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APPROXIMATION OF SOLUTIONS OF FRACTIONAL DIFFERENTIAL EQUATIONS OF VARIABLE ORDER WITH USING THE S-TRANSFORM

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Abstrakt. *Operational analogues of integral-differential operators of variable fractional order in the S-transform are proposed. The examples of application of operational matrices of integration with variable fractional order to integrate different signals and solution of fractional differential equations with variable order have been presented. Computational experiments are performed in the software environment of Mathematica®.*

Keywords: mathematical modeling, operational calculus, *S*-transform, approximation and signal processing, dynamic system, fractional calculus, Caputo's fractional derivative, Riemann-Liouville's fractional derivative, fractional integral of Riemann–Liouville.

Introduction. It is known that the mathematical models of complex dynamical systems in fractal media are integral-differential equations with derivatives and integrals of fractional order [2 – 4]. The development of fractional calculus and its applications to the solution of problems of mathematical physics have led to the development of multi – physical approach, in which the functions characterizing the intensity of one physical field affect the parameters of other physical fields in the given region of space. Therefore the order of integro-differential operator, as one of the parameters, may also be varied as a function of time, spatial variables and functions of the intensity of fields of different physical nature. An example is a dynamic system, which describes the process of anomalous diffusion in porous media, taking into account the changing temperature or aging processes. A similar approach naturally generates mathematical models of the dynamics of the fractional systems in the form of integral-differential equations with fractional orders, varying according to certain laws [5 – 9]. Methods for solving differential equations with derivatives and integrals of variable fractional order are currently being intensively developed.

This paper deals with the formation of operational matrices of integration of variable fractional order in the approximation-operational method of *S*-transform [1]. The paper is organized as follows. The second section discusses the operations of fractional differentiation by Caputo and Riemann–Liouville and fractional integration of Riemann–Liouville [2; 4], whose orders are given as functions of time determined by the time interval of the process of development, and the expressions for the operational matrix of integration and differentiation with variable fractional order.

In the third section examples are provided of such matrices for the S-transform based on local Legendre polynomials and their applications to integrate different functions and solutions of differential equations of fractional order. Computational experiments have been performed in the software environment of Mathematica® [10]. The conclusion includes the analysis of the results and recommendations for their use.

***S*-transform and operational analogues of integral-differential operators of variable fractional order.** *S*-transform [1], is based on polynomial approximation of the signals, basic expressions of approximation make up the operational calculus of a special type:

$$\mathbf{X} = \left(\int_0^T \mathbf{S}(t)\mathbf{S}(t)^* \, dt \right)^{-1} \cdot \left(\int_0^T \mathbf{S}(t) x(t) \, dt \right), \tag{1}$$

$$x_a(t) = \mathbf{X}^* \mathbf{S}(t). \qquad (2)$$

Direct *S*-transform (1) generates an operational image of a signal $x(t)$ as a vector approximating polynomial coefficients $\mathbf{X}$, whereas the inverse *S*-transform (2) restores the signal in the form of approximation $x_a(t)$. Signal and the system of basis functions $\mathbf{S}(t)$ are defined on the same interval of the argument $t \in [0, T)$. In applying the *S*-transform to solving systems of fractional dynamics, mathematical models which are the integral-differential equations of fractional order, the task of system dynamics is reduced to the solution of algebraic equations in the operational space, and the transition to the original space is made by constructing a polynomial of the form (2).

The most important relations for the *S*-transform as an operational method are operating analogues of mathematical operations of integration and differentiation of fractional order[2 – 4]. The most frequently used operations include the integration of fractional order of Riemann–Liouville (3) and differentiation of fractional order of Riemann–Liouville and Caputo (4), (5):

$$y(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} x(\tau) d\tau, \qquad (3)$$

$$y(t) = \frac{d^n}{dt^n}\left(\frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} x(\tau) d\tau \right), \quad n-1 < \alpha < n, \qquad (4)$$

$$y(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n x(\tau)}{d\tau^n} d\tau, \quad n-1 < \alpha < n. \qquad (5)$$

When building operational analogs of fractional differentiation and integration, which are known to be the matrices whose elements depend only on the system of basis functions, on the argument interval and order of the operator, it should be noted that the columns of the matrices are the vectors of coefficients of the approximating polynomials for integrals and (or) derivatives of the functions of the base system. This allows us to create the following expression for the operational matrix of integration and differentiation:

– operational matrix of integration of β-order in Riemann–Liouville:

$$\mathbf{P}_s^\beta = \mathbf{W}^{-1} \cdot \left(\int_0^T \left(\frac{1}{\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} \cdot \mathbf{S}(\tau) d\tau \right) \cdot \mathbf{S}(t)^* dt \right), \qquad (6)$$

