

**THEORY AND METHODS OF SIGNAL PROCESSING**

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**M. P. Mukhina****VISUAL SLAM ALGORITHM USING CORRELATION EXTREMAL PRINCIPLE**

Aviation Computer-Integrated Complexes Department, National Aviation University, Kiev, Ukraine

E-mail: [m\\_mukhina@inbox.ru](mailto:m_mukhina@inbox.ru)

**Abstract.** *Visual information features for simultaneous localization and mapping algorithm are considered. The problem of navigation for continuous observation of the ground surface is formulated. The algorithm of correlation search and coordinates estimation is proposed based on Kalman filtering.*

**Keywords:** simultaneous localization and mapping; statistical correlation; Kalman filter.

**Introduction**

Simultaneous localization and mapping (SLAM) is based on the processing of navigation information in the form of random functions. The correlation between the realization of random functions (geophysical fields) is used to find the coordinates of the object as the extremum of correlation function or of any other statistical estimate of random function realizations. The control of object motion is performed as the determination of its location by the comparison between the distribution of template and current geophysical fields. The current field is perceived by sensor of navigational information, e. g. radiolocation image of surface, laser range image, etc. The template field is measured beforehand and presented in the form of map with specified accuracy. It must be mentioned here that the template map may be absent and will be formed and adjusted simultaneously with motion start.

Since the distributions of current and mapped fields along the prescribed route are random processes, the degree of their matching can be determined by the quantity of correlation function [1]. The maximum of this function proves that the current realization of the field coincides with the definite region of the field map whose coordinates are known.

The structure of SLAM systems has definite features. The first is that the object captures and stores the features of environment (landmarks) and in this way decreases the uncertainty of the map. The greater the number of detected landmarks is, the less the uncertainty of the map will be. With the recurrent scanning of already detected landmarks the uncertainty will be further decreased.

If there are some unknown reference points, object puts them on the map with the high uncertainty. But uncertainty will decrease because of correlation between object and reference points when rescanning of all reference points is presented.

Another feature of SLAM problems is map difficulties. Considerable quantity of correlations can lead to increasing of calculating complication of

navigation problems. To save all possible correlation links calculation of  $O(n^3)$  and  $O(n^2)$  links should be done, where  $n$  is a number of functions.

So, for solving SLAM problems it is important to create an efficient map of geophysical field, that will content typical features of environment, that are necessary for object positioning.

Another SLAM problem is too important to check authenticity of the map, that leads to "loop close", i. e. returning of object to initial point. If map is compiled correctly, the object will be able to define initial point on the map and return to it with the minimal difference comparing with observing reference points.

**Related Works**

Reduction of calculating complication that appears during processing and map updating in the common case is possible by providing the map in the form of:

- set of the most informative areas;
- set of typical features (reference points);
- coded (brief) image.

Information of geophysical field in the common case can be given in the form of 2D-matrix with the values of the third coordinate, for example height.

The main point of proposed methods consisting in reduction of all available information about geophysical fields to common presentation in the form of 3D-matrix of image, and the methods of correlation searching of matches are developed for that [2; 3].

In case of optical (visual) field different variations of standard map are used in the form of:

- set of contours;
- distribution of field intensity (histogram);
- set of typical features (for example, coefficients of wavelet transform).

**Correlation extremal principle**

**Problem statement.** The working information is the image (frame) of the surface field. Let's denote

the observing field as  $f(x, y)$ , assuming that  $Oxy$  is a horizontal rectangular coordinate system.

Observation vector is given as a set of discrete measured values of the field  $z_i$  ( $i = 1, 2, \dots$ ) (fig. 1).

