basis of a small Fermat theorem in the theory of residues modulo a prime number [1, 2].

The mapping of "tent" in a non-symmetric form is represented by an iterative expression [5]:

$$x_{x+1} = \begin{cases} x_n / a & x_n \in [0, a), \\ (1 - x_n)/(1 - a) & x_n \in [a, 1]. \end{cases} \quad (3)$$

and in the symmetric form by the iteration expression:

$$x_{n+1} = \begin{cases} \mu x_n & x_n < 1/2, \\ \mu(1 - x_n) & x_n \geq 1/2. \end{cases} \quad (4)$$

These maps were studied in detail in [6], and the investigations are continuing at the present time because the values of a and μ significantly affect the structure of the diagrams and its investigation is not trivial. Suffice it to say that the information on these maps was corrected in the English Wikipedia in 2016 [5].

From these papers and the latest results it follows that the properties of the numbers $x \in [0,1]$ and their effect on the structure of the diagrams have not yet been investigated. Since the numbers of the form $1/p$ for $p \in P$ entirely belong to the class of primes, this is one of the incentives to investigate these mappings and structurally similar mappings on the set of numbers $\left\{ \frac{1}{p} \middle| p \in P \right\}$.

Classification of prime numbers on the basis of nonlinear mappings and the theory of residues

Since $1/p$ begins with $1/2$ and tends to zero, the following maps were investigated at this stage [8, 9]. Since for large p, the number $1/p$ rounds off in the computer, in order to avoid the effect of rounding, all three selected mappings have been reduced to an integer form. The possibility of reducing to integer form was one of the reasons for their choice. But not the only one. The mappings have the form $t_1(x), t_2(x), t_3(x)$ and the formulas given in [10].

The mapping $t_1(x)$ was chosen as the base mapping for the study of the properties of prime numbers on the basis of the theory of dynamical systems. The other two at this stage were mainly investigated in order to confirm that the properties of prime numbers significantly affect the structure of the iterative cycles and their lengths. These properties are preserved for even very different mappings.

The choice of mapping $t_1(x)$ in the original form of $f_1(x)$ [11] was due to one more circumstance. In the studies reported in OEIS A082654, an attempt was made by a group of American mathematicians to simulate prime numbers P by mapping to a certain degree congruent with $t_1(x)$. This mapping is given by the expression $x_{n+1} = ax_n(\bmod p)$ at $a = 4$. This mapping was used due to the fact that it is congruent on the set of prime numbers $t_1(x)$ along the length of the cycle, although the structures of the cycles turned out to be different. Since the structure of the iteration cycle in this paper was not the object of investigation, these two maps were used in parallel.

To study, taking into account the fact that the law of distribution of prime numbers has already been investigated by Valle Poussinam and Hadamard [9], only a set of primes was chosen. In the sequential study, blocks of 500 thousand numbers were allocated. In total, 20 such consecutive ones without gaps and intersections of blocks covering the first ten million prime numbers were examined. Each block of 500 thousand numbers was analyzed using the Excel system. In this system, a number of functions were implemented that allow filtering the properties of prime numbers that were of interest.

The concept of the class of a prime number p relative to a basis is introduced by calculating the value $k = (p-1)/l_a(p)$ for it, where a was chosen to be equal to four, as in A082654, $l_a(p)$ the iteration cycle length in the map $t_1(x)$.

For all blocks, we classify primes. At $a = 4$, class indexes are represented by even numbers. We will carry out a primary analysis of classes and study their basic properties.

The results of computer modeling allow us to conclude that in the modern theory of numbers (but really in all mathematics) two ways of solving many very complex mathematical problems are being formed,

– method of fundamental mathematical theory;

– method of computer modeling of solving mathematical problems;

This point of view is confirmed by numerous studies of a significant number of leading researchers in the field of modern mathematics [14].

It can be argued that if the results of computer simulation are mathematically matched with purely mathematical considerations and proven theorems, then the solution of complex problems can be constructed by reducing the results one to another. In essence, such an approach makes sense to be considered as the creation of a new paradigm for solving mathematical problems: a cyclic solution of mathematical problems by means of specific mathematical and computer iterations

– computer modeling;

– mathematical theory;

In this example, we are faced with a similar situation. Classification of prime numbers by base $a = 4$ is constructed. The first step in this direction is initiated by two different mapping classes:

1. Mappings given by continuous mappings beyond the data on a compact: $R \equiv (a,b) \equiv (0,1)$ that is, $(0,1) \subset R \,\&\, (a,b) \subset R$

