MATHEMATICAL MODELING OF PROCESSES AND SYSTEMS

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FRACTAL SLICING ROUTINE FOR ELECTROMAGNETIC COMPATIBILITY PROBLEMS SOLUTION

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Abstract. Research is dedicated to the questions of electromagnetic compatibility of n-th amount of loop antennas which are set on the current-conductivebody of free-form (for example, airplane). Frequency-spatial division of regular antennas of air-flyingvehicle must lean on previous calculations, which it is expedient to conduct yet on the stage of planning of airplane which will allow to get a corresponding economic effect because of costliness and great labour amount of model tests method.

Keywords: electromagnetic compatibility; finite elements method; fractal models.

Introduction

Solution of 3D electromagnetic problems seems to be a hard nut to crack even today. With all these superfast computers that modern scientists, using numerical methods, own and use around the world – it is very often still impossible to build a good 3D finite elements model (FEM), that can be treated with appropriate engineering software, like programs dealing with electromagnetic compatibility research. So, a questions arises, how to get rid of this “modeling jam”, keeping the object’s 3D geometry for FEM processing and at the same time being able to overcome those huge (sometimes hundred thousands or even millions elements sized) matrices factorization crashes, when inverting them in order to find out a desired solution.

Related Works

We want to propose an approach based on well known phenomenon of nature having fractal inner structure [1]. So, if to change in a moment - this harsh for computing 3D electromagnetic model into easy operated 2D clone without losing credibility and satisfactory (at least) verification of the solution got for the task dealt with, all it will give a great push for FEM research science, especially for PC simulation research, compared to experimental investigation algorythms. How to do this? Well, the answer looks to be obvious: of course by slicing initial 3D EMM to a number of 2D “images” [2] that could be understood as some “fractals” [1] of initial more complex topology.

Fractal slicing routine

Problem statement. To find out optimal placing of a couple of loop antennas on aircraft’s fuselage.

Step 1. The procedures may differ taking into consideration the peculiarities of FEM software code and design used, and the peculiarities of the task being investigated. A bunch of such 2D fractals of the initial 3D model (I3DM) should be considered as a matrix of resulting 2D images (R2DI) (elements) like this:

$$\mathbb{Z}=\left\langle \begin{matrix} \wp_{xy,1} & \wp_{xy,\ldots} & \wp_{xy,\mathrm{M}} \\ \wp_{xz,1} & \wp_{xz,\ldots} & \wp_{xz,\mathrm{M}} \\ \wp_{yz,1} & \wp_{yz,\ldots} & \wp_{yz,\mathrm{M}} \end{matrix} \right\rangle,$$

$\wp_{ij,k}$ contain here information not only about 2D topology and geometry of k-th (of M) 2D image having been sliced along ij-plane, but *all* initial data needed for the problem being solved, like positioning and radiation characteristics of an radiative antenna, the boundary conditions, FEM grid with all the coordinates, etc. Here xy, xy, yz are the 3 main planes in the Cartesian coordinates system, along which the slicing routine should be performed; M – any positive integer number, defined by modelist and being guided by practical necessity and advisability of the modeling problem. Custom projections can be applied, too, if needed for a particular problem.

A researcher should treat all or some number of these R2DIs, which can be described as using some FEM technique operator O *(that can be both integral and differential, or integral-differential, containing partial or fractional derivatives) application for obtaining some solution matrix* $\mathbb{Q}$ *keeping results got – it is the solution image matrix (SIM) (PC calculated using a chosen FEM technique).* The whole simulation process should be described as this:

$$\mathbb{Z}\cdot O=\mathbb{Q}. \tag{1}$$

The analysis been done, now a researcher should implement a FSR (fractal slicing routine) step two.

Step 2. It *consists in synthesis of initial 3D problem solution by sifting out unnecessary or doubtful SIM elements replacing them at the same time with zeros.* It’s a fully intuitive, heuristic process, and may be the most complicated step, because the decisions taken about the positive or negative values of appropriate SIM elements (*seeds*) – ought to be leant only on the judgment of an experimentalist, and, unfortunately this “human factor” seems not to be possible to get unbound of.

Finally, when having accomplished the second FSR step, one must perform the third and the last step.

