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KALMAN FILTERING IN ORIENTATION SYSTEMS

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The use of Kalman filter in system of orientation is considered. The block diagram of Kalman filter realization is proposed.

Keywords: component, orientation system, Kalman filter, block diagrams.

Introduction. For the accuracy increasing of angular velocity determination in orientation systems of Unmanned Aerial Vehicle (UAV), of any type is possible. The united block diagram of three-axis magnetometer, pyro horizon and module satellite navigation system (GPS, GLONASS) is presented in fig. 1.

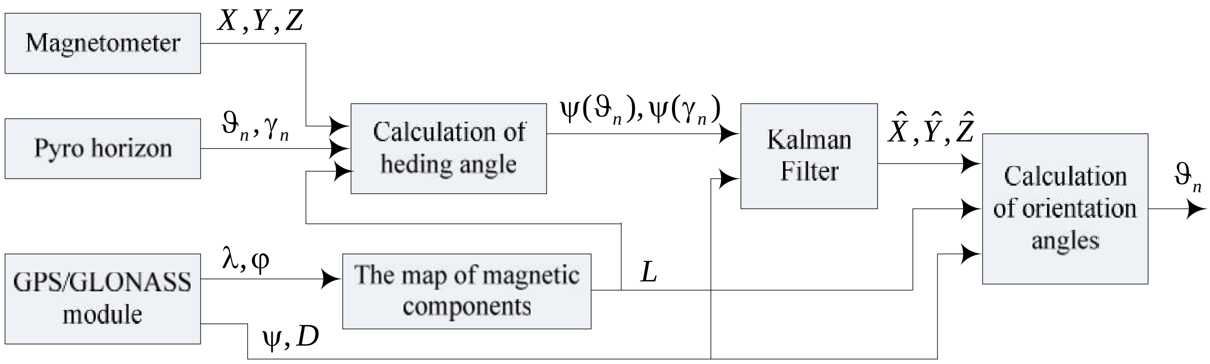

Fig. 1. Block diagram of the module of three-axis magnetometer, pyro horizon and satellite navigation system

The accuracy of this system depends on the realization of Kalman filter.

Problem statement. Kalman filter is synthesized from the following statement of filtration problem of dynamic object state variables in the presence of noise.

$$\hat{x} = \arg\left\{ \frac{\min}{\hat{x}} \operatorname{tr}\mathbf{P}_\varepsilon(t) \left| \begin{array}{l} \dot{x}(t) = A(t)x(t) + Bu(t) + G(t)w(t), \\ y(t) = C(t)x(t) + v(t), \\ M\hat{x} = Mx(t),\ Mw(t) = 0,\ Mv(t) = 0, \\ Cov[w(t),\ w(\tau)] = Q_w\delta(t-\tau), \\ Cov[v(t),\ v(\tau)] = R_v\delta(t-\tau), \\ Cov[w(t),\ v(\tau)] = 0, \\ Cov[\hat{x}(0),\ \hat{x}(0)] = P_0. \end{array} \right. \right\} \quad (1)$$

where $x(t)$ = $[x_1 \ldots x_n]^T$ – vector of state variables; $\hat{x}(t) = [\hat{x}_1 \ldots \hat{x}_n]^T$ – vector of the state variables estimates; $u(t)$ = $[u_1 \ldots u_r]^T$ – vector of input values; $y(t)$ = $[y_1 \ldots y_l]^T$ – vector of output; $A(t)$, $B(t)$, $C(t)$ – state, control and output matrices, respectively; $\operatorname{tr}\mathbf{P}_\varepsilon$ – trace of covariance matrix $\mathbf{P}_\varepsilon$ filtration error $\varepsilon(t) = x(t) - \hat{x}(t)$; w, v – disturbances on object input and output meter, m; Cov – operators of expectation and covariance; Q_w, R_v – intensity noise w, v; $\delta(t)$ – delta function of Dirac.

Kalman filter synthesis consists of two stages, namely:
– determining the structure of the filter;
– defining filter parameters.