2. Mappings that form in the theory of integers on the basis of Fermat's small theory.

In the first case, we start from topological dynamics, and in the second from the theory of recursion associated with the theorems of Fermat and Euler: If $GCD(a,n) = 1$, then $a^{n-1} \equiv 1(\bmod n)$, and hence $x_0 = d$, $x_{n+1} \equiv ax_n(\bmod n) \,\&\, a^{\varphi(n)} \equiv 1(\bmod n)$ generates a cycle of length $l_a(n)$ and always $l_a(n)$ divides $n-1$ and $\varphi(n)$, where $\varphi(n)$ is Euler's function. This fact means that the number-theoretic distribution generates for each $a \in N$ a virtually infinite set of recursions that, in accordance with the presented topological dynamics, are fixed points.

But in the second case the principle of forming a cycle is completely different than in the first case. It is all the more surprising that the mappings of $\{t_1(x), t_2(x), t_3(x)\}$ and $t_4(x)$ on the same natural numbers n relatively prime to a generate an iterative cycle. In this case, $t_1(x)$ and $t_4(x)$ according to the results of computer simulation always generate on the set of prime numbers an iterative cycle of the same length.

It is important to take into account that the symmetric "tent" mapping and on the basis of the Euler and Fermat theorem for all $n \in N$ and $GCD(a,n) = 1$ give an iterative cycle of the same length but different structure of iterations. Thus, these mappings generate the same classification of prime numbers for different values of the base a.

At $a=4$, we obtain a system of classes with even values of the indices $\{P_2, P_4, P_6, ..., P_{2k}, ...\}$, but in practice for any other $a \in N$ with $a>1$ we obtain another classification of primes in the form of classes with other values of the indices.

In particular, we consider some statements that can be considered as theorems, which at this stage are proved on the basis of the results of computer simulation.

Assertion 1. The set of classes $\{P_2, P_4, P_6, ..., P_{2k}, ...\}$ is infinite (the number of primes in each class is also infinite).

Hypothesis. (Generalized Artin conjecture) The distribution of primes in each of the classes is subject to the law:

$$\pi(x, a, P_i) = C(a,i)\pi(x) \quad (5)$$

where $C(a,i)$ is a constant, and also a function that depends on a and i. Here, for any $a>1$, the following relations hold:

$$\sum_{i=2,\& j=\overline{1,\infty}} C(a,i) = 1 \quad (6)$$

If a changes, then the same for any $a>1$, the set of indices $\{\lambda_1, ..., \lambda_k, ...\}$ is determined by its value. We can consider the assertion independent of assertion 1.

Assertion 2. For any $a>1$ as the basis of residues, the set of classes $\{P_\varsigma(a,1), P_\varsigma(a,2), ..., P_\varsigma(a,n), ...\}$ is infinite and $\varsigma(a,i) = \frac{p-1}{la_i(p)}$, for all $i=1,...,n,...$ where i the class number is ordered according to which the prime numbers of this class are distributed on a straight line with $tg(\lambda_i) = \left(\frac{p-1}{la(p,i)}\right)^{-1}$.

Assertion 3. The number of primes in each of the classes of the set $\{P_\varsigma(a,1), P_\varsigma(a,2), ..., P_\varsigma(a,n), ...\}$ is infinite.

Let's return to the case when $a=4$.

Assertion 4. If the number $n \in P_2$ then it is simple, i.e. $n=p$ and $p-1$ are divided into 2 and 4 from the set of even numbers. This is enough, but not necessary.

Assertion 5. All prime numbers of the form $p^* = 2^l p + 1$, where $p^*, p \in P$ and $l \in N$, belong to classes $P_2, P_4, P_8, P_{16}, ..., P_{2k}, ...$ and they are generalized numbers of Sophie Germain. It is proved that there are infinitely many such numbers.

Assertion 6. For any $a>1$, all the classes $P_\varsigma(1,1), ..., P_\varsigma(2,k)$ do not intersect.

The above statements concern only the prime numbers of the distribution between classes of composite numbers without simple ones having several other laws. But compound pseudo prime numbers should be treated separately.

Assertion 7. The prime numbers from the classes $P_6, P_{10}, P_{14}, P_{22}, ...$ have the property that $p \in P_{2q}$ satisfies the condition $(p-1)/2q$ completely.

Those. each $p-1$ in its decomposition into prime factors contains 2 and q.

Assertions 8. If the number $p \in P$ belongs to class P_{2m} then $p-1$ definition is divided by $2m$ and is the length of the cycle.

