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ALGORITHMS OF ROBUST OPTIMIZATION OF INERTIALLY STABILIZED PLATFORMS

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The features of the parametrical and structural robust optimization for systems of control by the initially stabilized platforms are considered. The main tasks of the initially stabilized platforms for the ground and marine vehicles are described. The statements of the parametrical and structural robust optimization problems for the initially stabilized platforms for the vehicles of the studied class are represented. The optimization results are given.

Keywords: component, ground and marine vehicles, robust parametrical and structural optimization, inertially stabilized platforms.

Introduction. In accordance with the modern scientific terminology [1] the initially stabilized platform (ISP) represents a platform equipped with the system of control by the spatial inertial orientation mounted at it payload. Usually the direction-finding (viewfinders, antennas, telescopes) and navigation (angle and rate gyro sensors, accelerometers, magnetometers) units and devices are considered as payloads of ISP. Application of the ISP allows solving such tasks as:

1) stabilization of the payload during angular motion of the object, on which it is mounted,

2) pointing at the unit line-of-sight to the given reference point;

3) the line-of-sight tracking in direction of the moving reference point.

Solution of the above listed tasks has great importance for the wide class of the vehicles such as ground, marine, aviation and space. The instrument making industry of the Ukraine has significant achievements in the area of the ISP attended for exploitation at the ground and marine vehicles.

The specific tasks of control by the ISP exploited at the ground vehicles include:

1) stabilization of the observation equipment (payload) in the vertical and horizontal planes during the vehicle angular motion;

2) pointing at the observation equipment line-of-sight to the immovable and movable reference points;

3) combination of the first and second modes that is the stabilized pointing of the observation equipment line-of-sight to the reference point.

The specific tasks of control by the ISP exploited at the marine vehicles include

1) the coarse stabilization of navigation sensors (the accelerometers) in the mode of the previous leveling;

2) the precise stabilization of navigation sensors (the gyros and accelerometers) in the mode of the precision leveling;

3) the initial alignment (setting to the meridian);

4) the combined stabilization and precision heading determination in the mode of the gyrocompass;

5) the combined stabilization and precision direction determination in the mode of the gyro-azimuth;

6) the calibration of navigation sensors during functioning and berthing.

Statement of the Parametrical robust optimization problem. Design of the ISP for solution of the above stated tasks is implemented in conditions of uncertainties caused both inaccuracies of the mathematical descriptions of the real systems and influence of the internal (parametrical) and external (coordinate) disturbances. Platforms used at the ground vehicles are characterized by the parameters which may change in the wide range. A moment of the platform inertia and the rigidity of the elastic connection between the movable platform with payload mounted at it and the actuator belong to such parameters.

Moreover, the ISP destined for exploitation at the ground and marine vehicles are subjected to influence of the external disturbances caused by the irregularities of the road or the ground surface profile and the sea irregular waves. The modern approach to solution of this problem lies in the synthesis of the robust systems able to operate in conditions of the parametrical structured and the coordinate disturbances in the modes of stabilization, pointing and tracking.

Improvement of the systems controlling by the ISP may be implemented in two directions including modification of the existing systems and creation of the new systems. The modification of the existing systems in conditions of uncertainties it is convenient to realize by means of the robust parametrical optimization. In the presence of sufficient a priori information about the analogous systems, creation of the new systems may be based on the parametric optimization too. But the modern approach to design of the new systems foresees using of the H_∞-synthesis that is the robust structural optimization providing design of a system in the conditions of uncertainties.

One of the modern approaches to the robust parametric optimization performance criteria is using the H_∞-norm of the matrix function of the closed loop system complementary sensitivity function [2; 3].

