MATHEMATICAL MODELING OF PROCESSES AND SYSTEMS

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MATHEMATICAL MODEL OF SATELLITE COMMUNICATION SYSTEMS' RADIOMONITORING MEANS DISTRIBUTION

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Abstract. *The mathematical model of distribution the means of radiomonitoring for observation data channels of satellite communications systems had been performed for consideration. The model is a formalized description of the distribution process using mathematical tools of fuzzy set theory, methods of dynamic and fuzzy mathematical programming.*

Keywords: dynamic programming; fuzzy mathematical programming; radiomonitoring; satellite communication systems.

I. PROBLEM STATEMENT

Satellite Communications System (SCS) for radiomonitoring are among the main sources of radio emission (SRE) [5], [8], [9].

Having a purpose of SCS radiomonitoring organization, posts are being deployed based on specialized means for radio emission analyses.

Given the trends in the development of communications and intensification of SCS use, the fulfillment of tasks on the radiomonitoring posts requires high efficiency and reliability of the information [8]. But putting the entire set of data channels into observation simultaneously is not possible because of the limited number of radiomonitoring means.

Providing the necessary efficiency of radiomonitoring can be done by increasing the number of posts, but this approach is not acceptable because of limited funding. Therefore, increasing the efficiency of available resources is a possible solution. This can be achieved through optimal distribution of SCS radiomonitoring means by the tasks, radiomonitoring objects (RMO) and SRE.

However, to date, the distribution of SCS radiomonitoring means on observation of satellite channels have been performed by empirical method, which is why it depends on the experience and qualifications of the person who makes decisions on radiomonitoring. In this regard, appears the problem of effective planning of targeted use of SCS radiomonitoring means. The outlined problem shows the necessity of solving the scientific task about optimization the process of planning according to the criteria of time and coverage of SRE under conditions of uncertainty.

II. RECENT PUBLICATIONS OVERVIEW

The task of means distribution by the RMO and SRE is a multi-criterion scientific problem [4]. In the known literature [4], [8], [9] it has being solved by: manual distribution method using the technique of successive approximation; a modified method of successive approximation; technique that implements heuristic quasi optimization procedure; the method of dynamic programming; techniques based on the use of scalar convolution by nonlinear scheme of compromise.

The analysis of existing distribution methods and techniques [4], [8] has shown that they have several disadvantages. Among them there are: impossibility of use in the uncertain conditions of functioning parameters, modes, intensity of functioning and the information content of satellite channels; a deterministic approach on solving distribution tasks; lack of technique that provides a solution to the complex problem of the search and observation; the complexity of the dynamic changes in the distribution for different radio-electronic environment.

The main disadvantage in methodology apparatus for SCS radiomonitoring means distribution is the lack of formal mathematical model that allows conducting this process under uncertain conditions.

Thus, the aim of the paper is the development of mathematical model for distribution of the SCS radiomonitoring means on observation for satellite data channels under uncertain conditions.

III. THE MAIN MATERIAL

Let us assume that the area of SCS radiomonitoring post's antenna systems contains M satellites with channels, identified for observation. Accordingly, the set of communications satellites (CS) is $Sp = \{Sp_1, Sp_2, ..., Sp_m, ..., Sp_M\}$, $m = \overline{1,M}$. At each mth satellite K_m channels were found. Thus, the total number of channels that require observation is $K = \sum_{m=1}^{M} K_m$.

Satellite communications system radiomonitoring post includes An antenna systems and N radiomonitoring means, while $N \geq An$, $An << M$ and $N << K$ [5], [8].

A well-known approach of solving the problem of radiomonitoring means distribution for observation is to choose the variants considering the SRE importance [4], [5], [8]. Thus, each SCS data channel can be associated with the coefficient of its relative importance $W_k^{Sp_m}, \forall k = \overline{1,K_m}, \forall m = \overline{1,M}$ [5].

Thus, it is necessary to distribute N radiomonitoring means for observation of K SCS data channels in order to cover the maximum number of objects with the most important channels.

In general, the problem of distribution can be solved by mathematical programming means [10]. In case if the value of the relative importance coefficients of the channel $W_k^{Sp_m}$ is known, the method of dynamic programming is used [1], [3], [6].

However, in the face of uncertain conditions and dynamic change of the situation clear values of the input parameters are not always available.

Let $W = \{0, 1, ..., I\}$ be the set that characterizes the channel's importance degree, where I is the meaning of the highest importance degree. Then each channel, specified for observation, is associated with a fuzzy set $W_i^{Sp_m}$ in W, which is a set of pairs

$$W_k^{Sp_m} = \left\{w, \mu_{W_k^{Sp_m}}(w)\right\},, \quad (1)$$

where $\mu_{W_k^{Sp_m}}(w)$ is a membership function of w to the $W_k^{Sp_m}$.

In this case the problem of distribution can be solved by using fuzzy mathematical programming [2], [7].