– operational matrix of differentiation of α-order in Caputo:

$${}^C\mathbf{D}_s^\alpha = \mathbf{P}_s^{n-\alpha} \mathbf{D}_s^n, \qquad (7)$$

– operational matrix of differentiation of α-order in Riemann–Liouville:

$${}^{RL}\mathbf{D}_s^\alpha = \mathbf{D}_s^n \mathbf{P}_s^{n-\alpha}. \qquad (8)$$

In (6) – (8) the following designations are introduced:

$$\mathbf{W} = \int_0^T \mathbf{S}(t)\mathbf{S}(t)^* dt \text{ – operational matrix approximation,} \qquad (9)$$

$$\mathbf{D}_s^n = \mathbf{W}^{-1} \left(\int_0^T \frac{d^n \mathbf{S}(t)}{dt^n} \mathbf{S}(t)^* dt \right) \text{– operational matrix of the } n\text{-}th \text{ order of differentiation.} \qquad (10)$$

Differentiation and integration of matrix-vector operands are performed element by element, * – the symbol of transposition of vector quantities. It is also assumed that the basic functions of the system allow the differentiation up to n-*th* order.

A distinctive feature of the S-transform is that the expression of operational matrices of differentiation and integration of fractional order given above can be generalized to the case of fractional order of integro-differential operators which are the functions of time or the other argument, which depends on the time. Therefore we can write the following expressions for the operational analogues of the corresponding operators with variable order:

$$\mathbf{P}_s^{\beta(t)} = \mathbf{W}^{-1}\left(\int_0^T\left(\frac{1}{\Gamma(\beta(t))}\int_0^t (t-\tau)^{\beta(t)-1}\mathbf{S}(\tau)d\tau\right)\mathbf{S}(t)^* dt\right), \tag{11}$$

$${}^{C}\mathbf{D}_s^{\alpha(t)} = \mathbf{P}_s^{n-\alpha(t)}\mathbf{D}_s^n, \tag{12}$$

$${}^{RL}\mathbf{D}_s^{\alpha(t)} = \mathbf{D}_s^n\mathbf{P}_s^{n-\alpha(t)}. \tag{13}$$

Examples of implementation and application of operational matrices of variable fractional order integration.

Example 1. Generate the operational matrices of integration by Riemann–Liouville with variable fractional order for the S-transform on the basis of the shifted Legendre polynomials of order zero (block-pulse of system functions) for the following parameters of S-transform:

$T = mh;\ m = 100;\ h = 0{,}01;\ \beta 1 = 2 - t/T;\ \beta 2 = 2 - \sin(\pi t/T);\ \beta 3 = 2e^{-t/T};\ \beta 4 = 1 + \sin(\pi t/T);$

Program 1.

– Set numerical values of the parameters:

```
m := 100; h := 0.01; T = m*h; β1 = 2 - (i - 0.5) / m;
β2 = 2 - Sin[π*(i - 0.5) / m];
β3 = 2 e^(-(i-0.5)/m); β4 = 1 + Sin[π*(i - 0.5) / m];
```

– Forming the operating matrix of integration:

```
H[β_, h_, m_] :=
  Table[h^β / Gamma[β + 2] * Which[i < j, 0, i == j, 1, i > j,
    (i - j + 1)^(β+1) - 2 (i - j)^(β+1) + (i - j - 1)^(β+1)], {i, m}, {j, m}];
```

– Definition of operational matrices of fractional integration with the various laws changing the order of the operator:

```
Po1 = N[H[β1, h, m]]; Po2 = N[H[β2, h, m]]; Po3 = N[H[β3, h, m]];  Po4 = N[H[β4, h, m]];
```

Example 2. Determine an approximation of the Riemann–Liouville variable order ($\alpha 4 = \beta 4$) integral for the signal $x2(t) = \sin(2\pi t)$,(Using S-parameters mentioned in Example 1).

Program 2.

– Definition of the basic system of functions S-transform:

```
S = Table[If[(i - 1)*h ≤ t < i*h, 1, 0], {i, m}];
```

– Setting the signal image:

```
F = Table[Sin[2 π*(i - 0.5)*h], {i, m}];
```

– Finding the image of the integral and its approximation:

```
Y2 = Po4.F; ya2 = Y2.S;
```

– Finding the exact value of the Riemann–Liouville integral (reference for comparison with the approximation):

```
y2 = ∫_0^t (t - τ)^(α4-1) * Sin[2 π*τ] / Gamma[α4] dτ
```

– *The result of integration:*

```
yo2 = (2 π t^(2+Sin[π t]) HypergeometricPFQ[{1}, {3/2 + 1/2 Sin[π t], 2 + 1/2 Sin[π t]}, -π^2 t^2]) /
   (Gamma[1 + Sin[π t]] (2 + 3 Sin[π t] + Sin[π t]^2))
```

