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PECULIARITIES OF MATCHED FILTERS SYNTHESIS

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Abstract. *The peculiarities of analogue matched filters synthesis are considered in the paper. The realization of matched filter on phase contours with creation of necessary delay change law is presented. Also recommendations on justified choice of the filter parameters are given*

Keywords: nonlinear frequency modulation; matched filter; phase contour; *Q*-factor; bandwidth.

I. INTRODUCTION

Usage of complex signal and its optimal processing with the help of matched filter makes it possible to obtain high resolution in case of signal with long duration. Creation of matched filter is rather complicated problem. That is why it is necessary to determine possibility of filter manufacturing, its complexity, and requirements for its parameters selection. Such procedures are considered in the paper.

II. ANALYSIS OF RESEARCH AND PUBLICATIONS ON THE SUBJECT

The main peculiarities of matched filters are described in the work [1]. Here it is possible to find theoretical information about matched filters and connections between parameters of a filter and signal. Possible ways of designing filters with any dependence of time delay on frequency are considered in the work [2]. But the materials, mentioned above, cannot help to make rational choice of the filter parameters for particular sample. That is why it is necessary to consider connection between parameters of filter and signal in order to provide minimum number of the filter elements and stability of its characteristics. These questions are scrutinized in the article.

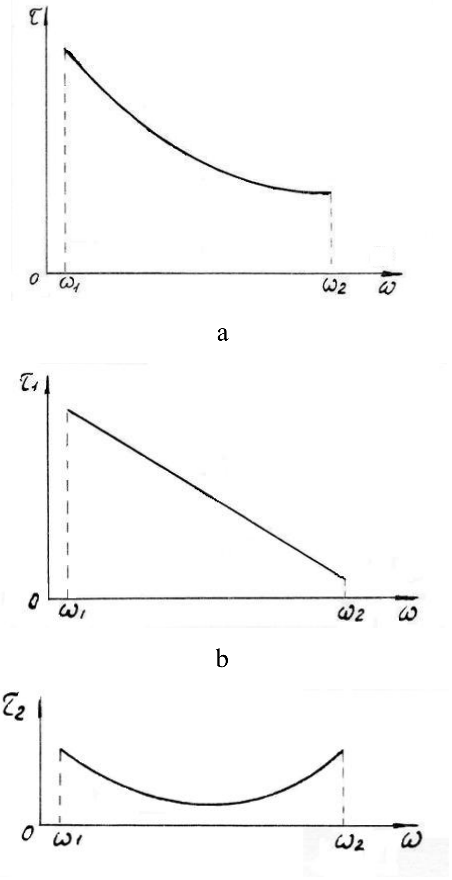
III. AIM

The aim of the paper is investigation of possibility for a matched filter implementation in case of long-term signals and appropriate requirements to the filter parameters and to reasonable choice of its elements.

IV. DEVELOPMENT OF ANALOG MATCHED FILTER AND ITS PARAMETERS SELECTION

There are features of analog matched filters synthesis, in the low frequency area in particular, considered in the paper. Filters, matched to signal with linear frequency modulation (LFM) are important to discuss. Filters for signals with non-linear frequency modulation can be obtained from the mentioned one by adding non-linear delay unit (Fig. 1). In this case the part of linear component of the delay in real circumstances turns out to be significant (over 90 %). Because of that the filter, matched with LFM-signal, is to be the main part of any matched filter.

With the help of set of non-linear delay units it is possible to obtain matched filter for signals with different modulation laws.



a

b

c

Fig. 1. Frequency responses of delay time for different filters: (a) filter for signal with non-linear frequency modulation; (b) filter for signal with linear frequency modulation (linear component of (a) filter); (c) filter with non-linear dependence of the delay on frequency (non-linear component of (a) filter)

The one of rational ways to create analog matched filters is their implementation on phase contours with forming the required delay change law. Transfer function of second order phase contour can be described as follows:

$$H(S)=\frac{S^2-b_1S+b_0}{S^2+b_1S+b_0},$$

where $S=\sigma+j\omega$, and coefficients of polynomials are connected with their roots by the following expressions:

$$b_1=2\sigma_i;\quad b_0=\sigma_i^2+\omega_i^2. \tag{1}$$

In formula (1) σ_i and ω_i are absolute values of real and imaginary parts of polynomial roots.

Magnitude frequency response (MFR) of phase contour doesn't depend on frequency, and frequency responses of phase and delay time can be described by the following formulae:

$$\varphi(\omega)=2\mathrm{arctg}\left(-\frac{\omega b_1}{b_0-\omega^2}\right),$$

$$\tau(\omega)=\frac{2\sigma}{\sigma_i^2+(\omega-\omega_i)^2}.$$

The responses, mentioned above, are illustrated in Figs 2 and 3.

