THEORY AND METHODS OF SIGNAL PROCESSING

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DATA FUSION SCHEMES IN AIDED NAVIGATION SYSTEMS

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Abstract. *The comparative analysis of different data fusion is done for navigation components of the state vector in aided navigation systems namely in inertial-satellite navigation systems constructed by the compensation method or based on Kalman filtering. The errors of coordinate and velocity components estimates are investigated by two data fusion algorithms.*

Keywords: data fusion; compensation scheme; optimal filtering; estimation of errors.

I. INTRODUCTION

ICAO Committee on future navigation systems recommended the obligatory use of satellite navigation systems (SNS) aided to inertial navigation systems (INS). When building inertial satellite navigation systems (ISNS) the data fusion of INS and SNS information is usually done based on optimal Kalman filtering. However, the practical implementation of Kalman filtering onboard the aircraft causes some difficulties. Basic one among them is divergence phenomenon which occurs when working with unknown stochastic signal at the input of filter, which is typical for most strapdown INS (SINS). Since in SINS due to nonstationarity the transformation matrices of sensor signals in navigation coordinate system the stationary random processes of main error sources (drifts of gyroscopic sensors and accelerometers) become non-stationary.

To avoid these difficulties, a number of modifications of Kalman filter is developed, in particular, the use of robust and adaptive filtering algorithms, Yazvynsky algorithm, etc. To reduce computations, reduced Kalman filter can be used.

However, in modern airborne complexes there are other algorithms to be used such as sub-optimal processing algorithms which are well proven themselves in practice. In particular, one of such method is method of mutual compensation. The expediency of using the method of compensation in the processing of information in ISNS is explained by the fact that the measurement of the navigation parameters is performed be sensors with errors in different frequency ranges. And if the data fusion does not include the estimation of state vector components which are not observed (e.g. orientation parameters in SINS are measured with acceptable accuracy and do not require additional estimation), the use of suboptimal methods of processing the navigation data is of some interest.

Quality of information processing with using the compensation method depends on the quality of the filtering procedure in the estimation of SINS error signal, corrupted by high-frequency error of SNS, which can be described by the Gaussian white noise. In [2] the filter of compensation scheme is proposed, which in authors' opinion provides the data fusion of observed navigational components of the state vector, with quality comparable to optimal Kalman filter. But the comparative characteristics of these two methods are not given.

Thus the comparative analysis of different navigation schemes of data fusion for the observed components of state vector becomes very urgent problem to develop and implement the processing algorithms of inertial satellite navigation systems.

II. PROBLEM STATEMENT

The problem statement of synthesis of optimal data fusion algorithms for INS and SNS can be formulated as following: to find the best (in terms of minimum of estimate error variance) estimates of the state vector by observed signals of inertial and satellite navigation systems.

It is known that for the dynamic system of this type

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{B}\xi_x;$$
$$\mathbf{Z} = \mathbf{HX} + \xi_z,$$

the optimal Kalman filter provides the minimum of error variance and consists of three blocks:

– main part

$$\dot{\hat{\mathbf{X}}} = \mathbf{A}\hat{\mathbf{X}} + \mathbf{K}_F\left[\mathbf{Z} - \mathbf{H}\hat{\mathbf{X}}\right];$$

– Kalman gain calculation

$$\mathbf{K}_F = \mathbf{P}\mathbf{H}^{\mathrm{T}}\mathbf{R}_z^{-1};$$

– solution of covariance equation

$$\dot{\mathbf{P}} = \mathbf{AP} + \mathbf{PA}^{\mathrm{T}} - \mathbf{PH}^{\mathrm{T}}\mathbf{R}_z^{-1}\mathbf{HP} + \mathbf{BR}_x\mathbf{B}^{\mathrm{T}}.$$

Here **X** is state vector; **A** is matrix of system coefficients; **B** is matrix of disturbances, which act on the input of dynamic system; **Z** is measurement vector; **H** is observation matrix; ξ_x is measurement noise vector; $\hat{\mathbf{X}}$ is estimates of state vector; $\mathbf{K}_F$ is Kalman gain matrix; **P** is covariance matrix, that must be calculated during the whole estimation process or beforehand if possible; $\mathbf{R}_x$, $\mathbf{R}_z$ are correlation matrices which characterize the dependences between components of disturbance vector ξ_x (or measurement noise vector ξ_z) and random functions of white noise type.