Step 3. *This step implies researcher’s return to I3DM but now with a strict logical decision made about current EMC problem solution derived from SIM seeds.* This decision is built on a comparison of SIM seeds to the results obtained with help of experimental method (EM) performed on natural object or (at least) natural object’s 3D model. Only those seeds have been accumulated into integral verdict formulation process, who well comply with EM results. The verdict is simply the rule:

$$\Re = \left\langle \begin{matrix} \mathbb{Q}_{xy,1} \wedge EM_{xz,1} & \dots & \mathbb{Q}_{xy,M} \wedge EM_{xy,M} \\ \dots & \dots & \dots \\ \mathbb{Q}_{yz,1} \wedge EM_{yz,1} & \dots & \mathbb{Q}_{yz,M} \wedge EM_{yz,M} \end{matrix} \right\rangle \cdot \left\langle \begin{matrix} 1 \\ \dots \\ 1 \end{matrix} \right\rangle, \tag{2}$$

where $\mathbb{Q}_{nm,M} \wedge EM_{nm,M}$ = 1 if they comply, and = -1 when they don’t. PC modeling is to be treated as satisfactory (there is a convergence of solution), if and only if $\Re_{ij} > 0$. With this FSR algorithm kept in mind, let’s proceed now to it’s practical realization for a EMC problem consisting in placing antennas on aircraft’s fuselage.

Electrodynamics

We’ve taken Comsol® modeling environment for evaluation of EMC of 2 antennas set on a fuselage of an aircarft. The differential equation in partial derivatives for description the physical processes for this model is:

$$\nabla \times (\mu_r^{-1} \nabla \times E) - (\varepsilon_r - j\sigma / \omega\varepsilon_0) k_0^2 E = 0,$$

where μ_r – relative magnetic permeability; E – intensity of electric field at the monitoring point, (*V/m*); ε_r – relative dielectric permittivity; σ – conductivity of a proxy element, (*Sm/m*); ω – circular frequency of oscillations, (Rad/s); $\varepsilon_0 = 10^{-9} / 36\pi$ – electric constant, (*F/m*); $k_0 = \dfrac{2\pi}{\lambda}$ – wave number; λ – wavelength of oscillations, (*m*). This ordinary differential equation in partial derivatives is derived from Maxwell’s equations. It is solved by Comsol® software with help of numerical methods.

Research

a) **100 MHz.** We place a loop antenna with current $I_0 = .1A$ on the keel of an *yz*-fractal slice of a hypothetic aircraft (we are not binding ourselves yet to some more practical model, which can be done comparatively easy later on). It’s nothing more than just a 2D fractal of initial 3D aircraft’s FEM model (not shown in the paper for no need). So, in our case the initial R2DI matrix will consist of only one element: *yz*-th plane’s slice, cut directly through the middle of the fuselage:

$$\mathbb{Z} = \left\langle \wp_{yz,1} \right\rangle,$$

SIM matrix, got by applying the differential operator O (1) to the problem described with $\mathbb{Z}$ as well will contain only one element: $\mathbb{Q} = \left\langle \mathbb{Q}_{yz,1} \right\rangle$. This is the simplest possible case. Let’s say the EM (for $EM_{yz,1}$) has reulted itself in the real placing of the pair of antennas on an aircraft, that has a *similar* (to the aircraft we treat) design, size, antennas working frequencies, radiated powers and directional characteristics. Then, by applying the rule $\Re_{yz} = \mathbb{Q}_{yz,1} \wedge EM_{yz,1}$ (2) we will sincerely discover the placing: if $\Re_{yz}$ =1, then a **positive decision** is being possibility or impossibility of a forecasted antennas made about antennas’ installation in fuselage’s spots desired; on the contrary, if $\Re_{yz}$ =0 (FSR and EM do not comply) – **no custom installation is advised** by the aircraft’s constructor. Having applied FSR to this problem (for the case of 100 MHz frequency of a loop radiative antenna placed on the aircraft’s keel), we’ve discovered the pattern shown below for *z*-th component of electric field, generated by keel’s radiating antenna (fig. 1) and currents distribution on 2D FEM model’s fuselage (fig. 2):

b) **120 MHz** (same configuration and same current in radiating keel’s antenna). Having used FSR to this problem, we’ve found the following results for *z*-th component of electric field, generated by keel’s radiating antenna (fig. 3) and currents distribution on fuselage (fig. 4):

c) **140 MHz** (same configuration and same current in radiating keel’s antenna). After having treated this problem with FSR method, we’ve got the predictable and understandable results for *z*-th component of electric field, generated by keel’s radiating antenna (fig. 5) and currents distribution on fuselage (fig. 6):