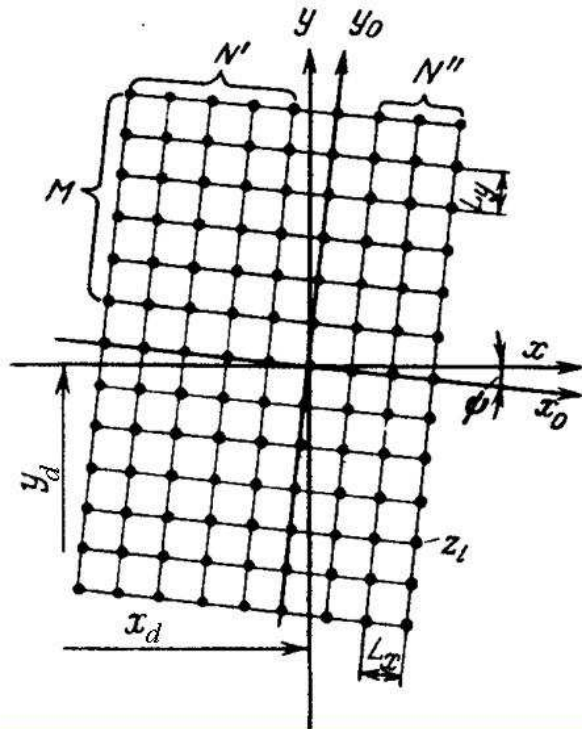


Fig. 1. Map of geophysical field

Let's suppose, that longitudinal axis  $x_0$  moves relatively axis  $x$ . If we had no heading error, so

$$z_i = fx_d + \left(-N' - 1 + i - N\right) \frac{i}{N} L_x, y_d + \left(M - \frac{i}{N}\right) L_y + \eta_i,$$

where  $x_d, y_d$  are coordinates of the point on the land surface, where the sensor axis of the surface field is directed (as we can have deviation  $\Delta\beta_x, \Delta\beta_y$  axes sensor field from vertical).

So coordinates  $x_d, y_d$  can differ from coordinates  $x_0, y_0$ ;  $L_x, L_y$  are scale distances between elements of the image;  $NL_x \times (2M + 1)L_y$  are the image size (frame); in general case let's consider the frame as asymmetrical [ $N = N' + N'' + 1, N' \neq N'', q = N(2M + 1)$ ];  $\eta_i$  is errors of measurement of the field by field sensor in  $i$  point:

$$\eta_i = \eta_i \left\langle x_d + \left(-N' - 1 + i - N\right) \frac{i}{N} L_x, y_d + \left(M - \frac{i}{N}\right) L_y \right\rangle.$$

Image elements are numbered in such way, that the first is considered to be the upper left element. Now if we consider constant heading error  $\psi$  (inconsistency between axis  $x_0$  i  $x$ ), which appears due to that, on the board of moving object rate is never known with absolute accuracy, the equation for observing quantities of the field is:

$$z_i = f \left\langle x_d + \left(-N' - 1 + i - N\right) \frac{i}{N} L_x \cos Y + \left(-M + \frac{i}{N}\right) L_y \sin Y, y_d + \left(-N' - 1 + i - N\right) \frac{i}{N} L_x \sin \Psi + \left(-M + \frac{i}{N}\right) L_y \cos \Psi \right\rangle + \eta \left\langle x_d + \left(-N' - 1 + i - N\right) \frac{i}{N} L_x \cos \Psi + \left(-M + \frac{i}{N}\right) L_y \sin \Psi, y_d + \left(-N' - 1 + i - N\right) \frac{i}{N} L_x \sin \Psi + \left(-M + \frac{i}{N}\right) L_y \cos \Psi \right\rangle$$

$(i = 1, 2, \dots, q).$

Image scale  $L_x$  i  $L_y$  are constant in time, but unknown on the moving object. In case of using the optical images on unmanned aerial vehicle (UAV) this uncertainty determines by inaccurate data of altitude and angles  $\Delta\beta_x, \Delta\beta_y$  of deviation of sensors axis field from vertical. In case of using the radar images of region and making sweep on horizontal range the need of scale is also explained by inaccurate data of altitude flight.

**One-dimensional SLAM.** Let's consider one dimension SLAM for uniform motion of the moving object (and a field sensor on it) along  $x$  axes:

$$x_d = V_x t, y_d = 0.$$

Equation of motion in scalar form is the following:

$$\dot{x}_d = V_x, \dot{V}_x = 0, \dot{L}_x = 0, \dot{\psi} = 0.$$

Let's assume that the state vector is:

$$\mathbf{X} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T,$$

where  $x_1 = x_d, x_2 = V_x, x_3 = L_x, x_4 = L_y, x_5 = \psi$ , so the equation of motion is:

$$\dot{\mathbf{X}} + \mathbf{A}\mathbf{X} = 0.$$

Matrix  $\mathbf{A}$  will be:

