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TEMPERATURE ERROR MODEL IN CORIOLIS VIBRATORY GYROSCOPES

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The effect of temperature variations on the output of Coriolis vibratory gyroscope is studied in this paper. Mathematical model of the temperature influences is developed and its parameters are identified using experimental data. The latter has been used to validate obtained model.

Keywords: Coriolis vibratory gyroscope, cross-damping, modeling, temperature errors.

Introduction. Vibratory gyroscopes that utilize Coriolis effect have been successfully used in vast amount of different applications since micro fabrication techniques made possible to reduce its cost in mass production along with significant reduction in size [1; 2].

At the same time, Coriolis vibratory gyroscopes (CVG) traditionally occupy niche of low accuracy sensors due to the low stability of its performance under influence of operational environment factors. One of the major sources of such instabilities is temperature variations that cause changes in all measurement characteristics of CVGs [3; 4]. In this paper we study the effect of temperature variations on CVG, develop empirical model of the temperature influences, identify its parameters, and develop model of angular rate measurement errors due to the temperature variations. Later we validate obtained models using experimental data obtained for CVG with cylindrical sensitive element.

Temperature related zero-rate output. Significant temperature related zero-rate output has been observed during experimental tests of CVG. For the temperature profile, shown in Fig. 1, and zero angular rate, CVG output is shown in Fig. 2 (uncompensated).

Temperature variations are assumed to cause this bias through the temperature dependent cross-damping. In this case excited primary oscillations of the sensitive element will induce secondary (output) oscillations even without external rotation applied to the sensor. Moreover, cross-damping induced oscillations will not be distinguishable from the oscillations due to the angular rate.

In order to develop mathematical model for this phenomenon let us first analyze how cross-damping affects dynamics of the CVS sensitive element.

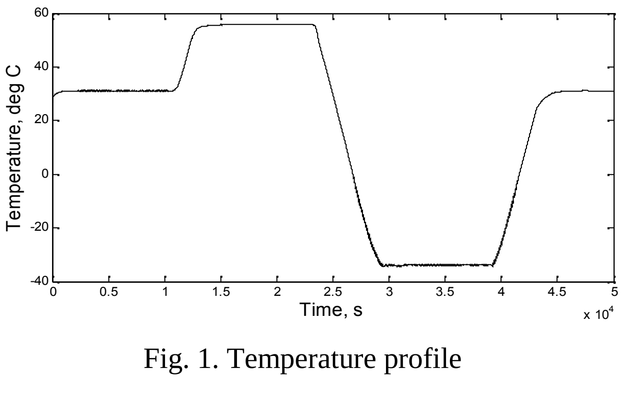

Fig. 1. Temperature profile

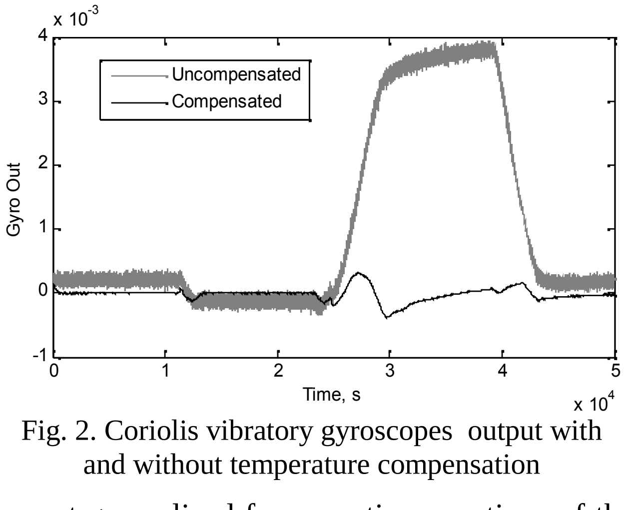

Fig. 2. Coriolis vibratory gyroscopes output with and without temperature compensation

Sensitive element motion equations. In the most generalized form, motion equations of the CVG sensitive element both with translational and rotational motion could be represented in the following form [5]:

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + (k_1^2 - d_1\Omega^2)x_1 + g_1\Omega\dot{x}_2 + d_3\dot{\Omega}x_2 = q_1(t), \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + (k_2^2 - d_2\Omega^2)x_2 - g_2\Omega\dot{x}_1 - \dot{\Omega}x_1 = q_2(t). \end{cases} \quad (1)$$