Henceforth the analysis of the implementation features of estimation algorithms will be limited only by filtering algorithms in separate channels (longitudinal or lateral), taking the assumption of their independence.

Block diagram of the compensation method realization is represented in Fig. 1.

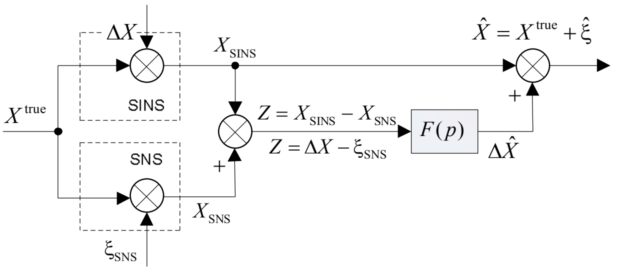

Fig. 1. Block diagram of the compensation method realization: $F(p)$ is dynamic filter of compensation scheme; X_{SINS}, X_{SNS} are navigation parameters (coordinates and velocity components) obtained from SINS and SNS; $\hat{X}$ is estimate of given navigation parameter; X^{true} *is* true value of navigation parameter; ΔX is error of SINS; ξ_{SNS} is noise component of SNS error; Z is navigation parameters of observation; $\hat{\xi} = [1 - F(p)]\Delta X + F(p)\xi_{\text{SNS}}$ is error of data fusion

The data fusion algorithm based on the compensation method has the following form:

$$\hat{X} = X_{\text{SINS}} - F(p)Z.$$

The equation of compensation scheme (Fig. 1) can be written as follows:

$$\hat{X} = X^{\text{true}} + \Delta X - F(p)(\Delta X - \xi_{\text{SNS}})$$

or

$$\hat{X} = X^{\text{true}} + [1 - F(p)]\Delta X + F(p)\xi_{\text{SNS}} = X^{\text{true}} + \hat{\xi}.$$

The error $\hat{\xi}$ will be decreasing with greater difference between spectral characteristics of sensor errors ΔX and ξ_{SNS}. If the filter $F(p)$ is selected to minimize the distortion of disturbance ΔX and to suppress the noise ξ_{SNS}, then the error of complex system will be minimal, i.e. the error $\hat{\xi}$ decreases depending on the difference in spectral characteristics of noises ΔX and ξ_{SNS}. With significant difference in frequency characteristics of noises at the output of filter $F(p)$ (see Fig. 1) the disturbance ΔX will be reproduced without any changes, and at the output of compensation scheme the exact value of measured parameter X^{true} is obtained, since

$$\hat{X} = X^{\text{true}} + \Delta X - \Delta X = X^{\text{true}}.$$

In [1] the dynamic filter of compensation scheme is developed, which is the third order link

$$F(p) = \frac{3T_F p + 1}{(T_F p + 1)(T_F p + 1)(T_F p + 1)}.. \tag{1}$$

One of the most important characteristic of any compensation scheme is its convergence and convergence time. To improve these characteristics it is propose to use the filter with variable structure in the compensation scheme. The structure of the filter varies with time and has the following form:

$$F(p) = \begin{cases} \dfrac{1}{T_{F1}p + 1} & \text{if } t \le 3T_{F1}; \\ \dfrac{3T_{F2}p + 1}{(T_{F2}p + 1)(T_{F2}p + 1)(T_{F2}p + 1)} & \text{if } 3T_{F1} < t \le 3T_{F2}; \\ \dfrac{3T_F p + 1}{(T_F p + 1)(T_F p + 1)(T_F p + 1)} & \text{if } 3T_{F2} < t. \end{cases}$$

The problem statement can be formulated as follows: to perform the comparative analyses of estimation results of SINS errors, in particular errors of dead reckoning of coordinates and velocity components, using algorithms of reduced Kalman filter and algorithms of compensation method.

III. PROBLEM SOLUTION

The comparative analyses of filtering methods and algorithms will be limited only by linear filtering algorithms of signal-to-noise components with not-overlapped frequencies. That is, they have significant difference in the frequency characteristics of errors and work in real-time systems. Algorithms of data fusion in ISNS are related to mentioned type.

