AUTOMATIC CONTROL SYSTEMS

UDC 681.5(045)

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SECONDARY WAVE CONTROL SYSTEM OF THE CORIOLIS VIBRATORY GYROSCOPE RESONATOR

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Abstrakt. *On the basis of transfer function linearization of vibratory gyroscope resonator, as a plant, the question of the resonator secondary wave control system stability is analyzed when the resonator parameters are changed, for example from temperature. The two cases are analyzed when controller is tuned on the bandwidth of 100 Hz and 200 Hz.*

Keywords: Coriolis vibratory gyro, Bode diagram, “slow” variables.

Introduction. Signals coming from the Coriolis vibratory gyroscope (CVG) resonator are amplitude-modulated. The carrier is resonant frequency (from several to tens of kHz), and the angular rate is the envelope (a few hundreds of Hz). To measure angular rate it needs to demodulate the resonator signals. The demodulated signal components have a low frequency, so the dynamic equations for the demodulated components are called the dynamic equations for the slow variables. Namely slow variables behavior determines CVG accuracy. Control signals are created using slow variables, but before to send them to the resonator they are re-modulated by the resonant frequency of the required phase.

Because of the presence of modulators and demodulators the control system is nonlinear, making it difficult to research and analysis. For the linearization of the control system in this work the dynamic equations for the demodulated, i.e. “slow” variables, are used and then based on these equations, a linear transfer function of the plant with the controller is derived, ensuring the required quality control. Sensitivity analysis to changing the basic resonator (plant) parameters is conducted.

Linearization of the control system. One technique of CVG control system linearization was considered in [1]. Here a different approach is considered, using the transition from the dynamic equations for the “fast” variables to the equation for the “slow” variables [2].

Coriolis vibratory gyroscope resonator dynamic equations in the “fast” variables are written as follows:

$$\begin{aligned} &\ddot{x} - 2k\Omega\dot{y} + c_{11}\dot{x} + c_{12}\dot{y} + k_{11}x + k_{12}y = f_x, \\ &\ddot{y} + 2k\Omega\dot{x} + c_{21}\dot{x} + c_{22}\dot{y} + k_{21}x + k_{22}y = f_y, \end{aligned} \tag{1}$$

where k is Bryan coefficient, that is equal to about 0,4, c_{11} is damping coefficient along X-axis, c_{12} is a cross-coupling damping, k_{11} is X-axis natural frequency, squared, k_{12} is cross-coupling stiffness, $c_{21} = c_{12}$, c_{22} is damping coefficient along Y-axis, $k_{21} = k_{12}$, k_{22} is Y-axis natural frequency, squared, f_x, f_y are forces acting along X and Y axes, respectively.

Let’s introduce transformation of the “fast” variables x, y into the slow ones X and Y, each of which has a sine component X_s, Y_s and cosine component X_c, Y_c:

$$\begin{aligned} &x = X_s \sin(\omega t) + X_c \cos(\omega t); \\ &y = Y_s \sin(\omega t) + Y_c \cos(\omega t). \end{aligned} \tag{2}$$

Differentiating (2) and determining the expressions for the derivatives of $\dot{x}$, $\ddot{x}$, $\dot{y}$ and $\ddot{y}$, after substituting in equation (1) the dynamic equation for the “slow” variables can be obtained:

$$
\begin{aligned}
\dot{X}_s &= -\frac{c_{11}}{2}X_s - \frac{k_{11}-\omega^2}{2\omega}X_c - \frac{c_{12}-2k\Omega}{2}Y_s - \frac{k_{12}}{2\omega}Y_c + \frac{F_{xc}}{2\omega};\\
\dot{X}_c &= \frac{k_{11}-\omega^2}{2\omega}X_s - \frac{c_{11}}{2}X_c + \frac{k_{12}}{2\omega}Y_s - \frac{c_{12}-2k\Omega}{2}Y_c - \frac{F_{xs}}{2\omega};\\
\dot{Y}_s &= -\frac{c_{21}-2k\Omega}{2}X_s - \frac{k_{21}}{2\omega}X_c - \frac{c_{22}}{2}Y_s - \frac{k_{22}-\omega^2}{2\omega}Y_c + \frac{F_{yc}}{2\omega};\\
\dot{Y}_c &= \frac{k_{21}}{2\omega}X_s - \frac{c_{21}-2k\Omega}{2}X_c + \frac{k_{22}-\omega^2}{2\omega}Y_s - \frac{c_{22}}{2}Y_c - \frac{F_{ys}}{2\omega}.
\end{aligned}
\tag{3}
$$