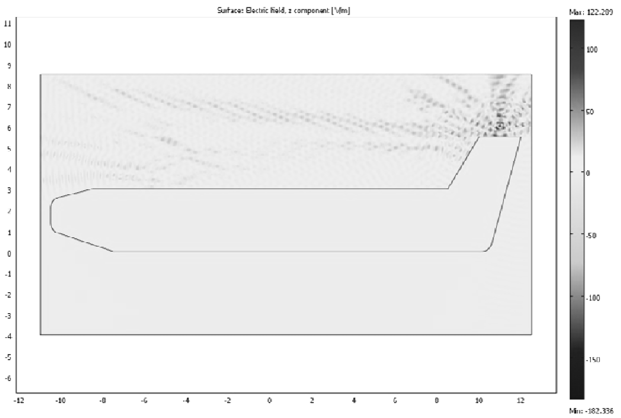

Fig. 1. Loop antenna on keel, 100 MHz

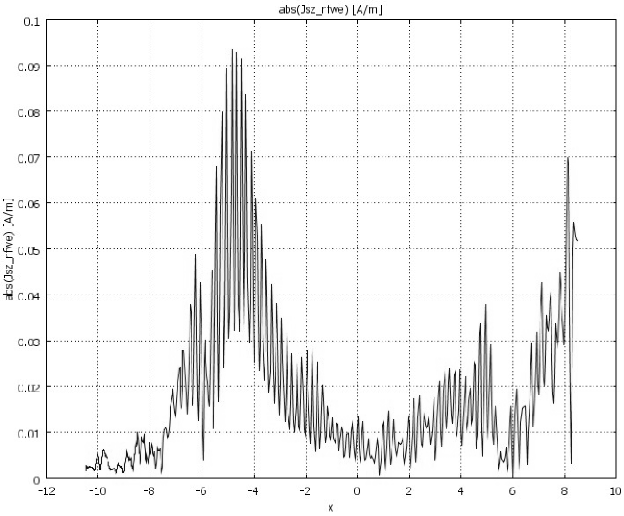

Fig. 2. Currents on fuselage, 100 MHz

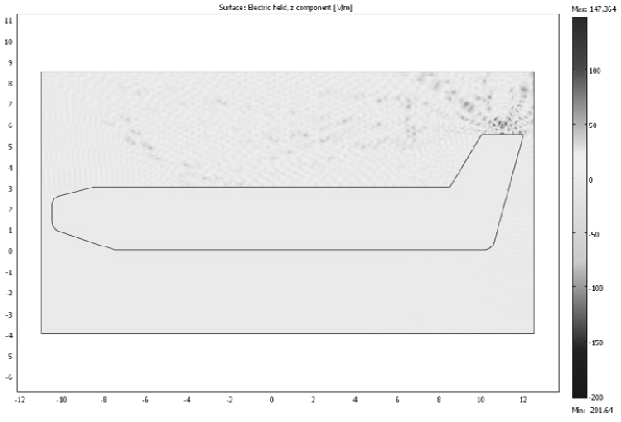

Fig. 3. Loop antenna on keel, 120 MHz

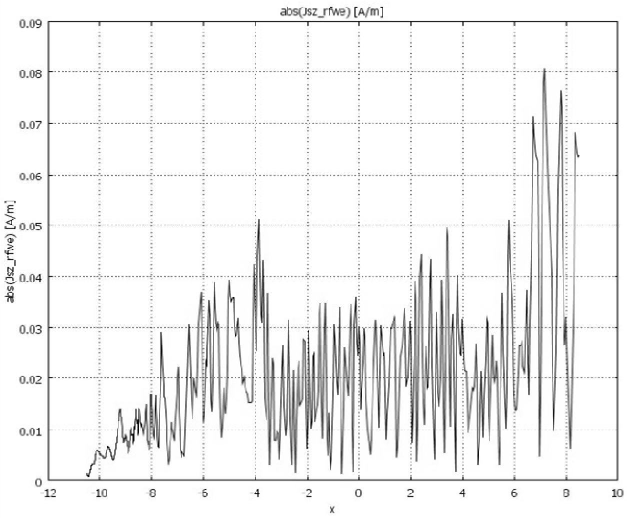

Fig. 4. Currents on fuselage, 120 MHz

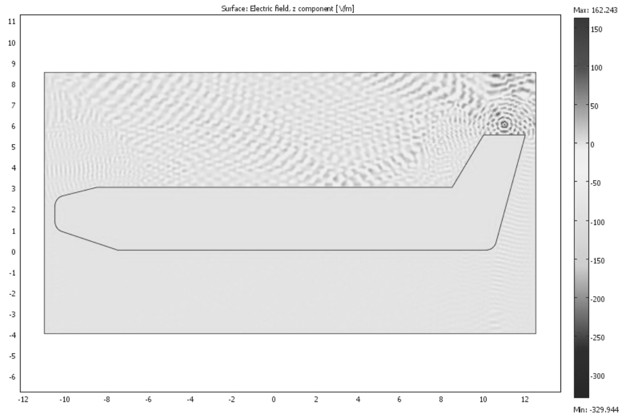

Fig. 5. Loop antenna on keel, 140 MHz

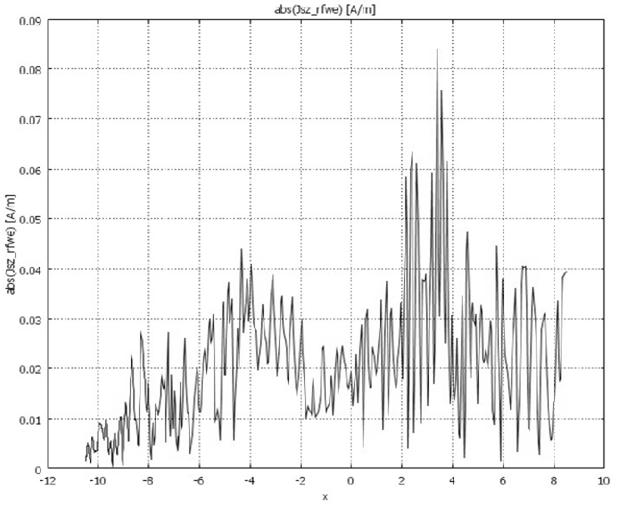

Fig. 6. Currents on fuselage, 140 MHz

Conclusions

From figs 1 – 6 it can be easily seen, that due to FSR method realization implemented in the paper, for 100 MHz it’s *undesirabe* to put a second antenna in any places between -5,5 m and -4 m; for 120 MHz *bad places* for installing it are between 6,5 m and 8 m; for 140 MHz it’s *unwise* to mount any antennas between 2m and 4m. This is so because the fuselage currents induced from keel antenna radiation are *the highest* in these regions. So, they will produce the highest electromagnetic fields around the fuselage and this will result in worsening of EMC situation in the Fresnel zone (close to the airplane's fuselage), if any allocation of a second antenna takes place. FSR method helps to overcome difficulties met when trying to deal with realistic-like 3D FEM models of aircrafts (possible number of FEM elements for such models can exceed hundreds of thousands and even millions, which means huge matrices and their very complex, PC-resource-consuming factorization needed for numerical methods application), helicopters, other multiple-antenna carrying vehicles. It establishes the rules of constructing equivalent 2D models of such systems, and, after doing so, gives the algorithms of making the final verdict about the antennas installations spots on the airplane's fuselage. This routine can save a lot of money on the stage of aircraft's construction.