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Let's also use vector representation for the noise vector  $\varepsilon = [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_q]^T$ . As the heading error is small, we can make the linearization by  $\psi$  and then observation matrix  $\mathbf{h}(\mathbf{X})$  will take the form:

$$\mathbf{h}_1(\mathbf{X}) = f \left\langle x_1 + \left( -N' - 1 + i - N \right) \frac{i}{M} \right\rangle x_3 + \left( -M + \right) \frac{i}{N} \left[ x_4 x_5 \right] = \\ = (-N' - 1 + i - N) i / N [x_4 x_5 + (M - i) / N] x_4 \rangle.$$

Let's assume that the matrix of spectral densities of noises is diagonal:

$$s_z = \begin{pmatrix} s_1 & \dots & 0 \\ \vdots & s_2 & \vdots \\ 0 & \dots & s_q \end{pmatrix},$$

where  $S_1, \dots, S_q$  are values of spectral densities of errors in different points at the frame at zero frequency.

Let's suppose that the measurements are done with the same accuracy ( $S_1 = S_2 = \dots = S_q = S_0$ ), then  $S_z = S_0 * I$ , where  $I$  is identify matrix of size  $q \times q$ :

$$R_0 = \begin{bmatrix} \sigma_{x_0}^2 & & & & 0 \\ & \sigma_{v_{x_0}}^2 & & & \\ & & L_x^2 \sigma_{\lambda_0}^2 & & \\ & & & L_y^2 \sigma_{\mu_0}^2 & \\ 0 & & & & \sigma_{\psi_0}^2 \end{bmatrix},$$

where  $\sigma_{x_0}^2, \sigma_{v_{x_0}}^2, \sigma_{\psi_0}^2$  are variances of initial errors of estimation in position, speed and heading of UAV, respectively;  $\sigma_{\lambda_0}^2, \sigma_{\mu_0}^2$  are variances of initial relative errors of estimation in scale  $L_x, L_y$ . Initial value of inverse matrix will be equal to

$$\Sigma_0 = \begin{bmatrix} \sigma_{x_0}^{-2} & & & & 0 \\ & \sigma_{v_{x_0}}^{-2} & & & \\ & & L_x^{-2} \sigma_{\lambda_0}^{-2} & & \\ & & & L_y^{-2} \sigma_{\mu_0}^{-2} & \\ 0 & & & & \sigma_{\psi_0}^{-2} \end{bmatrix}.$$

**Kalman estimation.** The algorithm of estimation will be based on Kalman filtering:

$$\dot{\hat{\mathbf{X}}} + \mathbf{A} \hat{\mathbf{X}} = \mathbf{k}_F [\mathbf{z} - \mathbf{h}(\hat{\mathbf{X}})];$$

$$\mathbf{k}_F = \mathbf{R} \left( \frac{\partial \mathbf{h}}{\partial \hat{\mathbf{X}}} \right)^T \mathbf{S}_z^{-1};$$

$$\dot{\Sigma} - \Sigma \mathbf{A} - \mathbf{A}^T \Sigma = \left( \frac{\partial \mathbf{h}}{\partial \hat{\mathbf{X}}} \right)^T \mathbf{S}_z^{-1} \left( \frac{\partial \mathbf{h}}{\partial \hat{\mathbf{X}}} \right);$$

$$\Sigma = \mathbf{R}^{-1}.$$

With the calculation and plotting the graphs (fig. 2) to reach the required accuracy of SLAM we will use the equation of one-dimensional cross of correlation function for the geophysical field  $f(x, y)$ :

$$R_f(\Delta) = \sigma_f^2 \exp(-\alpha^2 \Delta^2),$$

where  $\sigma_f^2$  is the variance of the field  $f(x, y)$ .

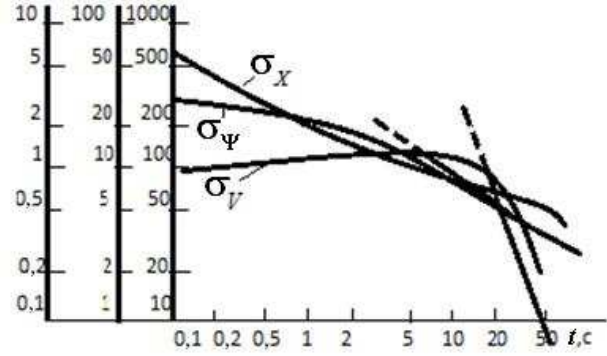


Fig. 2. Estimation of errors of UAV coordinates by SLAM

The correlation radius  $\rho$  of the field can be determined by the relationship