– *Visualization of integral and its approximation (fig. 1):*

```
Plot[{yo2, ya2}, {t, 0, 1}, PlotPoints → 250]
```

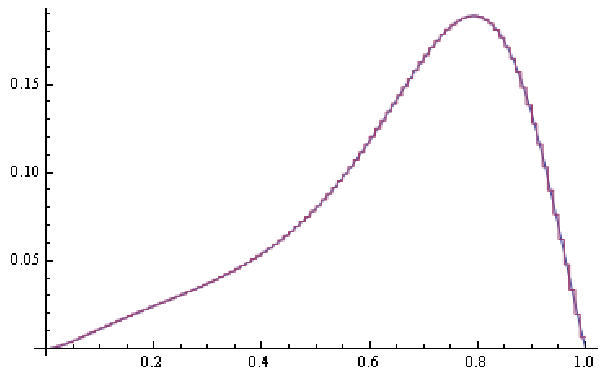

Fig. 1. The integral of signal: $x2(t) = \sin(2\pi t)$ of order: $\beta 4 = 1 + \sin(\pi t / T)$; and its approximation

– *Visualization of the error function of approximation (fig. 2):*

```
Plot[yo2 - ya2, {t, 0, 1}, PlotPoints → 250]
```

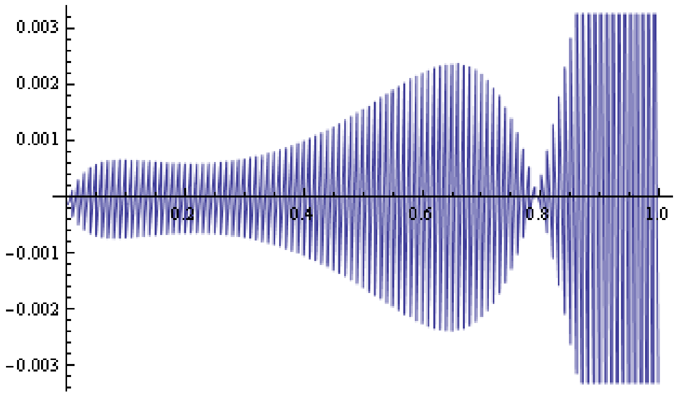

Fig. 2. The error function of approximation of an integral with a variable order of integration

Example 3. Determine an approximation of integral of Riemann–Liouville of variable order for the signal (Use *S*-transform's parameters specified in Examples 1 and 2). The code fragments are presented without comments (fig. 3, 4).

Program 3.

```
α3 = 2 e^-t;
y3 = ∫_0^t ((t - τ)^(α3-1) * Sin[2 π * τ]) / Gamma[α3] dτ
yo3 = (e^(2 t) π t^(1+2 e^-t) HypergeometricPFQ[{1}, {1 + e^-t, 3/2 + e^-t}, -π^2 t^2]) / ((2 + e^t) Gamma[2 e^-t])
Y3 = Po3.F;
y3a = Y3.S;
Plot[{yo3, y3a}, {t, 0, 1}, PlotPoints → 250]
Plot[yo3 - y3a, {t, 0, 1}, PlotPoints → 250]
```

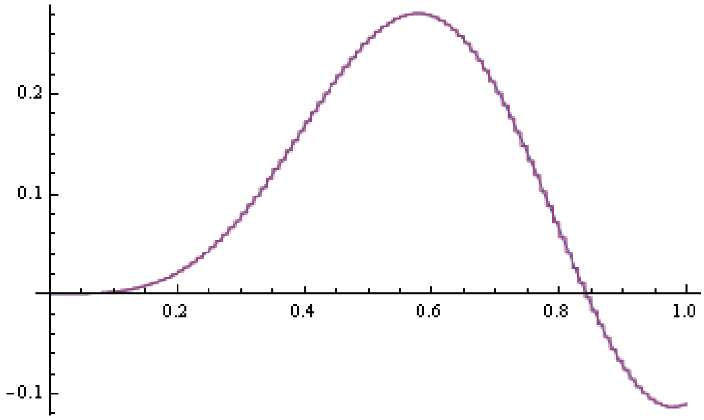

Fig. 3. The integral of the signal: $x2(t)=\sin(2\pi t)$ of order $\beta 3 = 2e^{-t/T}$; and its approximation

