AUTOMATIC CONTROL SYSTEMS

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VIRATORY GYRO ACCURACY PARAMETERS IMPROVING BY MEANS OF EXCITATION CONTROL

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Abstract. *On the basis of Coriolis vibratory gyro standing wave manipulation by applying corresponding control forces, bias is reduced to the value that allows the gyro to measure North direction using low-cost and small-sized Coriolis vibratory gyro in differential mode of operation. Condition is determined for control forces resulting in frequency mismatch reduction to minimum in wide temperature range during angle rate measurement.*

Keywords: Differential Coriolis vibratory gyro; bias; control forces; frequency mismatch; *Q*-factor mismatch.

Introduction

Coriolis vibratory gyro (CVG) accuracy and its manufacturing cost are mainly determined by its sensor quality of manufacture. Basic sensor parameters determining its quality are resonator *Q*-factor, *Q*-factor mismatch and resonant frequency mismatch. After sensor manufacture it is usually carried out mass balance procedure to reduce manufacturing imperfections expressed in *Q*-factor and resonant frequencies mismatches. Mass balance by mechanically removing small masses from the resonator is sufficiently complex, time consuming and low producible procedure. This procedure results in reduction of batch production and CVG cost increasing.

Differential CVG proposes to locate standing wave in the middle between two electrodes, under equal angle $\theta = 22.5°$ to both of them. Doing this frequency mismatch can be reduced to 10^{-4} Hz by applying corresponding control signals on the electrodes and in addition it can be realized differential mode of angle rate measurement.

Possibility to measure angle rate using differential mode of operation and to keep frequency mismatch at low level in wide temperature range were firstly demonstrated in [1]. Different and almost always unknown in practice, variable in time and temperature electrode transformation coefficients of mechanical deformation into voltage and vice-versa have not been taken into account in [1].

Problem statement

Differential CVG dynamic equations analysis taking into account different by values electrode transformation coefficients is given in this work. Condition which has to meet excitation control signals to compensate for frequency mismatch is derived. Angle θ under which standing wave should be aligned in order to compensate for cross damping as a bias component is determined.

Bias components calculation procedure allowing one to estimate these components for short time after switching on the gyro and thus to provide high bias repeatability of differential CVG from switch on to switch on is given and analyzed. Numerical experiment based on linearized CVG model [2] in slow variable [3] is also given.

Differential CVG measurement equations

Coriolis vibratory gyro resonator dynamic equation can be presented as follows [4]:

$$\ddot{x} - 2k\Omega\dot{y} + d_{xx}\dot{x} + d_{xy}\dot{y} + k_{xx}x + k_{xy}y = f_x;$$

$$\ddot{y} - 2k\Omega\dot{x} + d_{yx}\dot{x} + d_{yy}\dot{y} + k_{yx}x + k_{yy}y = f_y; \quad (1)$$

where k is Brian coefficient, approximately equal to 0.4, $d_{xx} = 2/\tau + \Delta(1/\tau)\cos\theta_\tau$ is X axis damping coefficient, $2/\tau = 1/\tau_1 + 1/\tau_2, \Delta(1/\tau) = 1/\tau_1 - 1/\tau_2$, where τ_1 is minimum resonator's damping time, τ_2 is maximum resonator's damping time, $d_{xy} = \Delta(1/\tau)\sin\theta_\tau$ is damping cross-coupling, $k_{xx} = \omega_1^2 - \omega\Delta\omega\cos 2\theta_\omega$ is normalized by mass resonator rigidity along X axis, $\omega\Delta\omega = (\omega_1^2 - \omega_2^2)/2$, where ω_1, ω_2 are maximum and minimum resonant frequency, $k_{xy} = -\omega\Delta\omega\sin 2\theta_\omega$ is rigidity cross-coupling, $d_{yy} = 2/\tau - \Delta(1/\tau)\cos\theta_\tau$ is Y axis damping coefficient, $d_{xy} = d_{yx}$, $k_{yy} = \omega_2^2 + \omega\Delta\omega\cos 2\theta_\omega$ is resonator rigidity along Y axis normalized by mass, $k_{xy} = k_{yx}$, f_x and f_y are normalized by mass control signals, θ_ω is angle between minimum frequency axis and standing wave (antinode) axis, θ_τ is angle between minimum damping axis and standing wave (antinode) axis. Control signals f_x and f_y can be presented as follows:

$$f_x = (K_x \dot{x} + K_f^x x) D_x G_x; \quad f_y = (K_y \dot{y} + K_f^y y) D_y G_y, \tag{2}$$

where G_x, G_y are transformation coefficients of voltages applied on X and Y electrodes into the forces, D_x, D_y are transformation coefficients of X and Y electrodes deformations into voltages, K_x, K_y are control signals components along X and Y axes responsible for damping, K_f^x , K_f^y are control signals components along X and Y axes responsible for rigidity.

In this case equation (1) can be rewritten as follows:

$$\begin{aligned} \ddot{x} + d_{xx}\dot{x} + k_{xx}x + k_{xy}y &= (2k\Omega - d_{xy})\dot{y} + (K_x\dot{x} + K_f^x x)D_x G_x; \\ \ddot{y} + d_{yy}\dot{y} + k_{xy}x + k_{yy}y &= (-2k\Omega - d_{xy})\dot{x} + (K_y\dot{y} + K_f^y y)D_y G_y . \end{aligned} \tag{3}$$

Grouping terms responsible for oscillation frequency in the left side of these equations, it can be obtained

$$\begin{aligned} \ddot{x} + d_{xx}\dot{x} + (k_{xx} - K_f^x D_x G_x)x + k_{xy}y &= (2k\Omega - d_{xy})\dot{y} + K_x D_x G_x \dot{x}; \\ \ddot{y} + d_{yy}\dot{y} + (k_{yy} - K_f^y D_y G_y)y + k_{xy}x &= (-2k\Omega - d_{xy})\dot{x} + K_y D_y G_y \dot{y} . \end{aligned} \tag{4}$$

Three last terms in the left side of these equations are responsible for oscillation frequency along X and Y axes, respectively.

Let's form control signals K_f^x and K_f^y so that to meet the following condition:

$$(k_{xx} - K_f^x D_x G_x)x + k_{xy}y = (k_{yy} - K_f^y D_y G_y)y + k_{xy}x = \omega_r^2. \tag{5}$$

It means that oscillation frequencies along X and Y axes are equal to each other at value ω_r. It can be reached when full phase difference of signals *x(t)* and *y*(*t*) is constant, *phase*{*x*(*t*)} – *phase*{*y*(*t*)} = const. Holding this phase difference, for example, with the aid of proportional and integral (PI) controller can significantly reduce frequency mismatch [5].

Under condition (5) equation (4) can be rewritten as follows:

$$\begin{aligned} \ddot{x} + d_{xx}\dot{x} + \omega_r^2 &= (2k\Omega - d_{xy})\dot{y} + K_x D_x G_x \dot{x}; \\ \ddot{y} + d_{yy}\dot{y} + \omega_r^2 &= (-2k\Omega - d_{xy})\dot{x} + K_y D_y G_y \dot{y}. \end{aligned} \tag{6}$$

Let's find stationary solution of these equations:

$$x = r\cos 2\theta \sin(\omega_r t); \; y = r \sin 2\theta \sin(\omega_r t + \varphi), \tag{7}$$

where φ is constant phase difference between X and Y electrode signals, r is standing wave amplitude, θ is an angle between X axis and standing wave oscillation axis.