Let it is necessary to determine the $m \times 1$ vector K of adjustable parameters of the ISP control system. In this case to increase efficiency of the robust parametric optimization it is possible to use the mixed H_2 / H_∞ approach, when the complex optimization performance criterion includes the closed loop system sensitivity function as a measure of performance and the closed loop system complementary sensitivity function as a measure of robustness. So, this criterion allows achieving compromise between performance and robustness. It may be described by the following expression [2]

$$\begin{aligned} J_{H_2/H_\infty}(K) &= \lambda_2^{\text{nom d}} \| \mathrm{S}(s,K) \|_2^{\text{nom d}} + \lambda_2^{\text{nom s}} \| \mathrm{S}(s,K) \|_2^{\text{nom s}} + \\ &+ \sum_{i=1}^{n} \lambda_{2_i}^{\text{dis s}} \| \mathrm{S}(s,K) \|_{2_i}^{\text{dis s}} + \sum_{i=1}^{n} \lambda_{2_i}^{\text{dis d}} \| \mathrm{S}(s,K) \|_{2_i}^{\text{dis d}} + \\ &+ \lambda_\infty^{\text{nom}} \| \mathrm{T}(s,K) \|_\infty^{\text{nom}} + \sum_{i=1}^{n} \lambda_\infty^{\text{dis i}} \| \mathrm{T}(s,K) \|_{\infty_i}^{\text{dis i}} + PF \end{aligned} \quad . \tag{1}$$

subject to

$$K(s) \in D, \; D: \operatorname{Re} | \operatorname{eig}(I + L(s)) | < 0, \tag{2}$$

where $\|\cdot\|_2^{\text{nom d}}$, $\|\cdot\|_2^{\text{nom s}}$, $\|\cdot\|_{2_i}^{\text{dis s}}$, $\|\cdot\|_{2_i}^{\text{dis d}}$ stand for H_2-norms of the sensitivity functions of a nominal system and a system disturbed by the parametrical structured disturbances for the deterministic and stochastic cases; $\|\cdot\|_\infty^{\text{nom}}$, $\|\cdot\|_\infty^{\text{dis i}}$ stand for H_∞-norms of the complementary sensitivity functions of a nominal system and a system disturbed by the parametrical structured disturbances; $\lambda_2^{\text{nom d}}, \lambda_2^{\text{nom s}}, \lambda_\infty^{\text{nom}}, \lambda_{2_i}^{\text{dis d}}, \lambda_{2_i}^{\text{dis s}}, \lambda_\infty^{\text{dis i}}$ are the weighting coefficients of the appropriate norms; n is a number of a system's models disturbed by the parametrical structured disturbances; PF is the penalty function satisfying the stability conditions during a process of the optimization; $S(s,K)$ is the matrix sensitivity function; $T(s,K)$ is the matrix complementary sensitivity function; $P(s)$ is the matrix of transfer functions of the controller; $W(s)$ is the matrix of transfer functions of the plant; $L(s) = P(s)W(s)$.

Then the statement of the H_2 / H_∞-optimization problem for the optimization criterion (1) subject to (2) becomes

$$K^* = \arg \inf_{K \in D} J_{H_2/H_\infty}(K)$$

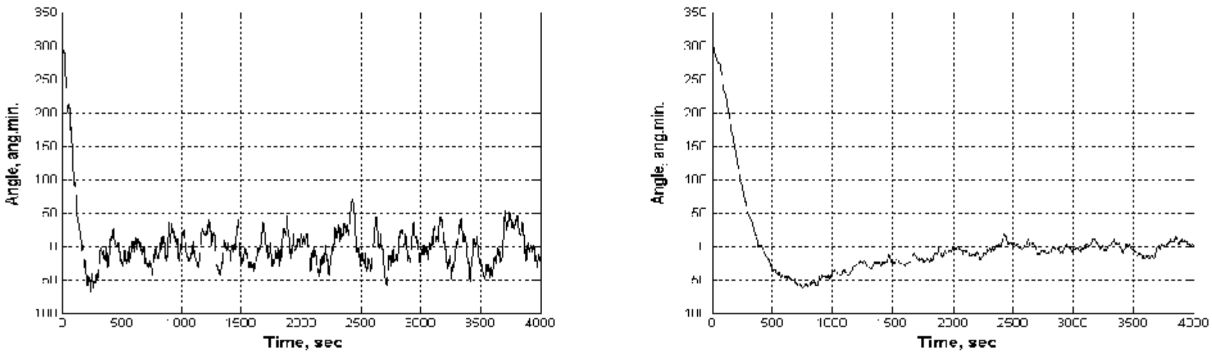
Such approach to the optimization is called the multipurpose as it allows to find a compromise between a system performance and robustness which represent the different conflicting purposes [2].