The required distribution is the formation of a plan D_{Ob}, which describes the appointment of a specific radiomonitoring means for observation SCS channels [4], [8]. Let the set of possible variants for the RMO observation plans be denoted as the vector [4]:

$$D_{Cn} = \left\{D_{Cn}^1, D_{Cn}^2, ..., D_{Cn}^q, ..., D_{Cn}^Q\right\}, q \in \overline{1,Q}, \quad (2)$$

where D_{Ob}^q is q th variant of the RMO observation plan; q is the ordinal number of variant; Q is the total number of variants.

Each variant of observation plan D_{Ob}^q corresponds the vector of the RMO selection list [4]:

$$D_{Cn}^q = \left\{R_1^q, R_2^q, ..., R_m^q, ..., R_M^q\right\}, \quad m \in \overline{1,M}, R_m^q \in 0{,}1, D_{Cn}^q \in \Omega_D, \quad (3)$$

where R_m^q is component of the D_{Ob}^q vector, which reflects the choice of mth RMO for observation; Ω_D are set of observation plans D_{Ob}^q variants.

Appointment variants of radiomonitoring means for SRE observation on mth RMO can be described as a vector [4], [8]:

$$R_m^q = \left\{B_{m1}^q, B_{m2}^q, ..., B_{mk}^q, ..., B_{mK_m}^q\right\}, \quad k \in \overline{1,K_m}, B_{mk}^q \in 0{,}1, R_m^q \in \Omega_R, \quad (4)$$

where B_{mk}^q is number of radiomonitoring means assigned for observation of kth SRE on the mth RMO; k is SRE number; Ω_R is the set of possible choices of R_m^q for SRE observation on m RMO.

Components $B_{mk}^q = 1$ if kth SRE on mth RMO in qth variant of plan should be observed, otherwise $B_{mk}^q = 0$. If all the components $B_{mk}^q = 0$, then $R_m^q = 0$ [4].

Obviously, there are many possible variants of D_{Ob} [0]. Among them it is necessary to choose the plan D_{Ob}^* which will ensure the monitoring of the most important channels and the most complete SRE coverage.

According to the Bellman-Zadeh approach [2], [7], the problem of plan D_{Ob}^* choice can be described as a multistep control process for the radiomonitoring means distribution for satellite data channels observation.

Let the iterative selecting procedure of a plan being presented as invariant deterministic system with a finite number of states M (by the number of satellites).

Let's denote the step's number as $t = 1, 2, \ldots, M$; x_t is the state of the system on the tth step of iterative distribution procedure, which characterizes the number of radiomonitoring means, which remained undistributed during previous steps. Accordingly $x_0 = N$; x_{M-1} shows the number of radiomonitoring means that will be assigned to observe the Mth satellite channels.

According to the procedure of dynamic programming control variables u_t in each tth step is the number of radiomonitoring means assigned to observe Sp_t satellite channels. Let $U = \{0, 1, \ldots, N\}$

be the set of possible control variables u_t. Then the equation of state can be written in the following form:

$$x_t = x_{t-1} - u_t\,,\ t = 1, 2, \ldots, M. \quad (5)$$

The observation plan D^*_{Ob} has being formed considering fuzzy constraints C_t that depend on the number of channels and their importance (1). Fuzzy sets C_t in W with a membership function $\mu_{C_t}(u_t)$ characterizes the importance of the t th satellite, in case that for observation after it assigned u_t radiomonitoring means.

Considering the definitions, we get the following expression:

$$\mu_{C_t}(u_t) = \max_{j=1,u_t}\left[\mu_{W_j^{Sp_t}}(w)\right]. \quad (6)$$

Let the fuzzy goal be given as a fuzzy subset G_M in W, that is fuzzy restriction on the x_M state of the system on the last step M. Thus, the task is to select a sequence of control parameters $u_1, u_2, \ldots, u_M$ that, considering fuzzy constraints, achieves fuzzy goal G_M [2], [7]. It is formulated as: "w should be approximately equal to I" which corresponds membership function [2]

$$\mu_{G_M}(w) = \left(1 + (w - I)^4\right)^{-1}. \quad (7)$$

So the state of the system x_M can be expressed as $x_M(x_0, u_1, u_2, \ldots, u_M)$ through solving the equations of state (5) for $t = 1, 2, \ldots, M$.

Using the rule of the fuzzy decision finding [2], [7], it is possible to write it in the following way:

$$D = C_1 \cap C_2 \cap \ldots \cap C_M \cap G_M\,. \quad (8)$$

For the membership functions the equation (8) goes as [2], [7]

$$\mu_D(u_1, u_2, \ldots, u_M) = \min\{\mu_{C_1}(u_1), \mu_{C_2}(u_2), \ldots, \mu_{C_M}(u_M), \mu_{G_M}(x_M)\}. \quad (9)$$

It is necessary to find a solution for the problem, which is to determine the sequence of control parameters $u_1^*, u_2^*, \ldots, u_M^*$ which has a maximum degree of membership to fuzzy decision D, that is

$$\mu_D(u_1^*, u_2^*, \ldots, u_M^*) = \max_{u_1,\ldots,u_M} \min\{\mu_{C_1}(u_1), \mu_{C_2}(u_2), \ldots, \mu_{C_M}(u_M), \mu_{G_M}(x_M)\}. \quad (10)$$