Under transformations it was taken into account that a standing wave has been established in the resonator, i. e. the following conditions are valid:

$$
\begin{aligned}
&\dot{X}_s\sin(\omega t) + \dot{X}_c\cos(\omega t) = 0;\\
&\dot{Y}_s\sin(\omega t) + \dot{Y}_c\cos(\omega t) = 0.
\end{aligned}
\tag{4}
$$

and, also, for the period of the “fast” oscillations the “slow” variables are not changed.

In equations (3) F_{xc}, F_{xs}, F_{ys}, F_{yc} are the cosine and sine components of forces f_x and f_y, respectively. Note that when working at the resonant frequency ω_r the condition $k_{11}-\omega_r^2 = k_{22}-\omega_r^2 = 0$ is true. In the absence of frequency mismatch $k_{12}=k_{21}=0$ and Q-factor mismatch $c_{12}=c_{21}=0$, a simple equation can be obtained that after the application of the Laplace transform determines the linear transfer function of the resonator. From the first equation of system (3) follows that:

$$
\dot{X}_s = -\frac{c_{11}}{2}X_s + \frac{F_{xc}}{2\omega_r}; \quad \rightarrow \frac{\Im\{X_s\}}{\Im\{F_{xc}\}} = \frac{\frac{1}{2\omega_r}}{s+\frac{c_{11}}{2}} = \frac{Q}{\omega_r^2}\frac{P_0}{\frac{2Q}{\omega_r}s+1}.
\tag{5}
$$

Here $\Im\{X_s\}$ and $\Im\{F_{xc}\}$ are Laplace transforms of the corresponding functions. In addition, the following relationships $c_{11}=2/\tau$ and $\tau = 2Q/\omega_r$ where Q is resonator Q-factor, τ is resonator time constant, have been taken into account, and was additional parameter P_0 introduced, which characterizes the conversion coefficient of the transducer that converts mechanical oscillation into electrical signal.

Control of the secondary wave. Now the control system of the secondary wave can be represented as equivalent linear system. As a controller, which generates control signal and provides it to the resonator, PIDF – controller (proportional – integral – differential with low-pass filter) is chosen. Block diagram of the equivalent control system is shown in fig. 1.

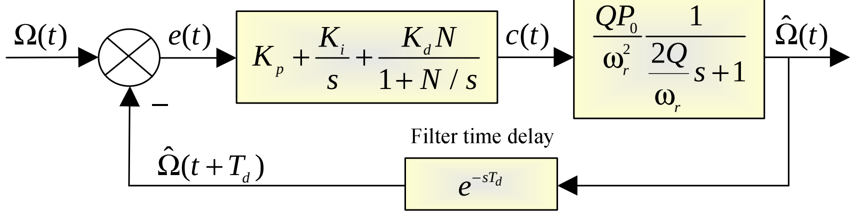

Fig. 1. Coriolis vibratory gyroscope angular rate control system block diagram

In this block diagram $\hat{\Omega}(t)$ is the measured angular rate and $\Omega(t)$ is the input angular rate, $e(t)$ is the measurement error. Time delay T_d, which exists in the system due to the presence of the demodulator filter was introduced in the block diagram. K_p, K_i, K_d and N denote the gains of proportional, integral, differential components of the PIDF – controller and its lowpass filter pole, respectively.