Substituting (7) in (6), it can be obtained after transformation:

$$\begin{aligned} &[d_{xx}\cos 2\theta - (2k\Omega - d_{xy})\sin 2\theta \cos\varphi - \\ &K_x G_x D_x \cos 2\theta]\cos\omega_r t = \\ &(2k\Omega - d_{xy})\sin 2\theta \sin\varphi \sin\omega_r t; \\ &[d_{yy}\sin 2\theta\cos\varphi + (2k\Omega + d_{xy})\cos 2\theta - \\ &K_y G_y D_y \sin 2\theta \cos\varphi]\cos\omega_r t = \\ &(d_{yy}\sin 2\theta \sin\varphi - K_y G_y D_y \sin 2\theta \sin\varphi)\sin\omega_r t. \end{aligned} \tag{8}$$

Right and left sides of these equations can be equal to each other for any t if amplitudes of the corresponding sine and cosine functions are equal to zero. Therefore, four equations for slow variables, which can be obtained after demodulation using two references $\sin\omega_r t$ and $\cos\omega_r t$:

$$\begin{aligned} X_c &= D_x d_{xx}\cos 2\theta - (2k\Omega - d_{xy})D_y \sin 2\theta\cos\varphi - K_x D_x \cos 2\theta = 0; \\ X_s &= (2k\Omega - d_{xy})D_y \sin 2\theta \sin\varphi = 0; \\ Y_c &= D_y d_{yy}\sin 2\theta + (2k\Omega + d_{xy})D_x \cos 2\theta\cos\varphi - K_y D_y \sin 2\theta \cos\varphi = 0; \\ Y_s &= D_y d_{yy}\sin 2\theta\sin\varphi - K_y D_y \sin 2\theta \sin\varphi = 0. \end{aligned} \tag{9}$$

These equations are valid for electrical signals.

From these equations follows that if to hold (using control system) phase difference $\varphi = 0$, then first and third equations of (9) remain nonzero and coefficient proportionality at angle rate Ω will be maximum because $\cos\varphi = 1$. In opposite case ($\varphi = \pi/2$) signal X_s carries information about angle rate Ω and to maximize coefficient at Ω wave angle should be put to $\theta = \pi/4$. It means that standing wave should be aligned along Y axis. The latter is equivalent to classic control algorithm of rate CVG.

Let's put in (9) $\varphi = 0$, and divide first and third equations of (9) by $\cos 2\theta$ and $\sin 2\theta$, respectively, for $\theta \neq 0, \pi/4$, it can be obtained the following equations:

$$\begin{aligned} -2k\Omega D_y \tan 2\theta + d_{xy} D_y \tan 2\theta + D_x d_{xx} &= K_x D_x; \\ 2k\Omega D_x \cot 2\theta + d_{xy} D_x \cot 2\theta + D_y d_{yy} &= K_y D_y, \end{aligned} \tag{10}$$

where

$$d_{xx}=\left[\frac{2}{\tau}+h\cos 2(\theta-\theta_\tau)\right];\ h=\Delta\left(\frac{1}{\tau}\right)=\frac{1}{\tau_1}-\frac{1}{\tau_2};$$
$$\frac{2}{\tau}=\frac{1}{\tau_1}+\frac{1}{\tau_2};\ d_{yy}=\left[\frac{2}{\tau}-h\cos 2(\theta-\theta_\tau)\right];$$
$$d_{xy}=h\sin 2(\theta-\theta_\tau).$$

There are measurement signals in the right side of equation (10).

It should be noted that in the first equation of (10) angle rate Ω has negative sign, and in the second one it has positive sign. Thus, control system holding standing wave between electrodes realizes additional, differential, mode of operation for CVG.

To effectively realize differential mode of operation for CVG it is necessary to align standing wave under angle θ^* at which the following condition is valid:

$$D_y\tan 2\theta^*=D_x\cot 2\theta^*,\text{or }\ \theta^*=\frac{1}{2}\arctan\sqrt{\frac{D_x}{D_y}}. \quad (11)$$

Angle θ^* can be determined through scale factors SF_x and SF_y of the two X and Y measurement channels, because, as can be seen from (10), $SF_x/SF_y=D_y/D_x\tan^2 2\theta$.