To do this, the dynamic programming method can be applied [1–3], [6], [7]. Using the equation of state (5), equation (10) can be written as:

$$\mu_D(u_1^*, u_2^*, \ldots, u_M^*) = \max_{u_1,\ldots,u_{M-1}} \max_{u_M} \min\{\mu_{C_1}(u_1), \mu_{C_2}(u_2), \ldots, \mu_{C_M}(u_M), \mu_{G_M}[f(x_{M-1}, u_M)]\}. \quad (11)$$

Given the known mathematical transformation, the equation (11) would be as follows:

$$\mu_D(u_1^*, u_2^*, \ldots, u_M^*) = \max_{u_1,\ldots,u_{M-1}} \min\Big\{\mu_{C_1}(u_1), \mu_{C_2}(u_2), \ldots, \mu_{C_{M-1}}(u_{M-1}), \max_{u_M} \min\{\mu_{C_M}(u_M), \mu_{G_M}[f(x_{M-1}, u_M)]\}\Big\}. \quad (12)$$

Let's introduce the equation

$$\mu_{G_{M-1}}(x_{M-1}) = \max_{u_M} \min\{\mu_{C_M}(u_M), \mu_{G_M}[f(x_{M-1}, u_M)]\}. \quad (13)$$

The function $\mu_{G_{M-1}}(x_{M-1})$ is a membership function of fuzzy goal for management task on the steps from 1 to $M-1$. Thus, it is the maximum degree of the goal G_M reaching, when on the $M-1$ step system comes into the x_{M-1} state.

Similarly we obtain a system of recurrent relations [2], [7]:

$$\begin{cases} \mu_{G_{M-v}}(x_{M-v}) = \max\limits_{u_{M-v+1}} \min\{\mu_{C_{M-v+1}}(u_{M-v+1}), \mu_{G_{M-v+1}}(x_{M-v+1})\}; \\ x_{M-v+1} = f(x_{M-v}, u_{M-v+1}). \end{cases}$$

Using these relations in sequence, starting with $v = 1$, we define the functions $u_M^*(x_{M-1})$, $u_{M-1}^*(x_{M-2})$, …, $u_1^*(x_0)$, and then in accordance to given initial state of the system, using the equation of state (5), we can calculate necessary management parameters backwards $u_1^*(x_0)$, $u_2^* = u_2^*(f(x_0, u_1^*))$, $u_3^* = u_3^*(f(f(x_0, u_1^*), u_2^*)), \ldots$.

Thus, considering the above, the mathematical model of the SCS radiomonitoring means distribution for observation of satellite data channels, considering their importance in the uncertain conditions, it is possible to write as the following system of equations:

$$\begin{cases} \mu_D(u_1^*, u_2^*, \dots, u_M^*) = \max\limits_{u_1,\dots,u_M} \min\left\{\mu_{C_1}(u_1), \mu_{C_2}(u_2), \dots, \mu_{C_M}(u_M), \mu_{G_M}\left(x_M\right)\right\}; \\ \mu_{G_{M-v}}\left(x_{M-v}\right) = \max\limits_{u_{M-v+1}} \min\left\{\mu_{C_{M-v+1}}(u_{M-v+1}), \mu_{G_{M-v+1}}\left(x_{M-v+1}\right)\right\}; \\ x_{M-v+1} = x_{M-v} - u_{M-v+1},\ v = \overline{1, M}; \\ \mu_{C_t}(u_t) = \max\limits_{j=1,u_t}\left[\mu_{W_j^{Sp_t}}\left(w\right)\right]; \\ \mu_{G_M}(w) = \left(1 + \left(w - I\right)^4\right)^{-1}; \\ \sum\limits_{t=1}^{M} u_t \le N. \end{cases}$$

IV. Conclusions

The mathematical model of the SCS radiomonitoring means distribution for observation the satellite data channels was developed. The proposed approach provides finding the rational distribution plan for the SCS radiomonitoring means and differs from the known methods by the using of the fuzzy mathematical programming and fuzzy set theory.

Future research can be dedicated to the development the method of SCS radiomonitoring means distribution for observation the satellite data channels.

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С. М. Гудзь, В. А. Шуренок, А. В. Родіонов. Математична модель розподілу засобів радіомоніторингу супутникових систем зв'язку
Запропоновано математичну модель розподілу засобів радіомоніторингу на спостереження каналів передачі даних систем супутникового зв'язку, яка є формалізованим описом процесу розподілу за допомогою математичного апарату теорії нечітких множин, методів динамічного та нечіткого математичного програмування.
Ключові слова: динамічне програмування, нечітке математичне програмування, радіомоніторинг, системи супутникового зв'язку.

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С. М. Гудзь, В. А. Шуренок, А. В. Родионов. Математическая модель распределения средств радиомониторинга систем спутниковой связи
Предложена математическая модель распределения средств радиомониторинга для наблюдения каналов передачи данных систем спутниковой связи, которая является формализованным описанием процесса распределения при помощи математического аппарата теории нечетких множеств, методов динамического и нечеткого математического программирования.
Ключевые слова: динамическое программирование, нечеткое математическое программирование, радиомониторинг, системы спутниковой связи.

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