The algorithmic support of the parametrical H_2 / H_∞-optimization includes creation of the mathematical descriptions of the control system in the space of states, and taking into consideration all non-linarites peculiar to a real system; determination of the minimal and balanced realizations of the linearized models and application of the optimization method based on the genetic algorithms. During the cyclic execution of this method at every its step the following operations are carried out such as calculation of the H_2, H_∞-norms of the synthesized system, analysis of the poles location at the plane of the complex variable and correspondingly indirect indexes of the transient performance, forming of the penalty function by this analysis results and determination of a system optimization complex criterion. Checking of the synthesis results is implemented by means of the non-linear model as much as possible close to the real system. In the case of the unacceptable results the parametrical optimization procedure must be repeated after change of the initial conditions or the weighting coefficients.

Results of the Robust parametrical optimization. Results of the parametrical H_2 / H_∞-optimization for the precision navigation marine system in the mode of the previous leveling are represented in [4]. As for other modes of a system operation, it is convenient study the navigation and stabilization contours separately taking into consideration complexity of the control systems for the marine initially stabilized platforms. The main purpose of the navigation contours optimization is determination of the control laws coefficients, as the form of the control laws for the studied systems is defined by the many years experience of such systems design. Therefore use of the H_2 / H_∞-optimization is the most convenient in the given case.

The results of the parametric optimization of the system navigation contours in the mode of precise leveling [5] are presented in fig. 1. Based on experience of the marine ISP production, the researched control laws were chosen based on the integral control with damping, regulating of the free oscillations and isodromic correction. The vector of the adjustable parameters for this mode consists of six components $K = [k_1, k_2, k_3, k_4, k_5, k_6]$, which correspond to damping (k_1, k_4), regulating of free oscillations (k_2, k_5) and isodromic correction (k_3, k_6) for the pitch and roll channels correspondingly. The values of the vector are

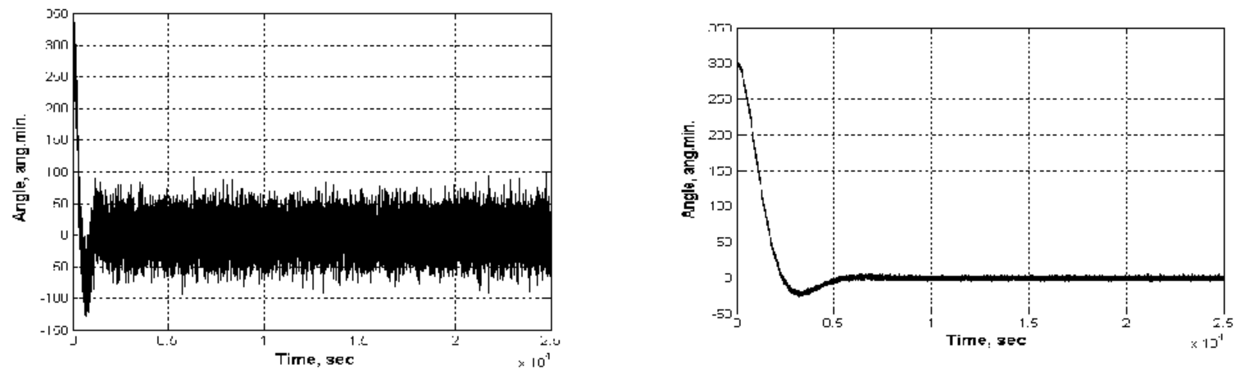
$$K = [1{,}83 \cdot 10^{-2}; 70{,}3; 2070; 1{,}83 \cdot 10^{-2}; 70{,}3; 2070].$$



a *b*

Fig. 1. The angle of platform deviation from the plane of horizon (an angle of the pitch) in the mode of the precise leveling under action of disturbances: *a* – for non-optimized system; *b* – for optimized system