Here x_1 and x_2 are the generalized coordinates that describe primary (excited) and secondary (sensed) motions of the sensitive element respectively, k_1 and k_2 are the corresponding natural frequencies, ζ_1 and ζ_2 are the dimensionless relative damping coefficients, Ω is the measured angular rate, which is orthogonal to the axes of primary and secondary motions, q_1 and q_2 are the generalized accelerations due to the external forces that act on the sensitive element. The remaining dimensionless coefficients are different for the sensitive elements that exploit either translational or rotational motion. For the vibrating cylinder sensitive element, for example, $d_1 = d_2 = 1$, $d_3 = 1$, $g_1 = 2$, $g_2 = 2$. For other sensitive elements designs expressions for these coefficients can be found in [5].

If cross damping is present in the system, the motion equations 1 are transformed to the following form

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + (k_1^2 - d_1\Omega^2)x_1 = \\ = -(g_1\Omega + 2\zeta_{21}k_1)\dot{x}_2 - d_3\dot{\Omega}x_2 + q_1(t), \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + (k_2^2 - d_2\Omega^2)x_2 = \\ = (g_2\Omega + 2\zeta_{12}k_2)\dot{x}_1 + \dot{\Omega}x_1 + q_2(t). \end{cases} \quad (2)$$

Here ζ_{12} and ζ_{21} are the relative cross-damping coefficients. Constant cross-coupling through the damping can be removed by calibration. However, calibration is unable to deal effectively with varying in time damping.

Cross-damping induced output. Let us first analyze cross-damping related components in the amplitude of the secondary oscillations. Transforming equations 2 using amplitude-phase complex variables is similar to what has been demonstrated in [6], the following first-order equation for the slowly varying amplitudes ($\ddot{A}_2 \approx 0$) can be produced:

$$\begin{aligned} &2(\zeta_2 k_2 + j\omega)\dot{A}_2 + (k_2^2 - \omega^2 + 2j\omega k_2\zeta_2)A_2 = \\ &= (j\omega g_2\Omega + 2j\omega\zeta_{12}k_1 + \dot{\Omega})A_1. \end{aligned} \quad (3)$$

Here A_1 is the constant (does not depend on time) complex amplitude of the primary oscillations

$$A_1 = \frac{q_{10}}{k_1^2 - \omega^2 + 2jk_1\zeta_1\omega}.$$

and complex amplitudes A_i are expressed in terms of real amplitudes and phases as $A_i(t) = A_{i0}(t)e^{j\phi_{i0}(t)}$, where i equals 1 or 2 for the primary or secondary oscillations correspondingly. It also should be noted that disturbances in primary oscillations caused by secondary are considered negligible comparing to the forces from the excitation system.

Applying Laplace transformation to both sides of the equation 3, and solving obtained algebraic equation for the secondary amplitude, results in

$$A_2(s) = \frac{A_1\left[(s + jg_2\omega)\Omega(s) + 2j\omega k_2\zeta_{12}(s)\right]}{k_2^2 - \omega^2 + 2j\omega k_2\zeta_2 + 2s(k_2\zeta_2 + j\omega)} \quad (4)$$

Solution 4 can be represented as a sum of the following two components:

$$A_2(s) = A_2^{\Omega}(s) + A_2^{\zeta}(s).$$

$$A_2^{\Omega}(s) = \frac{A_1(j\omega g_2 + s)}{(s+j\omega)^2 + 2\zeta_2 k_2 (s+j\omega) + k_2^2}\Omega(s). \tag{5}$$

$$A_2^{\zeta}(s) = \frac{2A_1 j\omega k_2}{(s+j\omega)^2 + 2\zeta_2 k_2 (s+j\omega) + k_2^2}\tilde{\zeta_{12}(s)}.$$