In conventional data fusion algorithms of ISNS the estimation of dead reckoning errors of SINS is done with the help of current information from SNS with further correction of output data of SINS. Such scheme is well known and called invariant integration scheme. Its feature is forming the measurements as the difference between corresponding readings of dead reckoning systems and correctors that provides practical independence (invariance) of estimated errors of dead reckoning systems on the properties of aircraft navigation parameters. With use of difference measurements the problem to estimate the errors of a subsystem taking into account errors of another is solved. In the invariant integration scheme the linear error equations are used as the dynamic models of estimated errors of dead reckoning systems. And therefore linear Kalman filters are used to estimate errors.

To synthesize optimal Kalman filter let's consider only longitudinal channel and take into account its independence on the lateral channel. Then the model of dead reckoning errors can be represented as

$$\Delta\dot{X} = \Delta V_x;\quad \Delta\dot{V}_x = \Delta a_x;\quad \Delta\dot{a}_x = \xi_x,$$

where ΔX, ΔV_x, Δa_x are errors of SINS in coordinate and its derivatives, respectively; ξ_x is accelerometer noise given as white noise with intensity S_{ax}.

Observation model will be written as

$$Z_1 = X_{\text{INS}} - X_{\text{SNS}} = \Delta X + \varsigma_x;$$

$$Z_2 = V_{\text{INS}} - V_{\text{SNS}} = \Delta V + \varsigma_v;$$

$$X_{\text{INS}} = X^{\text{true}} + \Delta X;\quad V_{\text{INS}} = V^{\text{true}} + \Delta V;$$

$$X_{\text{SNS}} = X^{\text{true}} + \varsigma_x;\quad V_{\text{SNS}} = V^{\text{true}} + \varsigma_v.$$

The solution of given problem is the continuous reduced Kalman filter:

$$\Delta\dot{\hat{X}} = \Delta\hat{V} + K_{F1}(Z_1 - \Delta\hat{X});$$

$$\Delta\dot{\hat{V}} = \Delta\hat{a} + K_{F2}(Z_2 - \Delta\hat{V});$$

$$\Delta\dot{\hat{a}} = K_{F3}(Z_1 - \Delta\hat{X}),$$

here X_{INS}, V_{INS} are coordinate and velocity measured by SINS; X_{SNS}, V_{SNS} is coordinate and velocity obtained from SNS receiver; X^{true} is true value of coordinate; ΔX, ΔV are errors of SINS which are considered as systematic errors caused by gyroscope drifts and inaccuracy of accelerometers; ς_x, ς_V are noise components of SNS receiver errors given as white noises with spectral densities Sx, Sv, respectively; K_{Fi}, $i = \overline{1,3}$ is coefficients of filtering, in particular coefficients K_1, K_2 are determined as

$$K_1 = \frac{R_{11}}{S_x},\quad K_2 = \frac{R_{12}}{S_v}.$$

Correlation moments R_{11}, R_{12} are determined from the solution of a set of differential equations (Riccati equations) of third order:

$$\dot{R}_{11} = 2R_{12} - \frac{R_{12}^2}{S_x},\qquad R_{11}(t_0) = \sigma_{x_0};$$

$$\dot{R}_{12} = 2R_{22} - \frac{R_{11}R_{12}}{S_x},\qquad R_{12}(t_0) = R_{12_0};$$

$$\dot{R}_{22} = S_x - \frac{R_{12}^2}{S_v},\qquad R_{22}(t_0) = \sigma_{V_0}^2.$$

Filtering coefficients can be obtained also as constants from steady-state Riccati equations to be converted to non-linear algebraic equations.

Analyses of Kalman filtering algorithms, e.g. of the first equation, shows that with absence of component $\Delta\hat{V}$ this equation describes the ordinary aperiodic filter – Butterworth filter of the first order which is used in integration schemes of inertial Doppler navigation systems and based on compensation scheme. Naturally that with integration of highly accurate SNS and coarse SINS the errors of such integration scheme are decreased in comparison with SINS error but simultaneously exceed several times SNS error (Fig. 2)

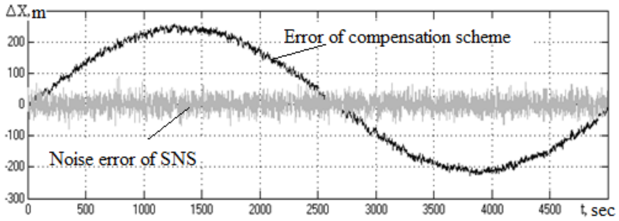

Fig. 2. Error of compensation scheme and noise errors of SNS

It is due to filtering of noise component of SNS error that in turn causes the corruption of systematic error of SINS because of sluggishness of aperiodic filter. The compensation of sluggishness of aperiodic filter in Kalman filter algorithms is provided by component $\Delta\hat{V}$ with estimation of error ΔX.