The brief summary of the results of computer analysis makes it possible to state that in substantiating by analytical methods, in essence, a new approach to the study of the properties of not only prime numbers and composite natural numbers, whole, rational, is formed.

In [1], the authors emphasize the Artin conjecture according to which, for any prime number q which is the primitive root of some set of prime numbers $P(q)$, the distribution of such primes has the form $\pi(x,q) = c(q)\pi(x)$,

where $c(q)$ is a constant, which depends on the choice of q.

We formulate a generalization of the Artin conjecture. Let n be an arbitrary natural number greater than one. According to the method of classification of prime numbers formed for any n, we obtain a system of classes of primes.

$$P_\varsigma(1,n), P_\varsigma(2,n), ..., P_\varsigma(l,n)... \quad (7)$$

where $\varsigma(1,n)$ is the class index function, while i is the class number, n is the number. Thus, for $a=4$ for classes $P_2, P_4, P_6, ..., P_{2k}, ...$, then the indices of classes determined by the functions $c(1,4)$, $c(2,4)$, $c(3,4)$, and so on.

The generalized Artin conjecture: for any natural $n \geq 2$, the distribution of prime numbers in classes and in the whole classification has the form:

$$\pi(x, \varsigma(i,n)) = c(\varsigma(i,n), n)\pi(x) \quad (8)$$

$$\sum_{i=1}^{\infty} c(\varsigma(i,n), n) = 1 \quad (9)$$

for any $n > 1$.

Table 1 and Table 2 show the results of computer simulation of this hypothesis. It is confirmed on the basis that in this case the whole set of prime numbers, P as well as in the proof, Valle Poussin and Hadamard are taken into account. Any prime number p belongs to one of the classes and the classes do not intersect.

Table 1

The distribution of prime numbers in 1 to 8 classes in the generalized Artin conjecture

Interval, ml	P2	P4	P6	P8	P10	P12	P14	P16
0.0–0.5	280631	46805	49832	35060	14176	8325	6662	8717
0.5–1.0	280598	46781	49941	35098	14188	8322	6610	8743
1.0–1.5	280371	46784	49732	35043	14200	8255	6724	8838
1.5–2.0	280216	46680	49802	35374	14186	8243	6853	8726
2.0–2.5	280681	46775	49938	34864	14251	8309	6656	8924
2.5–3.0	280328	46625	49748	35292	14349	8408	6733	8687
3.0–3.5	280900	46692	49867	34960	14090	8326	6792	8741
3.5–4.0	280397	47025	49904	34785	14157	8315	6773	8896
4.0–4.5	280329	46855	49977	35291	14150	8167	6555	8898
4.5–5.0	280196	46782	49653	35130	14359	8347	6697	8755
5.0–5.5	280376	46664	49971	34963	14139	8254	6585	8823
5.5–6.0	280331	46779	49905	35057	14096	8244	6819	8789
6.0–6.5	280491	46752	49912	35152	14209	8235	6545	8768
6.5–7.0	280653	46938	49850	34618	13991	8427	6670	8860
7.0–7.5	280456	46525	49619	35188	14006	8345	6777	8837
7.5–8.0	280523	46812	49833	34969	14110	8315	6687	8741
8.0–8.5	280794	46568	49907	35098	14059	8250	6627	8867
8.5–9.0	280373	46759	49705	35193	14082	8344	6822	8664
9.0–9.5	280228	46975	49810	34961	14234	8282	6824	8772
9.5–10.0	280547	46740	49839	35042	14137	8366	6576	8744

Consider the problem of distributing prime numbers with residues relatively prime to a given number m. The number $m \geq 3$. This problem has been investigated by many mathematicians. Chebyshev also claimed in the middle of the nineteenth century that the shares of primes with residues 1 and 3 are equal, $1/2$, but convergence to $1/2$ for the deduction 1 has a top view, and for a deduction of 3 from below. In [10, 11] this problem was investigated for $m = 4$ and $m = 24$. A filter system has been created in the work, which allows you to test a more general result for any $m \geq 3$. 0 is the following statement:

Assertion 9. For any $m \geq 3$, the set of prime numbers P is divided into residue classes with respect to a given module, and the relations $\pi(x,i,m) = \frac{1}{\varphi(m)}\pi(x)$, where $\varphi(m)$ is Euler's function, are valid.