Controller adjusmet to the bandwidth 100 Hz gives the following values for the gains: K_p = 3248, K_i = 2586, K_d = 0.88, N = 28626, with $T_d = 10^{-4}$ *s*. Bode diagram of the system for various parameters that can realized in the plant (CVG resonator) in the temperature range [-40 +85]°C is represented in fig 2. As can be seen phase and gain stability margins of the rate channel give reason to state that the system is stable, and the bandwidth does not change significantly [103 96] Hz. Figure 3 shows the unit step response of the control channel with various parameters of the plant. Time difference in settling time is approximately equal to 1 ms.

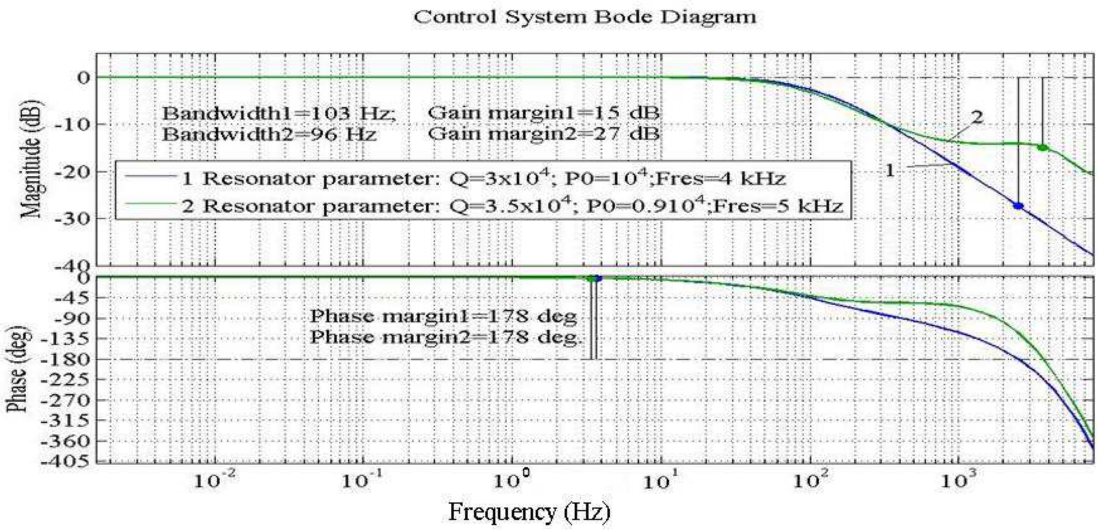

Fig. 2. Coriolis vibratory gyroscope angular rate feedback control channel with bandwidth 100 Hz Bode diagram

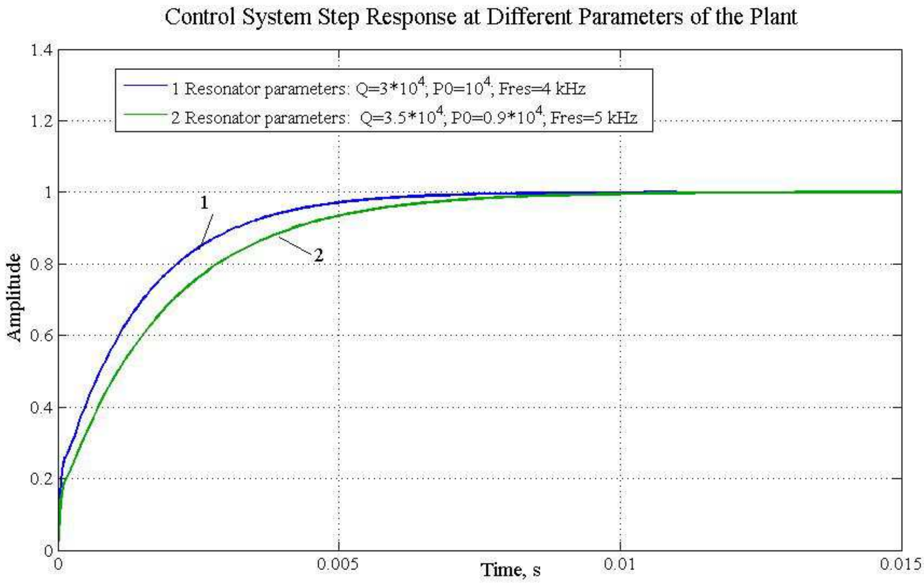

Fig. 3. Coriolis vibratory gyroscope rate channel unit step response in the temperature range [-40 +85] °C

Figure 4 shows Bode diagram of the CVG rate feedback control channel with bandwidth 200Hz. The controller parameters are the following: K_p = 5000; K_i = 33451; K_d = –3; N = 1157. In this case gain and phase margins are 33 and 23 dB, and 178 and 122 degrees, respectively.