Let's write equation (10) substituting variables d_{xx}, d_{yy}, d_{xy} by their expressions through τ and θ_τ, as:

$$\begin{gathered}-2k\Omega D_y\tan 2\theta^*+D_yh\sin 2(\theta^*-\theta_\tau)\tan 2\theta^*\\ +D_x\frac{2}{\tau}+D_xh\cos 2(\theta^*-\theta_\tau)=zx_1;\\ 2k\Omega D_x\cot 2\theta^*+D_xh\sin 2(\theta^*-\theta_\tau)\cot 2\theta^*\\ +D_y\frac{2}{\tau}-D_yh\cos 2(\theta^*-\theta_\tau)=zy_1,\end{gathered} \quad (12)$$

where $zx_1=K_xD_x$ and $zy_1=K_yD_y$ are X and Y channel measurements, respectively.

Let's state the problem of the following six two channels bias parameters determination:

$$\begin{gathered}x_1=D_yh\sin 2(\theta^*-\theta_\tau);\ x_2=D_x\frac{2}{\tau};\\ x_3=D_xh\cos 2(\theta^*-\theta_\tau);\\ x_4=D_xh\sin 2(\theta^*-\theta_\tau);\\ x_5=D_y\frac{2}{\tau};\ x_6=D_yh\cos 2(\theta^*-\theta_\tau).\end{gathered} \quad (13)$$

To determine $x_1...x_6$ in addition to couple of measurement equations (12) one more couple of measurement equations should be used. This one more couple of measurement equations can be obtained if to turn standing wave through the angle 90 deg. clockwise or counterclockwise by applying control signals on X and Y electrodes. In this case for $\theta=\theta^*+90°$ (or $\theta=\theta^*-90°$) the following equations can be written down:

$$\begin{gathered}-2k\Omega D_y\tan 2\theta^*-D_yh\sin 2(\theta^*-\theta_\tau)\tan 2\theta^*\\ +D_x\frac{2}{\tau}-D_xh\cos 2(\theta^*-\theta_\tau)=zx_2;\\ 2k\Omega D_x\cot 2\theta^*-D_xh\sin 2(\theta^*-\theta_\tau)\cot 2\theta^*\\ +D_y\frac{2}{\tau}+D_yh\cos 2(\theta^*-\theta_\tau)=zy_2.\end{gathered} \quad (14)$$

By addition of first equations of (12) and (14) as well as second ones, the following equations can be obtained after transformations:

$$\begin{gathered}-4k\Omega D_y\tan 2\theta^*+D_x\frac{4}{\tau}=zx_1+zx_2;\\ 4k\Omega D_x\cot 2\theta^*+D_y\frac{4}{\tau}=zy_1+zy_2.\end{gathered} \quad (15)$$

Taking into account relationship (11), it can be obtained for x_5 and x_2 bias components:

$$\begin{gathered}x_5=D_y\frac{2}{\tau}=\frac{zx_1+zx_2+zy_1+zy_2}{2(1+\tan^2 2\theta^*)};\\ x_2=D_y\frac{2}{\tau}=\frac{zx_1+zx_2+zy_1+zy_2}{2(1+\cot^2 2\theta^*)}.\end{gathered} \quad (16)$$

By subtraction of first equations of (12) and (14) as well as second ones, the following equations can be obtained after transformations:

$$\begin{gathered}2D_yh\sin 2(\theta^*-\theta_\tau)\tan 2\theta^*\\ +2D_xh\cos 2(\theta^*-\theta_\tau)=zx_1-zx_2;\\ 2D_xh\sin 2(\theta^*-\theta_\tau)\cot 2\theta^*\\ -2D_yh\cos 2(\theta^*-\theta_\tau)=zy_1-zy_2.\end{gathered} \quad (17)$$