Filter design is based on the linearity of filtration procedure. Because the information for the synthesis filter can be used in the input value $u(t) = \left[u_1\ ...\ u_r\right]^T$ of the object and the measured output value $y(t)$ = $\left[y_1\ ...\ y_l\right]^T$, then taking into account the filter linearity and the available information, filter structure takes the following form

$$\hat{x}(t) = \mathbf{L}_f(t)\hat{x}(t) + \mathbf{K}_f(t)\mathbf{y}(t) + \mathbf{B}_f(t)\mathbf{u}(t), \tag{2}$$

where $\mathbf{L}_f(t), \mathbf{K}_f(t), \mathbf{B}_f(t)$ – the matrices of state, inputs $y(t)$ and $u(t)$ respectively.

Filter matrix must be determined taking into account the requirements for estimates of the state variables, namely the requirements of unbiasedness and efficiency.

From the statement of filtration problem (1) it is followed that for its solution it is necessary to operate the covariance matrix $\mathbf{P}_\varepsilon(t)$ of filtration error

$$\varepsilon(t) = x(t) - \hat{x}(t).$$

Theoretical basis for the filters synthesis is the results of random processes transformation theory with help of linear dynamic operators. On base of this theory, the covariance matrix $\mathbf{P}_x(t)$ of the state vector x of dynamic system

$$\dot{x} = \mathbf{A}x + \mathbf{G}w,$$

when, its input is white noise w is determined by following equation

$$\mathbf{P}_x(t) = \mathbf{AP} - \mathbf{PA}^T + \mathbf{GQ}_w, \tag{3}$$

where Q_w is noise intensity.

For the fixation of this equation solution it is necessary to have the appropriate conditions. As such, it is often used the initial conditions of covariance matrix

$$\mathbf{P}_x(0) = Cov\left\{x(0)mx(0)\right\} - PA^T + GQ_w,$$

On based on differential equation (3) it can be obtained the corresponding differential equations of covariance matrix filter $\mathbf{P}_\varepsilon(0)$.

For this purpose it is necessary to find its dynamical transformation parameters, which determine the filtration error $L_f(t), K_f(t), B_f(t)$ – it can be realized on based on state equations (1).

$$\mathbf{x}(t) = \mathbf{Ax}(t) + \mathbf{Bx}(t) + \mathbf{Gw}(t),$$

$$\mathbf{y}(t) = \mathbf{Cx}(t) + \mathbf{v}$$

and filter model (2).

The differential equation of filtration error can be determined as

$$\dot{\varepsilon} = \dot{x} - \dot{\hat{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{Gw} - \mathbf{L}_f\hat{x} + \mathbf{K}_f\mathbf{Cx} - \mathbf{B}_f\mathbf{u} - \mathbf{K}_f\mathbf{v}, \tag{4}$$

or

$$\dot{\varepsilon} = (\mathbf{A} - \mathbf{K}_f\mathbf{C})\mathbf{x} + (\mathbf{B} - \mathbf{B}_f)\mathbf{u} - \mathbf{L}_f\hat{x} - \mathbf{K}_f\mathbf{v} + \mathbf{Gw}, \tag{5}$$

then, taking into account (3) and (5) the differential equation of covariance matrix of filtration error, can be written as

$$\mathbf{P}(t) = (\mathbf{A} - \mathbf{K}_f\mathbf{C})\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{K}_f\mathbf{C})^T + \mathbf{GQ}_w\mathbf{G}^T + \mathbf{K}_f\mathbf{R}_v\mathbf{K}_f^T,$$

Matrices $\mathbf{L}_f(t)$ (t) and $\mathbf{B}_f(t)$ are determined, taking into account the demands of unbiasedness of state variables estimates and $\mathbf{K}_f$ – from the condition of their effectiveness. From (4) the condition of estimates unbiasedness is followed

$$\mathbf{M}_{\varepsilon} = \mathbf{M}\left[(\mathbf{A}-\mathbf{K}_f\mathbf{C})\mathbf{x}+(\mathbf{B}-\mathbf{B}_f)\mathbf{u}-\mathbf{L}_f\hat{x}-\mathbf{K}_f\mathbf{v}+\mathbf{G}\mathbf{w}\right]=0. \qquad (6)$$