In the analyzed dynamic filters (1) the compensation schemes based on Butterworth filter of second order use [1] simple model of sluggishness compensation that actually is the aperiodic filter

$$W_{\text{comp}}(p) = (3T_F p + 1)/(T_F p + 1),$$

and the transfer function of low frequency filter has the form:

$$[1 - F(p)] = \frac{T_F^2 p^2 (T_F p + 3)}{T_F^3 p^3 + 3T_F^2 p^2 + 3T_F p + 1}. \tag{2}$$

That is, the dynamic filter (1) passes the constant and low frequency components of SINS errors without corruption. These components vary with time by laws of the first and second orders to be typical for Schuler oscillations. Simultaneously, the filter suppresses effectively the high-frequency noise components of SNS errors.

For comparative analyses of two data fusion schemes the filter coefficients K_{Fi}, $i=\overline{1,3}$ were synthesized, and also the time constant T_F was found for the intensities of white noise errors of SNS (Sx= 0,535 m^2/sec^3; Sv = 0,005 m^2/sec) and for the root mean square errors (σ_X = 50 m; σv = 0,55 m/sec).

The research of characteristics of complex ISNS was done by its mathematical simulation in software environment *Simulink*, as a part of Matlab.

The model of two-component SINS was created. It describes the aircraft motion in vertical plane. The input information for SINS model is signals of inertial sensors (taking into account theirs errors) and known data of the Earth radius, angular rate of the Earth rotation, acceleration of gravity force. The output information of SINS model is the calculated values of main flight and navigation parameters: pitch ϑ, horizontal $\dot{V}_E$ and vertical $\dot{V}_H$ components of aircraft center-of-mass acceleration, vertical speed V_H and rate of change of geodetic longitude $\dot{L}$, altitude H and geodetic longitude L.

Simultaneously the ideal navigation system was simulated identical by structure to SINS, but its input signals are signals of ideal sensors and actual values of the Earth radius, angular rate of the Earth rotation, acceleration of gravity force. Information from ideal navigation system is used to form true and not calculated values of navigation parameters, and also to estimate the accuracy characteristics of SINS by comparison the calculated and true values of navigation parameters.

With SNS simulation the output information of ideal navigation model is used that is corrupted by white noise components of SNS errors.

With simulation of inertial sensors, in particular angular rate sensor (ARS), let's suppose that ideal ARS measures: (component of angular rate of the Earth rotation, angular rate caused by aircraft motion relatively to the Earth, and component of angular rate caused by aircraft maneuvering by pitch. To form the model of real ARS the model of ideal ARS is used with output information corrupted by systematic component of sensor zero drift and by random component (measurement noise).

The same approach is used to simulate the accelerometers of SINS. With research the previously given data fusion algorithms are also simulated.

Simulation results demonstrate the estimation errors in coordinate and velocity component by two data fusion algorithms with SNS noise. They are represented in Fig. 3 (the increased zoom is used for detail consideration).