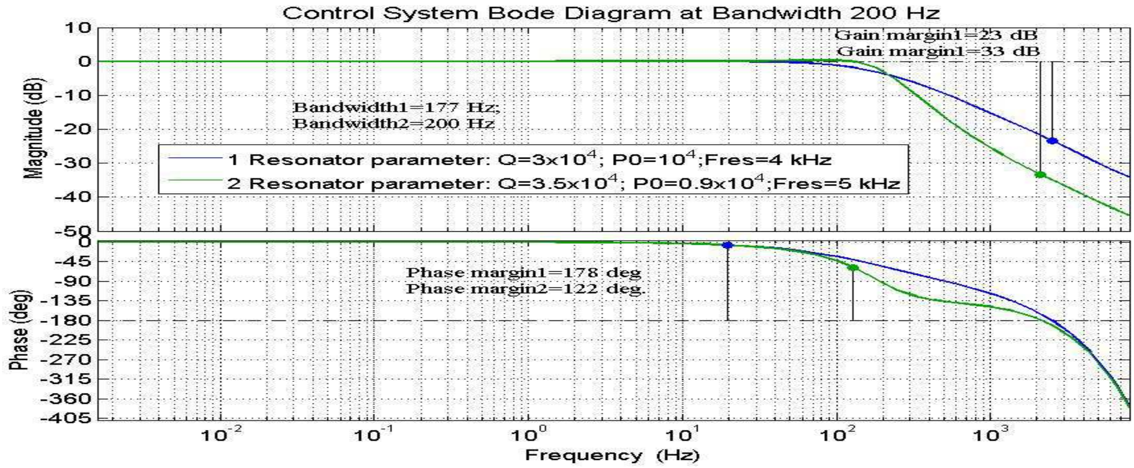

Fig. 4. Coriolis vibratory gyroscope angular rate feedback control channel with bandwidth 200 Hz Bode diagram

Conclusion. Thus, the possibility of tuning the rate control channel of CVG resonator secondary wave with bandwidth of 100 Hz and 200 Hz with a stable performance over a wide temperature range of [-40 +85]^{o}C was shown.

References

1. *Brooks L. G.* "Amplitude and Frequency Demodulation Controller for MEMS Accelerometer". Dissertation Bachelor of Science and Master of Engineering in Computer Science and Engineering, MIT, February 2000. 122 p.
2. *Loveday Ph. W.* "Analysis and compensation of imperfection effects in piezoelectric vibratory gyroscopes". Dissertation, Doctor of Philosophy, Virginia Polytechnic Institute, Blacksburg, Virginia, February 1999. 147 p.

В. В. Чіковані

Система керування вторинною хвилею резонатора коріолісового вібраційного гіроскопа.

На основі лінеаризації передавальної функції резонатора вібраційного гіроскопа, як об'єкта управління, досліджено питання стійкості системи при зміні, наприклад від температури, параметрів об'єкта управління у разі, коли регулятор каналу компенсації кутової швидкості налаштований на смугу пропускання 100 Гц і 200 Гц.

В. В. Чиковани

Система управления вторичной волной резонатора кориолисового вибрационного гироскопа.

На основе линеаризации передаточной функции резонатора вибрационного гироскопа, как объекта управления, исследованы вопросы устойчивости системы при изменении, например от температуры, параметров объекта управления в случае, когда регулятор канала компенсации угловой скорости настроен на полосу пропускания 100 Гц и 200 Гц.