Here $A_2^{\Omega}(s)$ is the part of the secondary amplitude due to the input angular rate, and $A_2^{\zeta}(s)$ is due to the cross-damping. Corresponding to (5) transfer functions are hence defined as

$$A_2(s) = W_2^{\Omega}(s)\Omega(s) + W_2^{\zeta}(s)\zeta_{12}(s).$$

$$W_2^{\Omega}(s) = \frac{A_1(j\omega g_2 + s)}{(s+j\omega)^2 + 2\zeta_2 k_2 (s+j\omega) + k_2^2}. \tag{6}$$

$$W_2^{\zeta}(s) = \frac{2A_1 j\omega k_2}{(s+j\omega)^2 + 2\zeta_2 k_2 (s+j\omega) + k_2^2}.$$

It is important to remember that the part of the secondary amplitude due to the cross-damping will be undistinguishable from the one caused by the angular rate. Let us therefore derive transfer function relating input cross-damping to the output angular rate as

$$\Omega^{\zeta}(s) = W_{\Omega}^{\zeta}(s)\zeta_{12}(s).$$

where $\Omega^{\zeta}(s)$ is the measured erroneous angular rate caused by the cross-damping. Quite apparently unknown transfer function $W_{\Omega}^{\zeta}(s)$ can be expressed using transfer functions from (6) as

$$W_{\Omega}^{\zeta}(s) = \frac{W_2^{\zeta}(s)}{W_2^{\Omega}(s \to 0)} = \frac{2k_2(k_2^2 - \omega^2 + 2jk_2\omega\zeta_2)}{g_2(k_2^2 - \omega^2 + 2k_2\zeta_2 s + 2j\omega(s + k_2\zeta_2))}. \tag{8}$$

Transfer function (8) can be further simplified using assumptions that are relevant to CVGs with cylindrical sensitive element, and are good approximations for other sensitive elements designs (see [6]). Namely, we can assume that natural frequencies are equal ($k_1 = k_2 = k$) as well as relative damping coefficients ($\zeta_1 = \zeta_2 = \zeta$), and primary oscillations excitation frequency is $\omega = k\sqrt{1-2\zeta^2}$. With these assumptions transfer function (8) becomes

$$W_{\Omega}^{\zeta}(s) = \frac{2k^2\zeta}{g_2(s + k\zeta)}. \tag{9}$$

Transfer function (9) allows efficient analysis of errors due to the cross-damping, which is not only present in the system, but can vary due to the different reasons.

Empirical modelling of cross-damping. If we assume that the cross-damping coefficient is a function of the temperature shift T from the calibration temperature, it can be approximated using polynomial as

$$\zeta_{12} = \zeta_{12}(T) \approx \sum_{i=0}^{n} \zeta_i^T T^i. \tag{10}$$

Number of terms *n* in representation (10) is selected to provide required accuracy.

Coefficients ζ_i^T can be determined experimentally when ambient temperature is known (measured) and angular rate is absent. Parameters of the cross-damping model (10) were found to have the following values: $\zeta_0^T = 1{,}0792 \cdot 10^{-3}$, $\zeta_1^T = -4{,}631 \cdot 10^{-5}$, $\zeta_2^T = 7{,}7044 \cdot 10^{-7}$, $\zeta_3^T = -5{,}8598 \cdot 10^{-9}$. Influence of the higher order components has been found negligible. In order to validate cross-damping model (10) we build temperature error compensation system using parameters mentioned above and a model (9) of cross damping errors.

Coriolis vibratory gyroscopes output after temperature error compensation is shown in Fig. 2 (compensated).

Conclusions. The model of temperature related errors in CVGs along with the empirical model of the cross-damping developed in this paper have been used to develop temperature error compensation system, which significantly (approximately 8 times) improved CVG performances in terms of zero-rate output. In future research we plan to use the model of cross-damping errors to develop stochastic system of temperature errors compensation.

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Температурна похибка моделі вібраційного гіроскопа Коріоліса

Вивчено вплив змін температури на вихідні сигнали коріолісових вібраційних гіроскопів. Розроблено математичну модель температурних впливів, параметри якої було ідентифіковано з використанням експериментальних даних. В подальшому експериментальні дані було використано для перевірки коректності отриманої моделі.

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Температурная погрешность модели вибрационного гироскопа Кориолиса

Изучено влияние изменений температуры на выходных сигналы кориолисовых вибрационных гиросокопв. Разработана математическая модель температурных воздействий, параметры которой были идентифицированы с использованием экспериментальных данных. В последствии экспериментальные данные были использованы для проверки корректности полученной модели.