By subtraction the second equation of (17) from the first one, taking into account (11), it can be obtained the following expressions for x_6 and x_3 bias parameters:

$$\begin{gathered}x_6=\frac{zx_1-zx_2-zy_1+zy_2}{2(1+\tan^2 2\theta^*)};\\ x_3=\frac{zx_1-zx_2-zy_1+zy_2}{2(1+\cot^2 2\theta^*)}.\end{gathered} \quad (18)$$

Multiplication of the first equation of (17) by D_y and second one by D_x and addition of them results in the following relationship:

$$\begin{gathered}2(D_y^2\tan 2\theta^*+D_x^2\cot 2\theta^*)h\sin 2(\theta^*-\theta_\tau)\\ =(zx_1-zx_2)D_y+(zy_1-zy_2)D_x.\end{gathered} \quad (19)$$

From (19), taking into account (11), the following equation can be derived

$$2D_y^2 h\sin 2(\theta^* - \theta_\tau)(\tan^2 2\theta^* + \tan^3 2\theta^* \\ = (zx_1 - zx_2)D_y + (zy_1 - zy_2)D_y \tan^2 2\theta^*. \quad (20)$$

After transformations of (20) the following expressions for x_1 and x_4 bias parameters can be obtained

$$x_1 = \frac{(zx_1 - zx_2)\cot 2\theta^* + (zy_1 - zy_2)\tan 2\theta^*}{2(1+\tan^2 2\theta^*)}; \quad (21)$$

$$x_4 = \frac{(zx_1 - zx_2)\cot 2\theta^* + (zy_1 - zy_2)\tan 2\theta^*}{2(1+\cot^2 2\theta^*)}. \quad (22)$$

Thus, all six parameters of X and Y channel biases for differential CVG can be calculated during pre-launch calibration.

Differential CVG measures angle rate using difference of X and Y channel signals, and sum of these signals has no information about angle rate, but has current information about linear combination of bias components. Actually, subtracting and adding of equations (12), it can be obtained

$$-4k\Omega D_y \tan 2\theta^* + (D_x - D_y)\frac{2}{\tau} \\ +(D_x + D_y)h\cos 2(\theta^* - \theta_\tau) = zx_1 - zy_1; \\ 2D_y h\sin 2(\theta^* - \theta_\tau)\tan 2\theta^* + (D_x + D_y)\frac{2}{\tau} \\ +(D_x - D_y)h\cos 2(\theta^* - \theta_\tau) = zx_1 + zy_1. \quad (23)$$

As can be seen from the first equation of (23), there is no cross damping term $d_{xy} = h\sin 2(\theta^* - \theta_\tau)$ in bias (second and third summands) components.

Let's write measurement equations (23) using designations (13)

$$-4k\Omega D_y \tan 2\theta^* + x_2 - x_5 + x_6 + \xi_1 \\ = zx_1 - zy_1; \\ 2x_4 \tan 2\theta^* + x_2 + x_5 + x_3 + \xi_1 \\ = zx_1 + zy_1, \quad (24)$$

where ξ_1 and ξ_2 are measurement noises.