Table 3 shows the results of computer simulation data analysis for $m = 16$, with $\varphi(m) = 8$

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Table 2

The distribution of prime numbers in 9 to 16 classes in the generalized Artin conjecture

Interval, ml	P18	P20	P22	P24	P26	P28	P30	P32
0.0–0.5	5507	2387	2523	6229	1778	1099	2515	2175
0.5–1.0	5556	2351	2544	6225	1790	1132	2530	2204
1.0–1.5	5675	2343	2533	6337	1799	1150	2530	2154
1.5–2.0	5391	2324	2546	6180	1804	1080	2559	2210
2.0–2.5	5573	2411	2545	6206	1743	1110	2555	2164
2.5–3.0	5519	2333	2510	6243	1846	1160	2555	2232
3.0–3.5	5424	2319	2549	6206	1670	1105	2542	2228
3.5–4.0	5560	2358	2544	6219	1810	1050	2477	2106
4.0–4.5	5505	2361	2614	6139	1862	1169	2442	2165
4.5–5.0	5652	2333	2603	6191	1838	1195	2494	2164
5.0–5.5	5544	2412	2614	6238	1844	1129	2552	2279
5.5–6.0	5569	2391	2544	6306	1854	1112	2590	2317
6.0–6.5	5626	2314	2559	6109	1766	1073	2446	2175
6.5–7.0	5594	2383	2586	6352	1773	1125	2537	2188
7.0–7.5	5535	2405	2569	6310	1760	1171	2504	2183
7.5–8.0	5642	2335	2557	6321	1790	1126	2598	2173
8.0–8.5	5654	2348	2463	6242	1851	1147	2396	2209
8.5–9.0	5508	2381	2546	6291	1805	1082	2541	2224
9.0–9.5	5574	2401	2558	6218	1745	1073	2559	2156
9.5–10.0	5483	2413	2523	6238	1849	1143	2549	2261

Table 3

The residue classes of relatively prime with 16

Interval, ml	1	3	5	7	9	11	13	15
0.0 – 0.5	62369	62515	62540	62526	62456	62581	62523	62489
0.5 – 1.0	62489	62601	62391	62475	62482	62394	62548	62620
1.0 – 1.5	62468	62546	62639	62498	62490	62539	62367	62453
1.5 – 2.0	62525	62380	62515	62450	62523	62569	62434	62604
2.0 – 2.5	62581	62495	62501	62482	62358	62495	62704	62384
2.5 – 3.0	62386	62468	62568	62631	62600	62500	62423	62424
3.0 – 3.5	62413	62471	62451	62571	62304	62431	62570	62789
3.5 – 4.0	62590	62563	62547	62572	62384	62522	62466	62356
4.0 – 4.5	62434	62624	62608	62419	62655	62516	62441	62303
4.5 – 5.0	62491	62599	62371	62521	62583	62430	62606	62399
5.0 – 5.5	62604	62372	62344	62712	62384	62458	62652	62474
5.5 – 6.0	62555	62739	62524	62294	62459	62525	62352	62552
6.0 – 6.5	62415	62246	62555	62546	62561	62548	62555	62574
6.5 – 7.0	62613	62649	62397	62522	62393	62390	62475	62561
7.0 – 7.5	62676	62400	62527	62395	62524	62633	62461	62384
7.5 – 8.0	62295	62450	62530	62591	62524	62391	62634	62585
8.0 – 8.5	62434	62574	62421	62511	62437	62509	62601	62513
8.5 – 9.0	62567	62428	62466	62680	62567	62574	62193	62525
9.0 – 9.5	62296	62506	62597	62320	62616	62748	62439	62478
9.5 - 10.0	62564	62366	62402	62471	62501	62559	62548	62589

Conclusions

From the tables above, it follows that if the values of each element of rows and columns are divided by 500 thousand, then in the first case we obtain the values of the generalized Artin hypothesis calculated as a result of computer simulation to the third decimal place. Obviously, if we move from blocks of 500 thousand prime numbers to blocks of 5 million consecutive prime numbers, we get estimates of Artin's coefficients at least to the fourth decimal place. Analysis of Table 3, allows us to assert that the residue classes modulo 16 constitute the $1/8$ fraction of the set of all primes. These estimates, as in Table 1 and Table 2, are obtained to the third decimal place. With an increase in the number of prime numbers in blocks, the accuracy of estimates is continuously increasing.

The paper presents the foundations of a new approach to research in the theory of numbers and dynamical systems starting with prime numbers. It is shown that the development of methods of computer modeling creates the basis for creating iterative procedures for solving complex mathematical problems in combination:

<computer model> $\Leftrightarrow$ <analytical theory>.