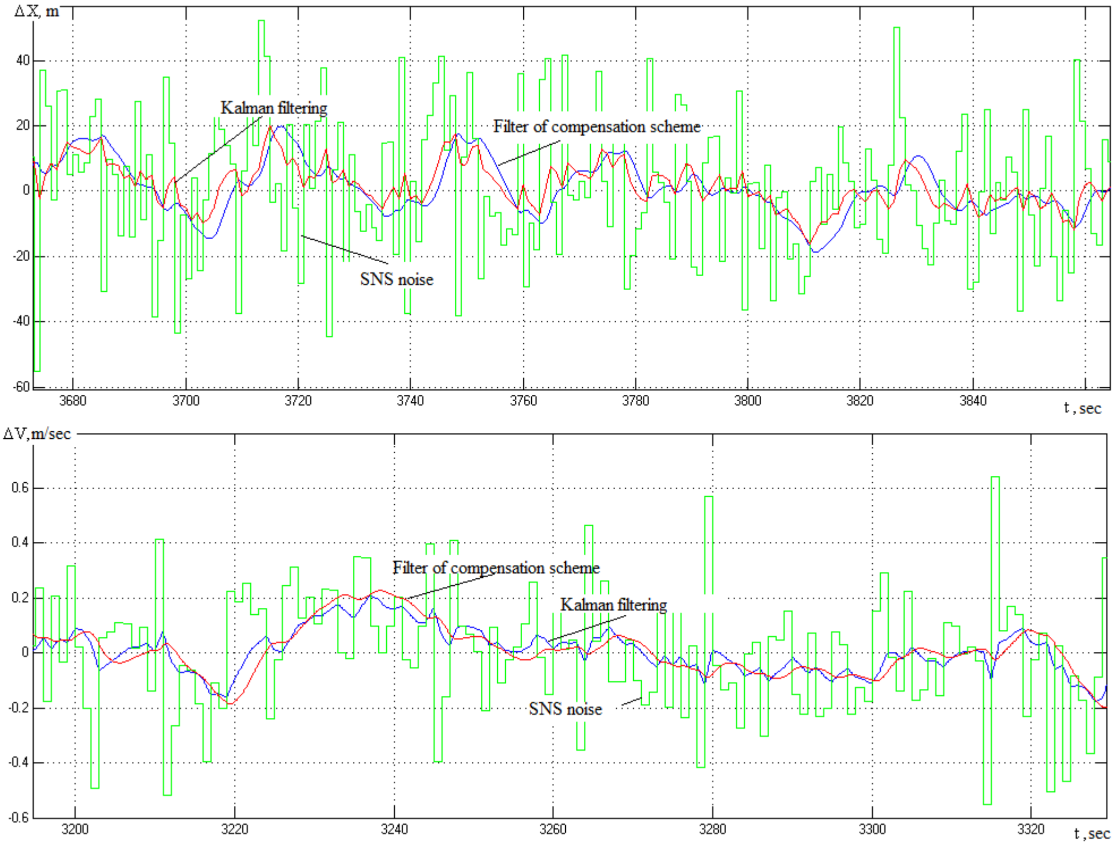

Fig. 3. Estimation errors in coordinate and velocity component by two data fusion algorithms with SNS noise

Data fusion algorithms were simulated for middle accurate SINS with systematic errors caused by gyroscope drifts and accelerometer inaccuracy ($\Delta X \approx$ 70 km per hour of flight, ΔV reaches 45 m/sec). Parameters of white noise components of SNS errors were the same as used for synthesis of filtering algorithms.

Analyses of simulation results proves the identity of two data fusion schemes though the presence of Butterworth filter of the second order allows providing the more effective smoothing of noise components of SNS errors. Especially this phenomena is observed in the speed channel.

One of the most important characteristic of any data fusion scheme is its convergence and convergence time. And these characteristics naturally depend on the time constant of filter. Obviously, there must be the reasonable compromise between the filtering properties of filter and settling time of estimate.

Transient processes of removal of initial errors in dead reckoning of coordinates in the compensation scheme with filter of variable structure and stationary Kalman filter are shown in Fig. 4.

In the moment of updating the information from SNS (beginning of compensation scheme operation) the aperiodic filter with small time constant T_{F1} is used which provides the minimal time of minimizing the error of compensation scheme to the level of SNS noises. Naturally, the filtering properties of such scheme are rather low, therefore the aperiodic filter is further replaced by the third order filter with time constant to be increased successively to the value of time constant T_F of stationary filter .