The results of the parametrical optimization of the system navigation contours in the mode of the initial alignment or setting to the meridian [5] are represented in fig. 2. Control in the alignment mode is provided by the integral correction with damping and regulating of the free oscillations and correction.



a *b*

Fig. 2. The angle of the azimuth in the mode of the initial alignment: *a* – for the non-optimized system; *b* – for the optimized system

– moments applied to the gyrocompass and providing setting to the meridian. The vector of the adjustable parameters is $K = [k_1, k_2, k_3, k_4]$, where k_1, k_2 are moments applied to gyrocompass, k_3 is damping coefficient, k_4 regulates free oscillations. At that $K = [0,69 \cdot 10^{-3}; 0,069 \cdot 10^{-4}; 0,6; 50]$.

Statement of The Structural robust Optimization problem and its Results. The robust structural optimization is based on solution of two Riccati equations, checking of some conditions [3] and minimization of the H_∞-norm of a system mixed sensitivity function.

The modern approach to solution of the robust structural H_∞-optimization is based on forming of the desirable frequency characteristics of a system. Such approach is implemented by means of forming of the augmented plant due to connection of the weighting transfer functions. The H_∞-norm of the mixed sensitivity function of the augmented system is used as the optimization performance criterion

$$J_{H_\infty s} = \left\| \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix} \right\|_\infty .$$

where W_1, W_2, W_3 are the weighting transfer functions, S, R, T are the sensitivity functions by the given signal, control and the complementary sensitivity function.

In the system of the computer-aided design Robust Control Toolbox the structural synthesis of the H_∞-controller by the method of the mixed sensitivity is implemented by means of functions, which provide creation of a system model in the space of states and execution of the H_∞-synthesis procedure. The successful solution of the H_∞-synthesis by the method of the mixed sensitivity essentially depends on the choice of the weighting transfer functions. In many cases the choice of these coefficients is carried out by the empiric methods taking unto account experience of the studied systems design.

Statement of the problem of the structural H_∞-optimization by the method of the mixed sensitivity is represented in fig. 3, *a* by example of system, which includes the plant $G(s)$, the controller $K(s)$ and is defined by the vector of outputs z, which characterises the system performance, the vector of inputs r and also the control u and observation y vectors [3].

To solve problems of the precise pointing and tracking it is necessary to use the two-degree-of-freedom controller [3]. Statement of the problem of the synthesis of the H_∞-optimal two-degree-of-freedom controller is shown in the fig. 3, *b*.