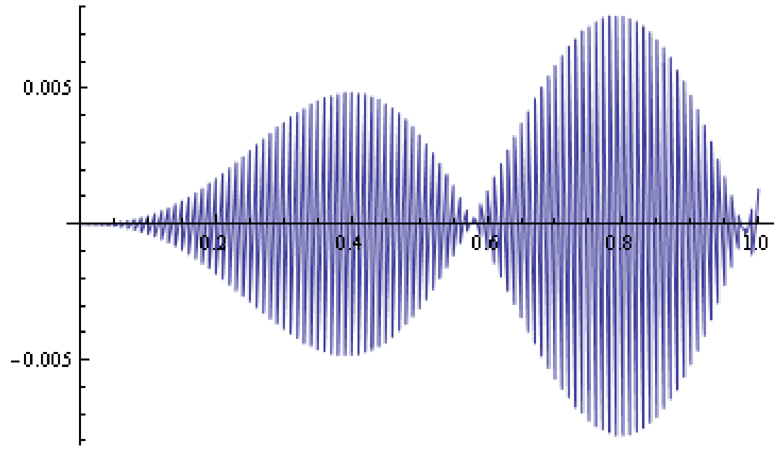

Fig. 4. The error function of approximation of an integral with a variable order of integration

Example 4. Find the approximate solution of fractional order differential equation with Caputo derivative: ${}_0^C D_t^{0,5}\left(x(t)\right)+2x(t)=\sin(2\pi t),\ x(0)=xo=5$. It is necessary to get an approximation of solution of the equation in the basis of Legendre polynomials of 10th order, using the method of *S*-transform based on block-pulse functions (the local version of the Legendre polynomials of order zero, $m = 100$), the interval of the argument $T = 2$.

Let's transform the given equation to a form suitable the operational method for the application. We denote the first derivative of the desired solution as $\frac{dx(t)}{dt}=u(t)$. Then $x(t)=\int_0^t u(\tau)d\tau + xo$. Using the definition of a derivative of fractional order by Caputo and implemented function $u(t)$, we obtain the following integral equation of fractional order, equivalent to a given differential equation: ${}_0J_t^{0,5}\left(u(t)\right)+2\int_0^t u(\tau)d\tau = f(t)-2xo$. Here ${}_0J_t^{0.5}$ – fractional integral operator by Riemann–Liouville. *S*-transform of this equation is: $\mathbf{P}^{0,5}\cdot\mathbf{U}+2\mathbf{P}^1\cdot\mathbf{U}=\left(\mathbf{F}-2xo\cdot\mathbf{1}\right)$. The solution in operational space is defined by following expressions: $\mathbf{U}=\left(\mathbf{P}^{0,5}+2\mathbf{P}^1\right)^{-1}\cdot\left(\mathbf{F}-2xo\cdot\mathbf{1}\right)$, $\mathbf{X}=\mathbf{P}^1\cdot\mathbf{U}+xo\cdot\mathbf{1}$. Transition to original space is performed in accordance with the formulas: $x_a(t)=\mathbf{X}^*\cdot\mathbf{V}$, $\frac{dx_a(t)}{dt}=\mathbf{U}^*\cdot\mathbf{V}$, where $\mathbf{V}$ – vector of basis functions of *S*-transform.

Program 4.

– Setting of S-transform's parameters and the input data:

```
h = 1/50; m = 100; n = 10; T = 2; β1 = 0.5; xo = 5; f[t_] := Sin[2 π * t];
```

– Forming of basic systems based on local and global versions of the shifted Legendre polynomials:

```
s[t_, h_, i_, j_] := If[(i - 1) * h ≤ t < i * h, LegendreP[j - 1, 1 - 2 i + 2 t / h], 0];
V = Table[s[t, h, i, 1], {i, m}];

S = Table[LegendreP[j - 1, -1 + 2 t / T], {j, n}];
```

– Forming of operational matrices of integration:

```
Pβ = Table[p[β1, i - j], {i, 0, m - 1}, {j, 0, m - 1}];
P1 = Table[p[1, i - j], {i, 0, m - 1}, {j, 0, m - 1}];
```

– Imaging the right part of the equation and the constant 1:

```
F = Table[f[(i - 0.5) * h], {i, m}]; One = Table[1, {i, m}];
```

– Forming the approximate matrix for signals defined in the table form:

```
tt = Table[(j - 0.5) * h, {j, m}];
w = Table[S[[i]] /. t → tt[[j]], {j, m}, {i, n}];
```

– Problem solution in operational space:

```
U = Inverse[Pβ + 2 P1].(F - 2 xo * One)
X = P1.U + xo * One
XL = PseudoInverse[w].X
```

– Transition into original space:

```
xa1 = X.V;
xa2 = XL.S;
```

– Visualization of solutions (fig. 5 – 8):

```
fig1 = Plot[xa1, {t, 0, m * h}, PlotPoints → 400]
fig2 = Plot[xa2, {t, 0, T}]
fig3 = ListPlot[Table[{tt[[i]], X[[i]]}, {i, m}]]
fig5 = Show[fig2, fig3]
```