Due to the errors centeredness $w(t)$ and $v(t)$, the expression (6) is followed

$$(\mathbf{A}-\mathbf{L}_f-\mathbf{K}_f\mathbf{C})\mathbf{x}+(\mathbf{B}-\mathbf{B}_f)\mathbf{u}=0.$$

As this condition must be existed, during the any values of state variables $x(t)$ and control variable $u(t)$, than it can be separated for two next expressions:

$$\left(\mathbf{A}-\mathbf{L}_f-\mathbf{K}_f\mathbf{C}\right)\mathbf{x}(t)=0, \qquad \left(\mathbf{B}-\mathbf{B}_f\right)\mathbf{u}(t)=0.$$

It is clear, that unbiasedness estimate condition will be satisfied if matrices $\mathbf{L}_f$, $\mathbf{B}_f$, are determined as follows

$$\mathbf{L}_f=\mathbf{A}-\mathbf{K}_f\mathbf{C}, \qquad (7)$$

$$\mathbf{B}_f=\mathbf{B}. \qquad (8)$$

The unbiasedness condition permits to determine matrix $\mathbf{B}_f$ of filter input by variable $u(t)$.

For the determination of matrix $\mathbf{L}_f$, it is necessary to find matrix $\mathbf{K}_f$. Taking into account founded matrices $\mathbf{L}_f$ (7) and B_f (8), the differential equation of the filter (2) is

$$\dot{\hat{x}}(t)=\left(\mathbf{A}-\mathbf{K}_{\Phi}\mathbf{C}\right)\hat{x}(t)+\mathbf{K}_{\Phi}(t)\mathbf{y}(t)+\mathbf{B}(t)\mathbf{u}(t). \qquad (9)$$

On based on object state differential equation (1) and filter state differential equation (9), the differential equation (5) of filtration error, takes place.

$$\dot{\varepsilon}=\left(\mathbf{A}-\mathbf{K}_{\Phi}\mathbf{C}\right)\varepsilon-\mathbf{K}_{\Phi}\mathbf{v}+\mathbf{G}\mathbf{w}.$$

The differential equation of filtration error covariation matrix follows

$$\dot{\mathbf{P}}(t)=\left(\mathbf{A}-\mathbf{K}_{\Phi}\mathbf{C}\right)\mathbf{P}+\mathbf{P}\left(\mathbf{A}-\mathbf{K}_{\Phi}\mathbf{C}\right)^T+\mathbf{G}\mathbf{Q}_{\mathbf{w}}\mathbf{G}^T+\mathbf{K}_{\Phi}\mathbf{R}_v\mathbf{K}_{\Phi}^T. \qquad (10)$$

Thus, to determine the matrix $\mathbf{K}_{\Phi}$ we need to find the covariance matrix $\mathbf{P}(t)$, this is the main difficulty in solving the problem of synthesis of optimal filters, namely the solution of the differential equation of the covariance matrix (10).

In the research paper the solution of the Riccati equation for the covariance matrix **P** are found by numerical integration. Accordingly, a block diagram of the generation of the covariance matrix by (10) is shown in fig. 2.

The problem of determining the matrix $\mathbf{K}_{\Phi}$ is greatly simplified if, instead of the solution (5) using an equivalent solution

$$\mathbf{K}_{\Phi}(t)=\arg\left\{\frac{d}{d\mathbf{K}_{\Phi}}\mathrm{tr}(P)=0\right\},$$

which was done by Kalman. Simplification is the case, as for the synthesis of the Kalman filter is not necessary to determine the covariance matrix, i. e in this case there is no need to address the Riccati differential equation (10).

State variable filters, synthesized in the Kalman and Wiener filtering to formulate problems, are dynamic operators. This implies that the results of the processing of the input signals depend not only on the past, but also on the initial conditions. It should be noted that the Wiener filter is not critical to the effect of initial conditions on the quality of the estimates of the state variables, as essentially in Wiener formulation of the problem is seen only stationary mode filters. However, this is not the case for the Kalman filter.