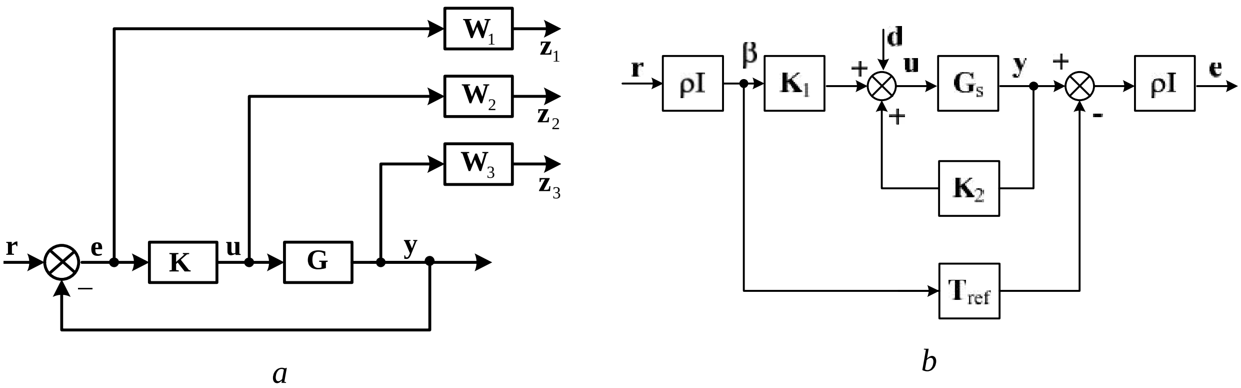


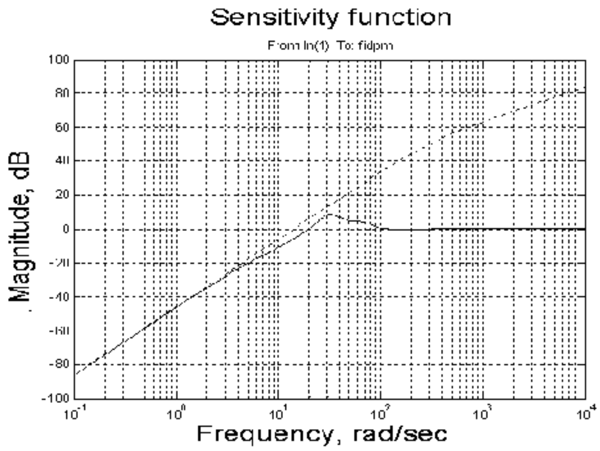
Fig. 3. Structural H_∞ -optimization: *a* – by the method of the mixed sensitivity; *b* – the system with two degrees of freedom

The synthesis of the two-degree-of-freedom controller may be carried out by the new method based on the robust stabilization taking into account both the parametrical structured disturbances and the coordinated disturbances. The optimization criterion of this method taking into consideration the sensitivity function by the coordinated disturbances is

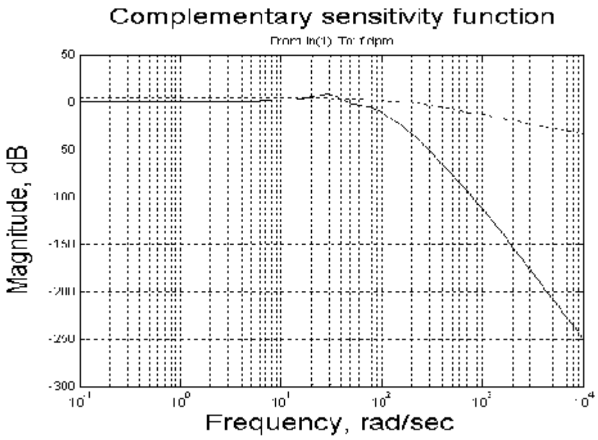
$$J_{H_{\infty ls}} = \left\| \begin{bmatrix} \circ(I-K_2G_s)^{-1}K_1 & K_2(\mathrm{I}-K_2G_s)^{-1}G_d & K_2(I-K_2G_s)^{-1}M_s^{-1} \\ \circ(I-G_sK_2)^{-1}G_sK_1 & (\mathrm{I}-K_2G_s)^{-1}G_d & (I-K_2G_s)^{-1}M_s^{-1} \\ \circ^{2}[(I-G_sK_2)^{-1}G_sK_1-T_{ref}] & \circ(I-K_2G_s)^{-1}G_d & \circ(I-K_2G_s)^{-1}M_s^{-1} \end{bmatrix} \right\|_\infty .$$

In the general the problem of the structural H_∞ -optimization leads to search of the stabilizing controller $K=\left[K_1(s) \quad K_2(s)\right]$ for the control object bounded by the transfer weighting functions $G_s = W_2GW_1$ and represented as result ob the normalized comprise factorization $G_s = M_s^{-1}N_s$. As result the synthesised feedback controller $K_2(s)$ provides the robust stability and attenuation of the parametrical structured and coordinate disturbances acting on the system. The prefilter $K_1(s)$ provides correspondence of the closed loop system reaction to the reference signal depending on the given reference model T_{ref} .The sensitivity and complementary sensitivity functions bounded by the weighting transfer functions are represented in fig. 4.