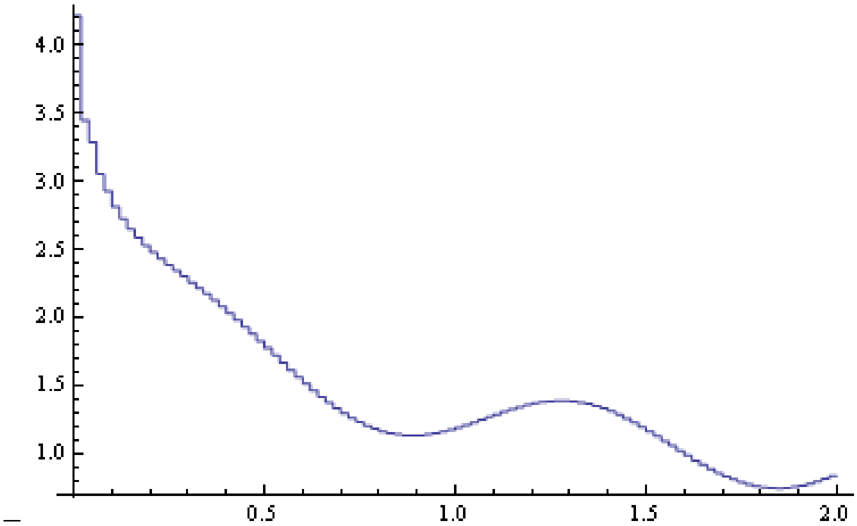

Fig. 5. Approximation of solution of the equation in the basis of the local version of the Legendre polynomials, m=100

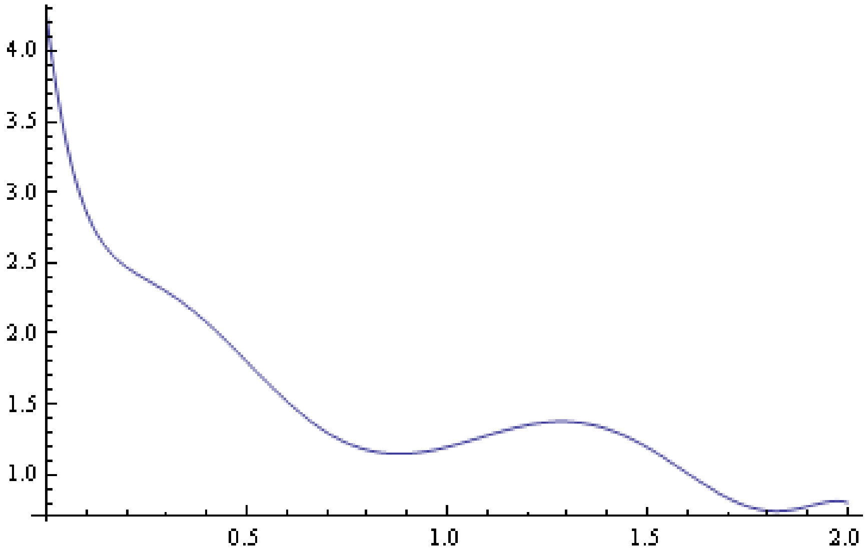

Fig. 6. Approximation of solution of the equation in the basis of the global version of the Legendre polynomials, n=10

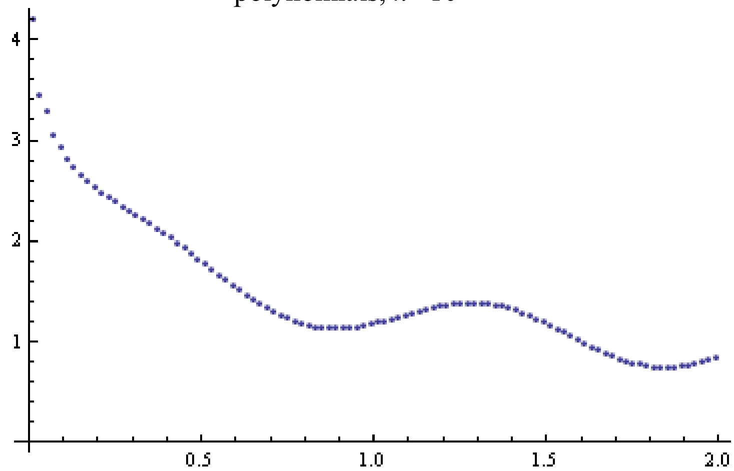

Fig. 7. Solution in the operational space (m=100)