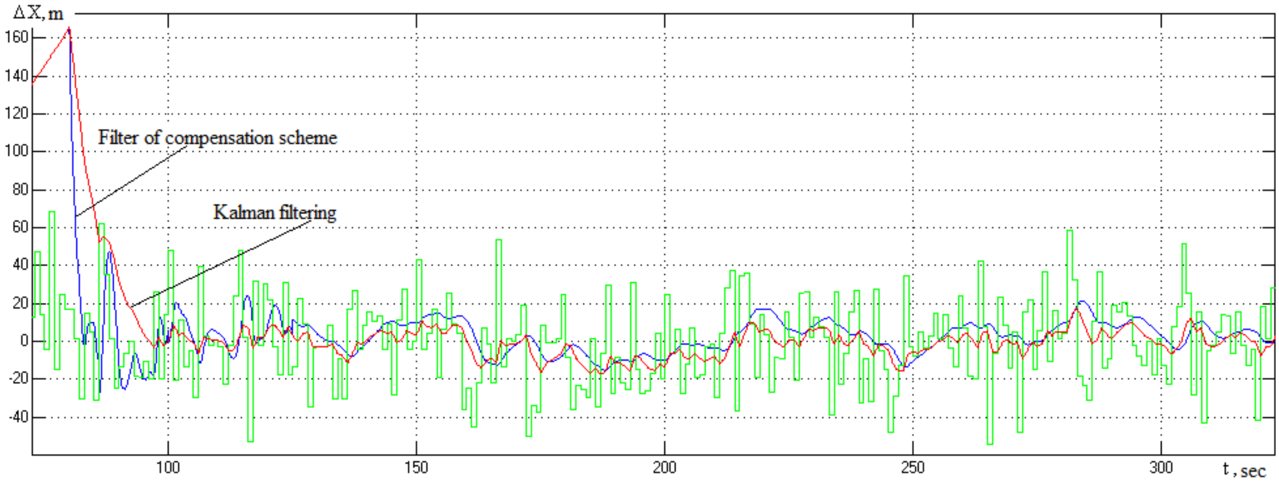

Fig. 4. Transient processes of removal of initial errors in dead reckoning of coordinates in the compensation scheme with filter of variable structure and stationary Kalman filter

The same approach can be realized in the algorithms of Kalman filtering by discrete change of filter coefficients K_{F_1}, K_{F_2} in the algorithm of reduced filter.

Data fusion algorithms were also simulated to very high coarse SINS with systematic errors caused by gyroscope drifts and accelerometer inaccuracy ($\Delta X \approx$ 25 km per 10 minutes of flight, ΔV reaches 600 m/sec.

The simulation results prove that the quality filtering is kept in the speed channel as before (estimation of SINS errors with comparison to highly accurate SNS). However, in the coordinate channel the corruption of estimate is observed in the compensation scheme (Fig. 5) though the estimate error does not exceed the error of SNS.

This phenomena is explained by the fact that for the dynamic filter (1) the transfer function of low-frequency filter (2) with large values of time constant T takes the form:

$$[1-F(p)] = \frac{T^3 p^3}{T^3 p^3 + 3T^2 p^2 + 3Tp + 1}.$$

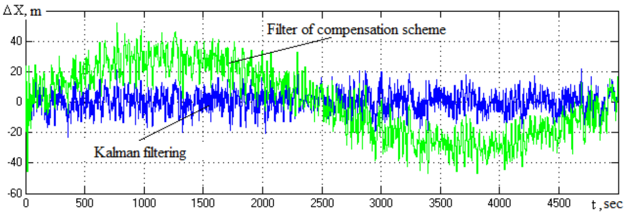

Fig. 5. Corruption of estimate in coordinate channel in the compensation scheme

Such low-frequency filter provides the third order astaticism and also does not pass the constant component of SINS error together with errors which vary by laws of the first and second orders. But in errors of coarse SINS there are also components which vary by laws of the third order. And this causes the corruption of errors in the compensation scheme.

To improve the quality of estimation procedure for ΔX it is recommended to use again Kalman filtering with aperiodic filter in compensation scheme, and the compensation of filter sluggishness can be done by component $\Delta\hat{V}$ from the channel of velocity error estimation.

The structure of dynamic filter $F_1(p)$ in the channel of coordinate error estimation in this case will be the following (Fig. 6).

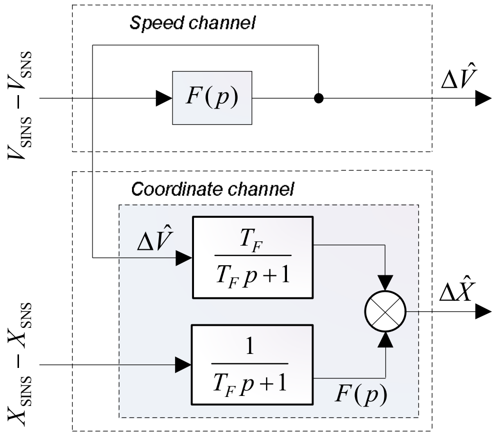

Fig. 6. Structure of dynamic filter $F_1(p)$ in the channel of coordinate error estimation

But if in the compensation scheme there is Butterworth filter of the second order then the structure of dynamic filter $F_2(p)$ and the sluggishness compensation in the channel of coordinate error estimation will be different (Fig. 7).