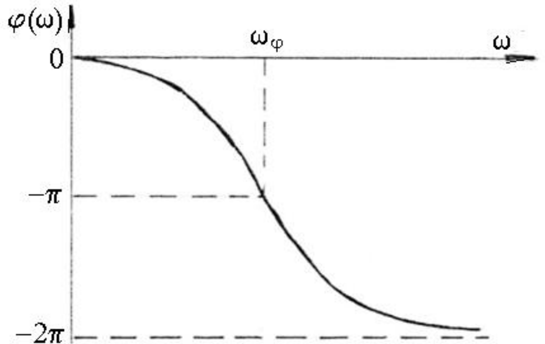


Fig. 2. Phase frequency response of the second order phase contour

Phase shift, caused by the circuit on the frequency $\omega_\varphi=\sqrt{b_0}$, is equal to ($-\pi$). Delay time curve is symmetrical in relation to frequency ω_i, on which it has its maximum:

$$\tau_{\max}=\frac{2}{\sigma_i}.$$

At frequencies $\omega_{1,2}=\omega_i\pm\sigma_i$ the delay is equal to

$$\tau_{1,2}=\frac{1}{\sigma_i}=0.5\tau_{\max},$$

that is decrease of σ_i leads to increase of $\tau_{\max}$ and sharpens the curve of delay.

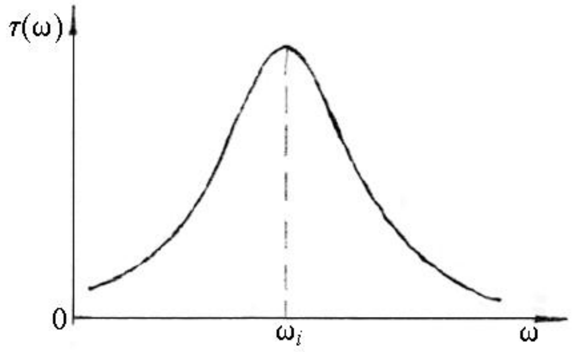


Fig. 3. Frequency response of delay time of the second order phase contour

The value, which is described by the following formula

$$Q_n=\frac{\sqrt{b_0}}{b_1}=0.5\sqrt{1+\frac{\omega_i^2}{\sigma_i^2}}$$

is a pole *Q*-factor. The pole *Q*-factor is one of the most important parameters in matched filters design because rise of pole *Q*-factor leads to increase of requirements to stability and quality factor of the circuit elements. That is why it is reasonable to provide minimum of the pole *Q*-factor at the design of matched filters.

Values of roots for transfer function polynomials of second order phase contour, which form a section of matched filter for LFM-signal, are given in the work [2]. The roots are defined for interval of normalized frequencies $1\le\Omega\le 2$ as follows

$$\lambda_{i_{1,2}}=-\hat{\sigma}_i\pm j\Omega_i.$$

To convert them into required frequency range it is necessary to use such ratio:

$$\nu_{i_{1,2}}=\hat{\sigma}_i(\omega_2-\omega_1)\pm j\left[\Omega_i(\omega_2-\omega_1)+2\omega_1-\omega_2\right] \\ =-\sigma_\mathrm{i}\pm j\omega_i \tag{2}$$

when decreasing delay during growth of frequency, and this statement

$$\nu_{i_{1,2}}=\hat{\sigma}_i(\omega_2-\omega_1)\pm j\left[\Omega_i(\omega_2-\omega_1)-2\omega_2+\omega_1\right] \\ =-\sigma_\mathrm{i}\pm j\omega_i \tag{3}$$

in case of the delay increase with rising frequency. In the mentioned expressions frequencies ω_1 and ω_2 are lower and higher bounds of operating frequency band (or bandpass) of the matched filter, which corresponds to frequency band of signal under processing.

Using statements (2) and (3) it is possible to obtain expressions for Q-factor

$$Q_n = 0.5\sqrt{1+\left(\frac{\frac{\omega_0}{\Delta\omega}-1.5+\Omega_i}{\hat{\sigma}_i}\right)^2} \quad (4)$$

at decrease of the delay, and

$$Q_n = 0.5\sqrt{1+\left(\frac{\frac{\omega_0}{\Delta\omega}+1.5-\Omega_i}{\hat{\sigma}_i}\right)^2} \quad (5)$$

in case of its increase. In the obtained formulate the frequency $\omega_0 = 0.5(\omega_1+\omega_2)$ represents central frequency of the filter bandpass; $\Delta\omega$ is the bandpass. At the assumption that $\omega_0 >> \Delta\omega$ expressions (4) and (5) can be represented as follows

$$Q_n \approx \frac{0.5\omega_0}{\hat{\sigma}_i\Delta\omega} = \frac{0.5 f_0}{\hat{\sigma}_i\Delta f} = \frac{0.5}{\hat{\sigma}_i\delta}.$$

From this it is obvious that at the prescribed frequency band $\Delta\omega$ of the signal the Q-factor grows with rising of central frequency of the filter bandpass (or with decrease of relative bandpass δ). On the assumption of this point of view it is reasonable to implement matched filter in the frequency range as lowest as possible. For this purpose heterodyning of the signal should be used.