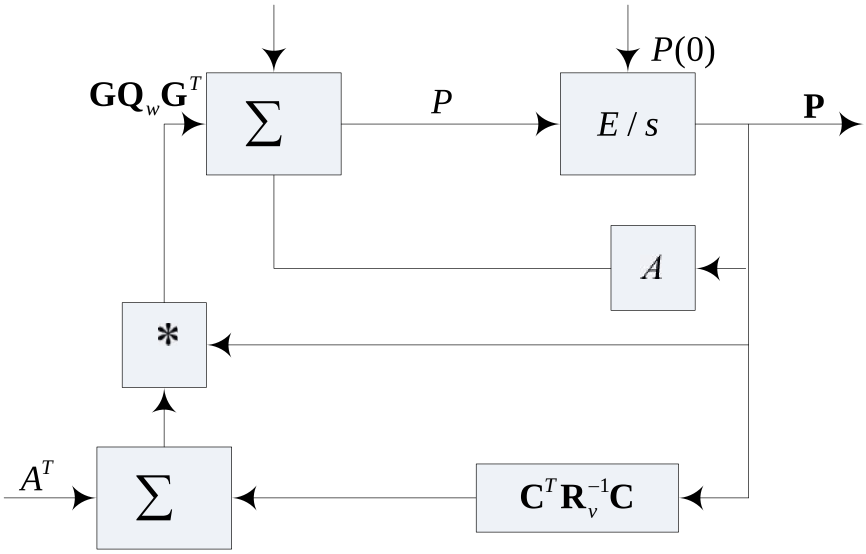

Fig. 2. Block diagram of the generation of the covariance matrix where (*) – multiplication inputs operation

The estimates $\hat{x}(t)$ of the state $x(t)$ are as follows:

$$\hat{x}(t) = \hat{x}_{y,u,v}(t) + \hat{x}_{cb}(t),$$

where $\hat{x}_{y,u,v}(t)$ – is the evaluation $\hat{x}(t)$; depending on the input signals $u(t)$, $w(t)$ and $y(t)$, $\hat{x}_{cb}(t)$ – a free component, depending on the initial estimates $\hat{x}(0)$ estimates of the state variables. When a mismatch of the initial conditions of the state variables of the object $x(0)$ and filter $\hat{x}(0)$ in the filter error $\varepsilon(t)$ will contain the corresponding free component $\varepsilon_{cb}(t)$, which according to the differential equation error filter (5) is equal to

$$\varepsilon_{cb}(t) = e^{\Phi(t)}\varepsilon(0),$$

where $\Phi(t)$ – is the transition matrix of the filter is equal to

$$\mathbf{\Phi}(t) = \mathbf{A} - \mathbf{K}_{\mathbf{\Phi}}\mathbf{C},$$

Thus, for non-zero detuning $\varepsilon(0) \neq 0$ detuning of the initial conditions of the state variables of the object and the filter error filter $\varepsilon_{cb}(t)$ is not equal to zero.

Obviously, neglecting the error of the initial conditions of the object and in the filter will result in the problem statement to the fact that the Kalman filter will be suboptimal, it really is and takes place.

This implies that the main objective of the Kalman filter in contrast to the Wiener filter is to ensure the effectiveness of filtering in the time interval before its release at steady state. However, because of the above error when account is taken of the influence of the initial conditions of the object and the filter Kalman filter with its main objective of this can not cope!

In this case, the differential equation of the covariance matrix of filtering error is (10).

To determine $\mathbf{K}_\Phi$ in the problem, there is one condition, namely the condition of efficiency.

Thus, the matrix $\mathbf{K}_\Phi$ is defined as:

$$\mathbf{K}_\Phi(t) = \arg\left\{\min_{K_\Phi} \operatorname{tr}\mathbf{P}_\varepsilon(t)\right\},$$

Conclusion. The procedure of orientation task solution is shown. The use of Kalman filter is proposed, the block diagram of Kalman filter realization is developed.

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Фильтр Калмана в системах ориентации

Рассмотрено использование фильтра Калмана в системе ориентации. Предложена реализация блок-схемы фильтра Калмана.