The comparative analyses of this compensation scheme with reduced Kalman filter was done by the mathematical simulation and proved (Fig. 8) their total identity.

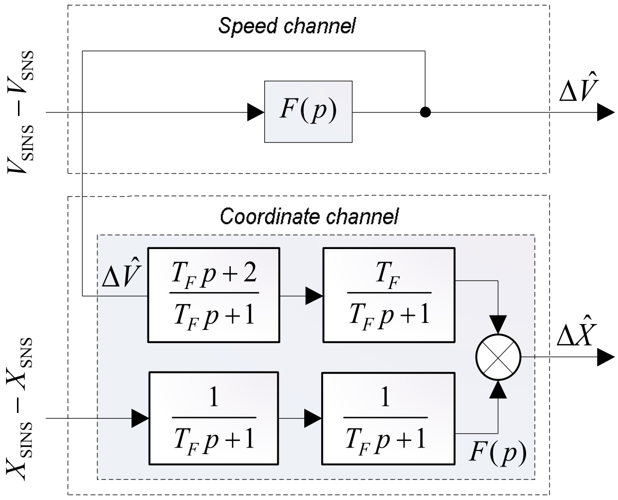

Fig. 7. Butterworth filter of the second order with variable structure

The simulation results which show the coordinate estimation errors for the different variants of integration in comparison with SNS are represented with increased zoom in Fig. 8.

The comparative analyses of simulation results shows that the accuracy of coordinate error estimation of the proposed scheme with dynamic filter $F_2(p)$ (based on Butterworth filter) is almost the same as the accuracy of optimal Kalman filter, and simultaneously the quality of filtering of SNS noise components is higher.

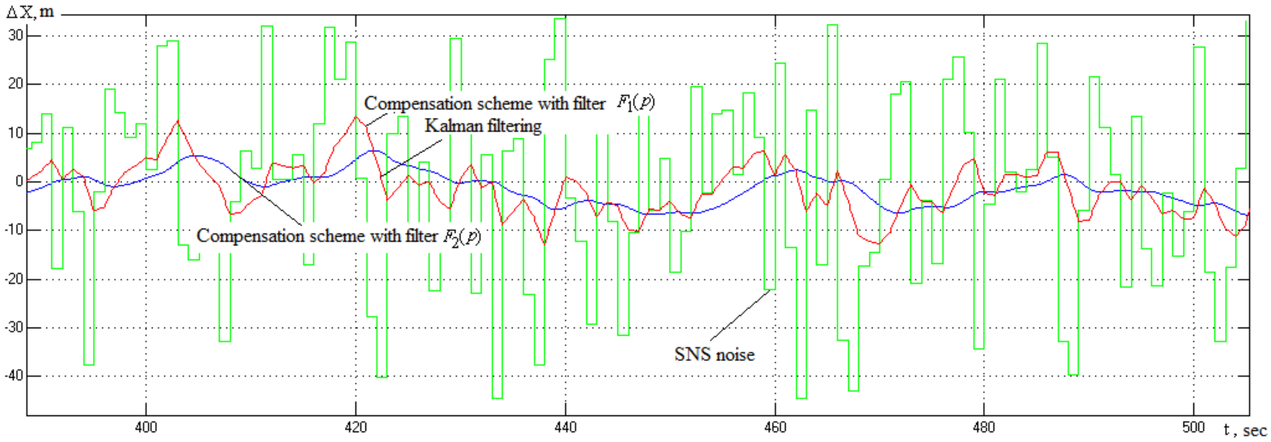

Fig. 8. The simulation results of coordinate estimation errors for the different variants of integration in comparison with SNS

The research of navigation information estimation was done for different aircraft maneuvers. The zoom maneuver was simulated (normal overload was not greater than 5 m/sec^2, longitudinal acceleration is 0,8 m/sec^2, pitch angle varies in the range + 15 ... – 8°), and also simulation was done for highly energetic maneuver (not used for civil aircraft) – normal overload exceeds 2,5 g, and pitch angle can reach 90°.