$$\frac{\int_0^{\infty} R_f(\Delta) d\Delta}{R_f(0)}$$

And equals

$$\rho = \sqrt{\pi} / (2\alpha).$$

The correlation function  $R_{\Delta_f}(\Delta)$  of the gradient of initial field is connected with  $R_f(\Delta)$  by the following equation:

$$R_{\Delta_f}(\Delta) = -\frac{\partial^2 R_f(\Delta)}{\partial \Delta^2}.$$

Thus,

$$R_{\Delta_f}(\Delta) = 2\alpha^2 \sigma_f^2 (1 - 2\alpha^2 \Delta^2) \exp(-\alpha^2 \Delta^2).$$

From this the variance  $\sigma_{\Delta}^2$  of the gradient of the used field becomes:

$$\sigma_{\Delta}^2 = \frac{\pi \sigma_f^2}{4 \rho^2}.$$

The main source of error is the error of field sensor used during the primal mapping. This error is the function of the spatial coordinate:

$$\tau_s = \frac{\rho_s}{V_x},$$

where  $V_x$  is the speed of motion,  $\rho_s$  is the correlation radius of the field sensor error.

The improvement of estimation of object speed is absent for the beginning because such information is absent in initial data. And only with further estimation we see decreasing of error in speed  $\sigma_v(t)$ .

### Conclusions

The behavior of the curve  $\sigma_x(t)$  is explained as following. During the first 5 seconds the initial error of positioning is decreased significantly due to the use of primal information from map. Then during 95 seconds the error  $\sigma_x(t)$  is also decreased but less rapidly and finally the further decreasing of error is done only due to the increasing of equivalent area of the frame with the object motion.

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**Mukhina Maryna Petrovna.** Candidate of Engineering. Associate Professor. Aviation Computer-Integrated Complexes Department, National Aviation University, Kyiv, Ukraine. Education: National Aviation University, Kyiv, Ukraine (2002). Research interests: Correlation extreme navigation, non-linear estimation, Gaussian particle filtering. Publications: 52. E-mail: [m\\_mukhina@inbox.ru](mailto:m_mukhina@inbox.ru)

### **М. П. Мухіна. Візуальний алгоритм одночасного картографування та позиціонування на основі кореляційно-екстремального принципу**

Розглянуто характерні особливості візуального алгоритма одночасного картографування та позиціонування. Сформульовано постановку задачі навігації для безперервного спостереження поверхні землі. Запропоновано Алгоритм кореляційного пошуку й оцінювання координат на основі калманівської фільтрації.

**Ключові слова:** одночасне картографування та позиціонування; статистична кореляція; фільтр Калмана.

**Мухіна Марина Петрівна.** Кандидат технічних наук. Доцент. Кафедра комп'ютерно-інтегрованих комплексів, Національний авіаційний університет, Київ, Україна. Освіта: Національний авіаційний університет, Київ, Україна (2002). Напрямок наукової діяльності: Кореляційно-екстремальна навігація, нелінійне оцінювання, гаусовський багатоточковий фільтр. Кількість публікацій: 52. E-mail: [m\\_mukhina@inbox.ru](mailto:m_mukhina@inbox.ru)

### **М. П. Мухина. Визуальный метод одновременного картографирования и позиционирования на основе корреляционно-экстремального принципа**

Рассмотрены характерные особенности визуального алгоритма одновременного картографирования и позиционирования. Сформулирована постановка задачи навигации для непрерывного наблюдения поверхности земли. Предложен алгоритм корреляционного поиска и оценки координат на основе калмановской фильтрации.

**Ключевые слова:** одновременное картографирование и позиционирование; статистическая корреляция; фильтр Калмана.

**Мухина Марина Петровна.** Кандидат технических наук. Доцент. Кафедра компьютерно-интегрированных комплексов, Национальный авиационный университет, Киев, Украина. Образование: Национальный авиационный университет, Киев, Украина (2002). Направление научной деятельности: Корреляционно-экстремальная навигация, нелинейное оценивание, гауссовский многоточечный фильтр. Количество публикаций: 52. E-mail: [m\\_mukhina@inbox.ru](mailto:m_mukhina@inbox.ru)