Results of the structural identification for the system controlling by the ground vehicle are represented in fig. 5. Here reaction on the impulse response for the nominal system and parametrically disturbed systems are represented. The disturbances are connected with the moment of the plant inertia and rigidity of elastic connection between the actuator and plant, which have great influence on the system performances and change in the wide range for the researched type of plants. The parametrical disturbances were sufficiently large. So, in the first case the moment of inertia was increased on 50 % and decreased on 50 %. In the second case the rigidity of elastic connection between the actuator and plant were changed on +20 % and –20 % correspondingly. These changes are shown in fig. 5, *a*, *b*.



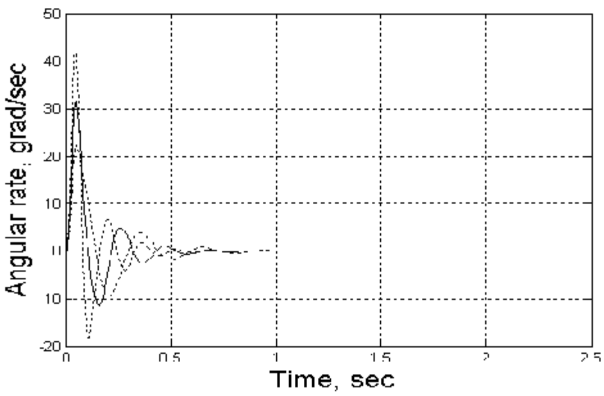
a



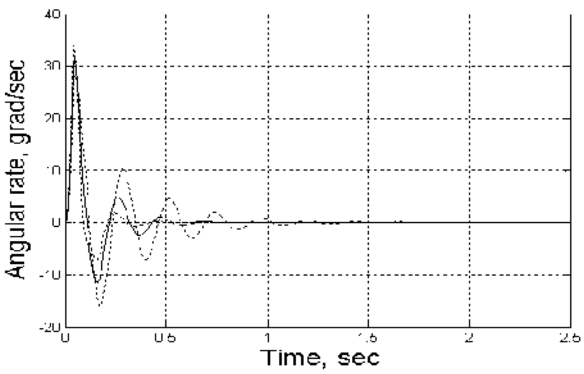
b

Fig. 4. The sensitivity and complementary sensitivity functions bounded by the frequency response $1/W_S$ $- a$; $1/W_T$ $- b$

The comparative analysis has been shown that in this case the reduction of the synthesized controller is convenient to carry out after H_∞-synthesis procedure termination. The reduction of the controller of the studied system may be carried out by means of the function, which performs balanced-truncation reduction of the controller.



a



b

Fig. 5. Results of the structural optimization: the impulse responses for decrease and increase of the stabilization object inertia moment (*a*) and rigidity of elastic connection between actuator and plant (*b*)

Results of the two-degree-freedom controller synthesis for the ground ISP are represented in fig. 6.

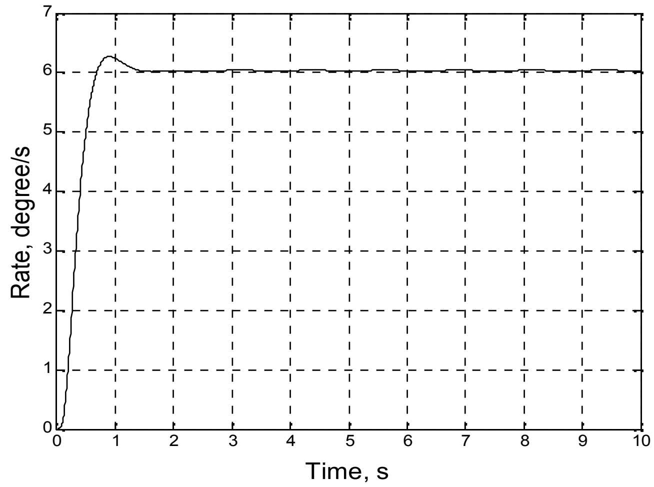


Fig. 6. Results of the two-degree freedom controller synthesis

Structure of the regulator in the space of states after reduction from eights order to fifth one looks like