References

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О. В. Вишнівський. Процедура фрактальної нарізки для розв'язання задач електромагнітної сумісності

Дослідження присвячено питанням електромагнітної сумісності *n*-ї кількості петлевих антен, які встановлюються на струмопровідне тіло довільної форми (наприклад, літак). Частотно-просторовий розділення штатних антен повітролітального апарату повинне спиратися на попередні розрахунки, які доцільно проводити ще на етапі проектування літака, що дозволить отримати відповідний економічний ефект зважаючи на дорожнечу і велику трудомісткість методу натурних випробувань.

Ключові слова: електромагнітна сумісність; метод кінцевих елементів; фрактальне моделювання.

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А. В. Вишневский. Процедура фрактальной нарезки для решения задач электромагнитной совместимоти

Исследование посвящено вопросам электромагнитной совместимости *n*-го количества петлевых антенн, которые устанавливаются на токопроводящее тело произвольной формы (например, самолёт). Частотно-пространственное разделение штатных антенн летательного аппарата должно опираться на предварительные расчёты, которые целесообразно проводить ещё на этапе проектирования самолёта, что позволит получить соответствующий экономический эффект ввиду дороговизны и большой трудоемкости метода натурных испытаний.

Ключевые слова: электромагнитная совместимост; метод конечных элементов; фрактальное моделирование

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