It is established that in mathematical theories in the study of dynamic processes it is necessary to take into account the properties of objects of sets over which the mathematical theory is constructed.

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КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ДИНАМИЧЕСКИХ ПРОЦЕССОВ В АНАЛИТИЧЕСКОЙ ТЕОРИИ ЧИСЕЛ

Г. Н. Востров, Р. Ю. Опята

Одесский национальный политехнический университет

Аннотация. *В работе методами компьютерного моделирования исследованы свойства простых чисел. Как динамической развивающейся системы. Построены методы классификации простых чисел на основе малой теоремы Ферма. Доказано, что по результатам компьютерного моделирования созданы основы развития аналитической и алгебраической теории чисел как динамических систем. В теории чисел сформировались такие направления исследований: аналитическое, алгебраическое, вычислительное, комбинаторное, геометрическое, вероятностное. Несмотря на их многообразие и огромное количество публикаций количество не решенных проблем весьма впечатляет и растет непрерывно. В исследованиях по теории чисел значительные усилия связаны с простыми и натуральными числами. В работах авторы весьма основательно представили полную картину научных исследований и ее проблематику.*

Ключевые слова: *Простые числа, малая теория Ферма, динамические системы, динамика развития структур простых чисел.*

КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ ДИНАМІЧНИХ ПРОЦЕСІВ В АНАЛІТИЧНОЇ ТЕОРІЇ ЧИСЕЛ

Г.М. Востров, Р.Ю. Опята

Одеський національний політехнічний університет

Анотація. *В роботі методами комп'ютерного моделювання досліджені властивості простих чисел. Як динамічної системи, що розвивається. Побудовано методи класифікації простих чисел на основі малої теореми Ферма. Доведено, що за результатами комп'ютерного моделювання створено основи розвитку аналітичної і алгебраїчної теорії чисел як динамічних систем. У теорії чисел сформувалися такі напрямки досліджень: аналітичне, алгебраїчне, обчислювальний, комбінаторний, геометричне, розподіл усіх. Незважаючи на їх різноманіття і величезна кількість публікацій кількість не вирішених проблем вельми вражає і росте безперервно. У дослідженнях з теорії чисел значних зусиль пов'язані з простими і натуральними числами. У роботах автори досить грунтовно представили повну картину наукових досліджень і її проблематику. Незважаючи на вражаючі результати, залишається не вирішеними, важливі як з теоретичної, так і практичної точок зору, багато проблем. У зазначених книгах стан досліджень в напрямку поглиблення досягнутих результатів представлені досить повно. Теорія чисел, починаючи з простих чисел і закінчуючи комплексними числами, активно використовується практично у всіх предметних областях сучасної чистої і прикладної математики. Однією з таких областей є математична теорія динамічних систем. З одного боку досліджується властивості динамічних систем певних над деякими підмножинами чисел з точки зору розподілу нерухомих точок функцій, що визначають динамічну систему, в залежності від їх властивостей. З іншого боку функції динамічних систем дозволяють досліджувати властивості чисел з області визначення динамічної системи. Система при цьому розглядаються з точки зору впливу їх властивостей на закони поведінки динамічної незалежно від вибору функції визначальною динамічну систему.*

Ключові слова: *Прості числа, мала теорія Ферма, динамічні системи, динаміка розвитку структур простих чисел.*

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George Vostrov, Ph. D. of Technical Sciences, Associate Professor of the Department of Applied Mathematics and Information Technologies, Odessa National Polytechnic University. Shevchenko ave., 1, Odessa, Ukraine.
E-mail: vostrov@gmail.com, mob. +380503168776

Востров Георгий Николаевич, кандидат технических наук, доцент кафедры прикладной математики и информационных технологий Одесского национального политехнического университета. Просп. Шевченко, 1, Одесса, Украина.
Эл. адрес: vostrov@gmail.com, тел. +380503168776

ORCID ID: 0000-0003-3856-5392

Roman Opiata, PhD student of the Department of Applied Mathematics and Information Technologies, Odessa National Polytechnic University. Shevchenko ave., 1, Odessa, Ukraine.
E-mail: roma.opyata@gmail.com, mob. +38095249753

Опята Роман Юрьевич, аспирант кафедры прикладной математики и информационных технологий Одесского национального политехнического университета. Просп. Шевченко, 1, Одесса, Украина.
Эл. адрес: roma.opyata@gmail.com, тел. +38095249753

ORCID ID: 0000-0001-5806-9615