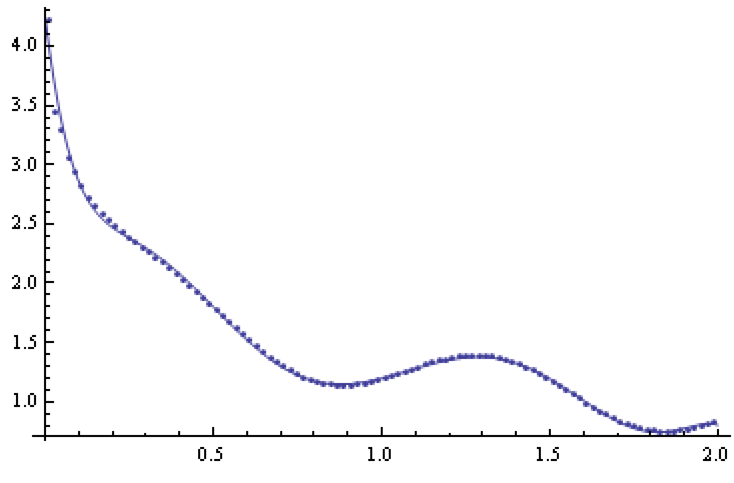

Fig. 8. Combined graph of the solutions in operational space and approximate solution in global version of the Legendre polynomials (n=10)

Example 5. Let us use the following differential equation involving derivatives of fractional variable order by Caputo: ${}_0^C D_t^{\alpha_1} y(t) + a \cdot {}_0^C D_t^{\alpha_2} y(t) + by(t) = f(t)$. Orders of the differential operators are limited to the following limits: $1 < \alpha_1 < 2,\ 0 < \alpha_2 < 1,$ the range of variation of the argument t

and the initial conditions are defined by expressions: $0<t<T$, $y(0)=y_0$, $y'(0)=y_{10}$. It is necessary to find the approximate solution of differential equation for the following orders of differential operators: $\alpha_1 = 1.5$, $\alpha_2(t)=e^{-t/T}$.

Let us use *S*-transform after elementary transformations of the equation.

$$y''(t)=u(t),\ y'(t)=\int_0^t u(\tau)d\tau + y_{10},\ \ y'(t)=\int_0^t u(\tau)d\tau + y_{10},\ \ y(t)=\int_0^t\int_0^\tau u(\tau_1)d\tau_1 d\tau + y_{10}t + y_0 \ ,$$

$$J^{2-\alpha_1}u(t) + aJ^{1-\alpha_2}u(t) + b\int_0^t\int_0^\tau u(\tau_1)d\tau_1 d\tau = f(t) - b(y_0 + y_{10}t) - aJ^{1-\alpha_2}y_{10}.$$

Turning to the operational space, we get

$$(\mathbf{P}^{2-\alpha_1} + a\cdot\mathbf{P}^{1-\alpha_2}\cdot\mathbf{P}^1 + b\cdot\mathbf{P}^2)\cdot\mathbf{U} = \mathbf{F}+\mathbf{\Phi},\ \mathbf{\Phi} = -(by_0 + ay_{10}J^{1-\alpha_2})\cdot\mathbf{1} - by_{10}\cdot\mathbf{t},$$

$$\mathbf{U} = (\mathbf{P}^{2-\alpha_1} + a\cdot\mathbf{P}^{1-\alpha_2}\cdot\mathbf{P}^1 + b\cdot\mathbf{P}^2)^{-1}\cdot(\mathbf{F}+\mathbf{\Phi}),\ \mathbf{Y}_1 = \mathbf{P}^1\cdot\mathbf{U} + y_{10}\cdot\mathbf{1},\ \mathbf{Y} = \mathbf{P}^2\cdot\mathbf{U} + y_0\cdot\mathbf{1} + y_{10}\cdot\mathbf{t}.$$

The transition to the original space is made by the usual formula for the inverse *S*-transform: $y_a(t)=\mathbf{Y}^*\cdot\mathbf{S}(t)$, $y'_a(t)=\mathbf{Y}_1^*\cdot\mathbf{S}(t)$, $y''_a(t)=\mathbf{U}^*\cdot\mathbf{S}(t)$.

Program 5.

– Definition of the parameters of S-transform and the initial data of problem to be solved:

```
m := 200; h := 0.025; T = 5; n = 10; α1 = 1.5; α2 = 1 - e^(-(i-0.5)/m); y00 = 1.25;
y10 = -0.5;
a = 2; b = 1.1;
s[t_, h_, i_, j_] := If[(i - 1) * h ≤ t < i * h, LegendreP[j - 1, 1 - 2 i + 2 t / h], 0];
V = Table[s[t, h, i, 1], {i, m}];
S = Table[LegendreP[j - 1, -1 + 2 t / T], {j, n}];
tt = Table[(j - 0.5) * h, {j, m}];
w = Table[S[[i]] /. t → tt[[j]], {j, m}, {i, n}];
H[β_, h_, m_] :=
  Table[h^β / Gamma[β + 2] * Which[i < j, 0, i == j, 1, i > j,
     (i - j + 1)^(β+1) - 2 (i - j)^(β+1) + (i - j - 1)^(β+1)], {i, m}, {j, m}];
```