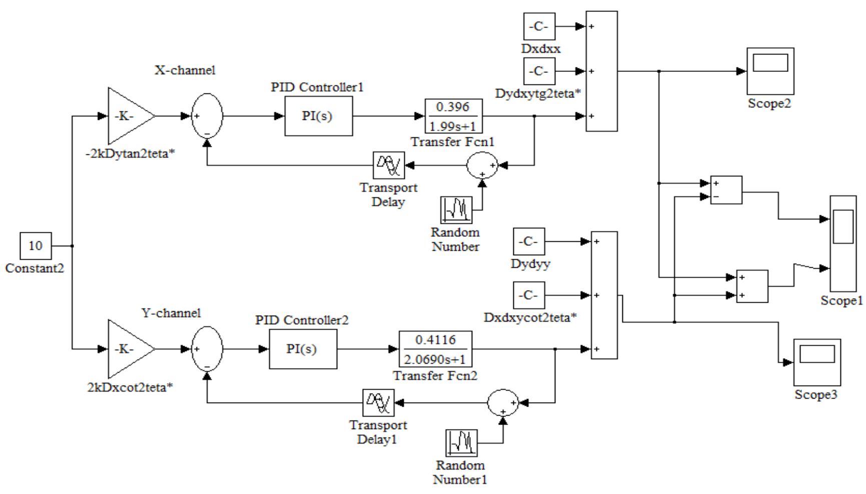
Numerical experiment

Let's determine bias parameters $x_1 ... x_6$ using expressions (16), (18), (21) and their errors in the figure. Linearized differential CVG model presence of measurement noise on the basis of simplified linearized simulink differential CVG model presented in view of two X and Y measurement channels in the figure. For the modeling procedure the following magnitudes of CVG parameters are used: k = 0,4, D_x = 1 μm/мВ; D_y = 1,1 μm/мВ; h = 0,0386658 s^{-1}; $\theta_\tau = 4°$; $2/\tau = 1{,}97195\ s^{-1}$, $\theta^* = \frac{1}{2} a\tan\sqrt{D_x / D_y}$; X and Y axes Q-factors are Q_x = 25000, Q_y = 26000, $\omega_r = 2\pi \cdot 4000$ rad/s.

Computation results of bias terms $x_1 ... x_6$ relative errors for two value of RMS measurement noises at averaging time of 1 s are presented in the table.

Relative errors of differential CVG bias components determination

Parameter	Error for σ_ξ = 6, deg/h %	Error for σ_ξ=20, deg/h %
x_1	5×10^{-12}	$1{,}5\times10^{-11}$
x_2	$4{,}2\times10^{-4}$	$1{,}3\times10^{-3}$
x_3	$3{,}5\times10^{-11}$	$7{,}4\times10^{-11}$
x_4	$4{,}7\times10^{-12}$	$1{,}5\times10^{-11}$
x_5	$4{,}2\times10^{-4}$	$1{,}3\times10^{-3}$
x_6	$3{,}5\times10^{-11}$	$7{,}2\times10^{-11}$



Linearized differential CVG model

Conclusion

Coriolis vibratory gyro differential mode of operation has a possibility to compensate for the frequency mismatch in wide temperature range during angle rate measurements.

Differential CVG prelaunch calibration procedure, based on standing wave reorientation can provide high bias repeatability at the level of 0.03deg/h and even higher for 3–5 s.

This result can be used to measure azimuth angle for short time.

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В. В. Чіковані, Г. В. Цірук. Збільшення параметрів точності вібраційного гіроскопа за допомогою управління збудженням

На основі маніпуляції стоячою хвилею коріолісового вібраційного гіроскопа застосуванням відповідних управляючих сил, зміщення нуля зменшується до значень, що дозволяє гіроскопу вимірювати напрямок на північ використовуючи дешеві і малогабаритні коріолісові вібраційні гіроскопи у диференціальнім режимі роботи. Визначено умови для управляючих сил, які призводять до зменшення різночастотності до мінімуму у широкому діапазоні температур в процесі вимірювання кутової швидкості.

Ключові слова: диференційний коріолісовий вібраційний гіроскоп; зміщення нуля; управляючі сили; різночастотність; різнодобротність.

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В. В. Чиковани, А. В. Цирук. Увеличение параметров точности вибрационного гироскопа посредством управления возбуждением

На основе манипуляции стоячей волной кориолисового вибрационного гироскопа приложением соответствующих управляющих сил, смещение нуля уменьшается до значений позволяющих гироскопу измерять направление на Север, используя дешевые и малогабаритные кориолисовы вибрационные гироскопы в дифференциальном режиме работы. Определены условия для управляющих сил, которые приводят к уменьшению разночастотности до минимума в широком диапазоне температур в процессе измерения угловой скорости.

Ключевые слова: дифференциальный кориолисовый вибрационный гироскоп; смещение нуля; управляющие силы; разночастотность; разнодобротность.

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