$$\mathrm{A}_p = \begin{bmatrix} -21,93 & -108,1 & 0,025 & 0,017 & 0 \\ -140,2 & -691,3 & 0,156 & 0,107 & 0 \\ 2,89\cdot10^{-5} & 6,85\cdot10^{-4} & -38,6 & -86,3 & 0 \\ 4,15\cdot10^{-4} & 3,59\cdot10^{-4} & 86,3 & -46,96 & 0 \\ 0 & 0 & 0 & 0 & 1,98\cdot10^{-9} \end{bmatrix};$$

$$\mathrm{B}_p = \begin{bmatrix} 543 & -110,1 & -32,71 & 22,43 & -3,08 \end{bmatrix}^T;$$

$$\mathrm{C}_p = \begin{bmatrix} 5,82\cdot10^{4} & 2,89\cdot10^{5} & -32,71 & -22,43 & -9,97\cdot10^{4} \end{bmatrix};$$

$$\mathrm{D}_p = 0 .$$

In this figure the given step response for system disturbed by the unbalanced moment with additional random disturbance is shown. The obtained controller $K = [K_1(s) \quad K_2(s)]$ in the space of states after reduction to the fifth order looks like

$$B_1^T = \begin{bmatrix} 0,965 & -0,058 & 14,48 & 0,23 & -0,92 \end{bmatrix};$$

$$C_1 = \begin{bmatrix} 0,965 & 0,058 & 14,48 & -0,23 & -0,92 \end{bmatrix};\ D_1 = 0.$$

$$A_1 = \begin{bmatrix} -1,6 & -0,3 & -35,3 & 0,9 & 2,8 \\ 0,3 & -0,01 & -49,3 & 0,1 & 0,5 \\ -35,3 & 49,3 & -1061,8 & 73,1 & 115,6 \\ -0,9 & 0,1 & -73,1 & -0,5 & -5,7 \\ 2,8 & -0,5 & 115,6 & 5,7 & -25,8 \end{bmatrix};$$

$$A_2 = \begin{bmatrix} -1067,9 & -117,9 & 122,2 & -47,9 & -12,3 \\ 117,9 & -0,01 & 0,1 & -0,01 & -0,01 \\ 122,2 & -0,1 & -24 & 16,6 & 4 \\ 47,9 & -0,1 & -16,6 & -27,7 & -15 \\ -12,3 & 0,1 & 4 & 15 & -64,6 \end{bmatrix};$$

$$B_2^T = \begin{bmatrix} -64,81 & 0,02 & 4,51 & 1,43 & -0,37 \end{bmatrix};$$

$$C_2 = \begin{bmatrix} 64,81 & 0,02 & -4,51 & 1,43 & 0,37 \end{bmatrix};\ D_2 = 0.$$

The controller synthesized by the structural method represents a system of sufficiently high order. There are some approaches [3] to reduction of the synthesized controllers such as reduction of a system's model before the H_∞-synthesis procedure, reduction of a system's model after the H_∞-synthesis procedure and use of the special methods which provide creation of a system with the reduced order.

Conclusion. Specific tasks for the ground and marine initially stabilized platform are listed. The possibilities of the robust optimization to the problem of design and modernization of the systems controlling by the initially stabilized platforms for the ground and marine vehicles are considered. The statements of problems and simulation results are given.

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О. А. Сущенко, А. А. Тунік

Робастна оптимізація інерціальних стабілізованих платформ

Розглянуто особливості параметричної і структурної робастної оптимізації інерціальних стабілізованих платформ. Описано основні завдання інерціальних стабілізованих платформ для наземних і морських рухливих об'єктів. Представлено постановки проблем параметричної і структурної робастної оптимізації інерціальних стабілізованих платформ для рухомих об'єктів розглянутого класу. Показано результати оптимізації.

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Робастная оптимизация инерциальных стабилизированных платформ

Рассмотрены особенности параметрической и структурной робастной оптимизации инерциальных стабилизированных платформ. Описаны основные задачи инерциальных стабилизированных платформ для наземных и морских подвижных объектов. Представлены постановки проблем параметрической и структурной робастной оптимизации инерциальных стабилизированных платформ для подвижных объектов рассматриваемого класса. Показаны результаты оптимизации.