– Development of operational matrices of integration of necessary orders, images of constant 1, the argument t and external influences:

```
Pa2 = N[H[1 - α2, h, m]];  P1 = N[H[1, h, m]];
Pa2 // MatrixForm;          P1 // MatrixForm;
P2 = N[H[2, h, m]];
P2 // MatrixForm;
One = Table[1, {i, m}];  Tim = Table[(i - 0.5) / m, {i, m}];
F = Table[e^(-2 (i-0.5)/m), {i, m}];
Φ = -3 y00 * One - 2 y10 * Pa2.One - 1.1 y10 * Tim;
```

– The solution of equation in the operational space:

```
U = Chop[Inverse[Pa1 + 2 Pa2.P1 + 1.1 P2].(F + Φ)];
Y1 = P1.U + y10 * One;  Y = P2.U + y00 * One + y10 * Tim;
```

– Finding solutions of the equation in the operational space of global version of Legendre polynomials:

```
YL = PseudoInverse[w].Y
```

– Transition to the original space of solutions:

```
yav = Y.V; yaw = YL.S;
```

– *Visualization of solutions and the phase portrait in the operational area:*

```
p11 = ListPlot[Y, PlotRange → All]  p12 = ListPlot[Y1, PlotRange → All]
p4 = ListPlot[YL, Filling → Axis]
ListPlot[Table[{Y[[i]], Y1[[i]]}, {i, m}], PlotRange → All]
```

– *Visualization of the approximation of solutions in the space of originals:*

```
p16 = Plot[{yav, yaw}, {t, 0, h*m}, PlotRange → All, PlotPoints → 400]
```

Visualization of results is shown in fig. 9 – 12.