The simulation results illustrate the errors of coordinate and velocity estimation by two algorithms for coarse SINS and are represented in Fig. 9.

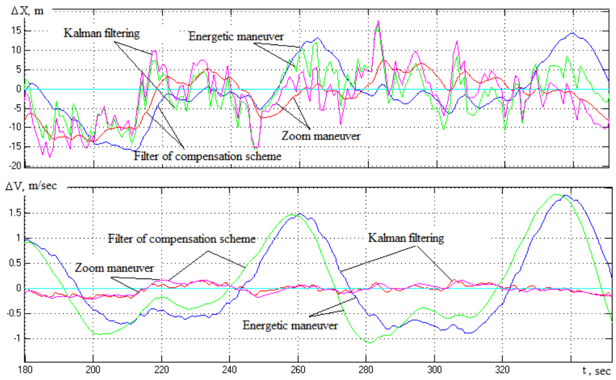

Fig. 9. The simulation results of errors in coordinate and velocity estimation by two algorithms for coarse SINS

Analysis of simulation results shows that with the energetic maneuvers in estimates of navigation parameters of coarse SINS both algorithms have dynamic errors, which is most evident in the channel of velocity estimation. With performing zoom maneuvers this effect is practically not manifested.

The same researches were done for middle accurate SINS (Fig. 10).

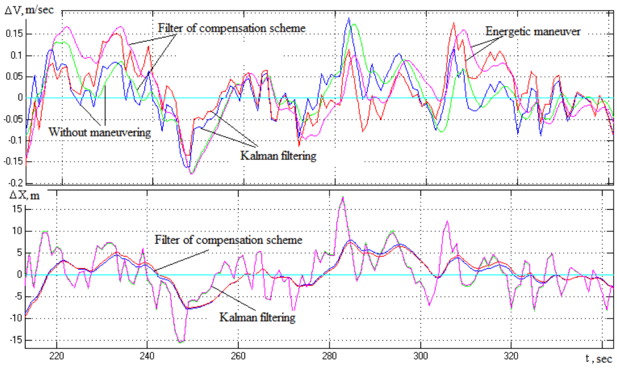

Fig. 10. The simulation results of errors in coordinate and velocity estimation by two algorithms for middle accurate SINS

Analysis of simulation results shows that in the coordinate channel there is no difference at all between the estimates of navigation parameters with flight without maneuvering and by energetic maneuvering. In the velocity channel this difference exists but it is negligible small.

IV. CONCLUSIONS

The comparative analyses of data fusion schemes of aided SINS and SNS shows that the accuracy of navigation parameters estimation of compensation scheme with proposed dynamic filters is not inferior to algorithms of Kalman filtering however the quality of filtering of SNS noise component is even higher. The significant drawback of compensation scheme to Kalman filtering is impossibility to estimate the non-observed components of state vector, therefore here it is necessary to have alternative methods of estimation of angular orientation parameters.

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М. К. Філяшкін, М. П. Мухіна. Схеми комплексування в інерціально-супутникових системах навігації

Проведено порівняльний аналіз різних схем комплексування навігаційних складових вектора стану в інерціально-супутникових системах, побудованих за методом компенсації або на основі калманівської фільтрації, досліджено похибки оцінювання координат і складових вектора швидкості двома алгоритмами комплексування.

Ключові слова: комплексування; схема компенсации; оптимальна фільтрація; оцінювання похибок.

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Напрям наукової діяльності: кореляційно-екстремальна навігація, нелінійне оцінювання, гаусовський багатоточковий фільтр.
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Напрям наукової діяльності: кореляційно-екстремальна навігація, нелінійне оцінювання, гаусовський багатоточковий фільтр.
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Н. К. Филяшкин, М. П. Мухина. Схемы комплексирования в инерциально-спутниковых системах навигации

Проведен сравнительный анализ различных схем комплексирования навигационных составляющих вектора состояния в инерциально-спутниковых системах, построенных по методу компенсации или на основе калмановской фильтрации, исследованы погрешности оценивания координат и составляющих вектора скорости двумя алгоритмами комплексирования.

Ключевые слова: комплексирование; схема компенсации; оптимальная фильтрация; оценивания погрешностей.

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