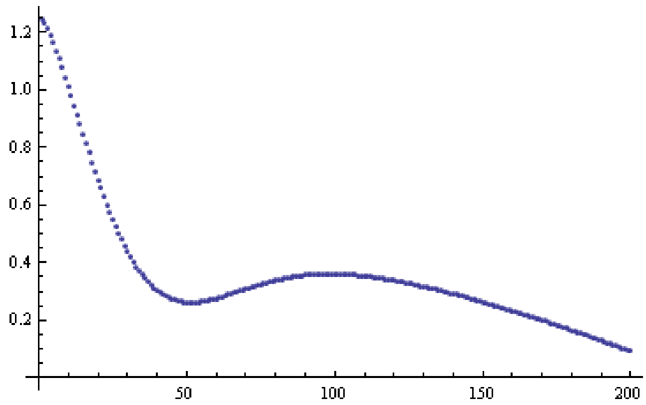

Fig. 9. Solution in operational space

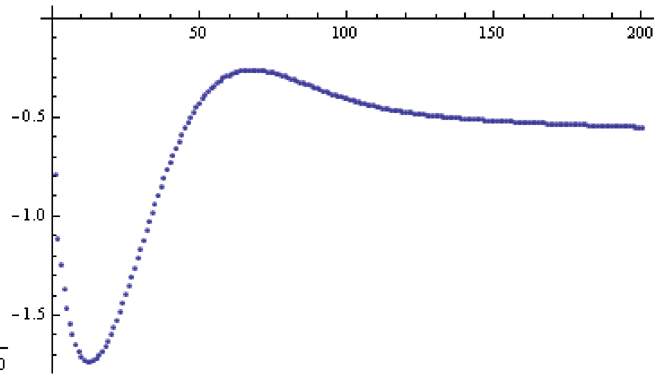

Fig. 10. The first derivative of solution in operational space

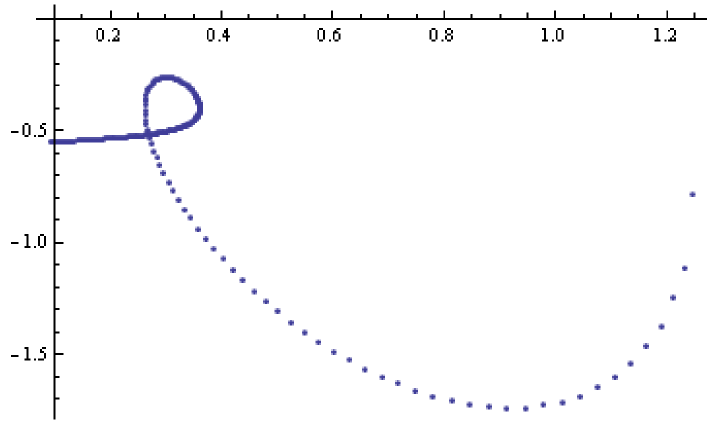

Fig. 11. The phase portrait of the solution of the equation in operational space

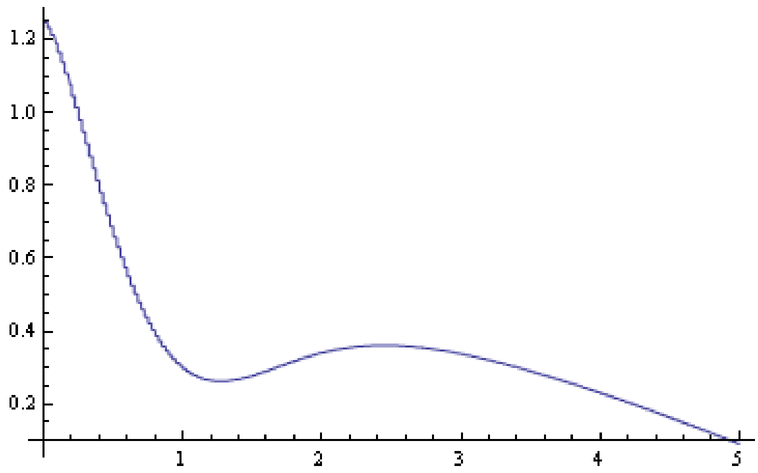

Fig. 12. Approximation of solution of the equation

Conclusions. Approach, proposed to the implementation of operational analogues of integral-differential operators of fractional order which is changed during the solution, extends the scope of the *S*-transform on fractional differential equations involving derivatives and integrals of non-integer variable orders. The experimental results and numerical experiments confirm the efficiency of the method. Programs listed in the paper can be adapted to changing the parameters of the operators, the coefficients of equations, initial conditions, as well as variations in the types and parameters of basic systems, and can be useful for researchers and engineers involved in modeling problems of fractal dynamics of systems, including also multiphysics problems.

References

1. *Vsevolod Vasylyev, Lilija Simak.* 2012. The polynomial approximation of signals as operational calculus: application to the modeling of dynamic systems of fractional order. Information Technology, No. 3 (187). P. 55–60. (in Russian).
2. *Uchaikin V. V.* 2008. The method of fractional derivatives. Ulyanovsk, Publishing House “Artichoke”. 512 p. (in Russian).
3. *Oldham K. B., Spanier J.* 1974. The Fractional Calculus. Academic Press. 234 p.
4. *Podlubny I.* 1999. Fractional Differential Equations. Academic Press. 340 p.
5. *Sheng Y., Chen Y. Q., Qiu T. S.* Fractional Processes and Fractional-Order Signal Processing: Techniques and Applications. Springer, 2012. 294 p.
6. *Sierociuk D., Podlubny I., Petras I.* Experimental Evidence of Variable-order Behavior of Ladders and Nested Ladders. arXiv:1107.2575v3 [math.DS] 17 July 2011. 8 p.
7. *Sun H. G.*, *Chen W., Chen Y. Q.* 2009. Variable – order fractional differential operators in anomalous diffusion modeling. Physica A, 388. P. 4586–4592.
8. *Sun H. G., Chen W., Sheng H., Chen Y. Q.* 2010. On mean square displacement behaviors of anomalous diffusion with variable and random orders. Physics Letters A, 374. P. 906–910.
9. *Sun H. G, Sheng H., Chen Y. Q.*, *Chen W.* On Dynamic-order Fractional Dynamic System. Proc. Of FDA’ 10, Article no. FDA10-073. 7 p.
10. *Wolfram Stephen.* 1996. The Mathematica book. Wolfram Media. Cambridge University Press. 1403 p.
11. *Vasyliev V. V., Simak L. O.* 2008. Fractional calculus and approximation methods for modeling of dynamic systems. Kyiv, NAN Ukraine. 256 p. (in Russian).

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Апроксимація рішень дробових диференціальних рівнянь змінного порядку з використанням *S*-перетворення

Запропоновано операційні аналоги інтегро-диференціальних операторів змінного дробового порядку в рамках *S*-перетворення. Розглянуто приклади застосування операційних матриць інтегрування зі змінним дробовим порядком до інтегрування різних сигналів і розв’язання диференціальних рівнянь дробового і змінного порядків. Обчислювальні експерименти виконані в програмному середовищі системи Mathematica®.

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Аппроксимация решений дробных дифференциальных уравнений переменного порядка с использованием *S*-преобразования

Предложены операционные аналоги интегро-дифференциальных операторов переменного дробного порядка в рамках *S*-преобразования. Рассмотрены примеры применения операционных матриц интегрирования с переменным дробным порядком к интегрированию различных сигналов и решению дифференциальных уравнений дробного и переменного порядков. Вычислительные эксперименты выполнены в программной среде системы Mathematica®.