In the range of lower frequencies (and higher values of δ accordingly) the value of Q-factor, which corresponds to drop-down law of delay change is less than the Q-factor, conforming to the rising one. In this case for phase circuit with $\hat{\sigma}_i = 0.1$ and $\Omega_i = 1$ at $\Delta f = 300$ Hz and $f_0 = 500$ Hz the Q-factors for drop-down and rising delay change laws are equal to 6 and 10 accordingly. This fact shows, that in the lower frequencies range it is worthwhile to built matched filters with decaying of time delay as frequency increases (which corresponds to LFM-signal with growing modulation law).

The dependence of pole Q-factor on the value of relative bandpass of matched filter at drop-down law of the delay change is shown in Fig. 4.

Each section of the matched filter provides processing of signal with frequency band $\Delta\omega$ and duration T_1, that is basis $D_1 = \frac{\Delta\omega T_1}{2\pi}$. To process a signal with duration T with the same frequency band $\Delta\omega$ it is necessary to connect N of such sections $\left(N = \frac{T}{T_1}\right)$ in cascade.

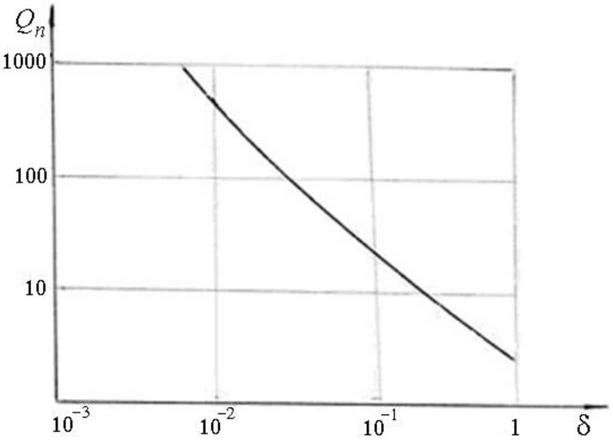


Fig. 4. Dependence of the pole Q-factor on the relative bandpass value of matched filter

When selecting sections of the matched filter it is important to take into account the following considerations. Increase of number of phase contours in a section is accompanied either by extension of D_1 or decrease of approximation error of the matched filter phase frequency response, which leads to certain contradictions between requirements for minimal complexity of the filter and permitted phase deflections. Analysis of the matched filter's sections shows that the requirements, mentioned above, are satisfied in the best way in the section with $\frac{D_1}{n} \approx 1$ (n is number of phase circuits in a section). These sections correspond with acceptable value of the pole Q-factor. That is why it is advisable to head for the sections with $\frac{D_1}{n} \approx 1$ at the matched filters design.

At the same time the total amount of phase circuits is approximately equal to the basis of signal. So, we can conclude that complexity of the matched filter (total amount of its elements) depends on the basis of signal.

However, higher stability of its parameters, better adjustability and more acceptable requirements to quality of its elements can be achieved only if the signal basis is obtained on wider signal frequency band, and this case corresponds to receiver with high resolution by distance.

V. CONCLUSIONS

Thus, considering all mentioned information it is possible to make reasonable choice of parameters for processing route of complex signal.

REFERENCES

[1] Gonorovskiy I. S. Radio circuits and signals. Moscow, Sovetskoe radio. 1997. 608 p. (in Russian).

[2] Trifonov I. I. Synthesis of reactive circuits with prescribed phase characteristics. Moscow, Sviaz'. 1984. 216 p. (in Russian).

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Т. Ю. Шкварницька. Особливості синтезу аналогових узгоджених фільтрів

Розглянуто особливості синтезу аналогових узгоджених фільтрів. Наведено реалізацію узгодженого фільтра на фазових контурах з формуванням необхідного закону зміни затримки. Подано рекомендації відносно доцільного вибору параметрів фільтра.

Ключові слова: нелінійна частотна модуляція; узгоджений фільтр; фазові контури; добротність полюса; смуга пропускання.

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Освіта: Київський політехнічний інститут, Київ, Україна (1994).
Напрям наукової діяльності: теорія і методи обробки сигналів.
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Т. Ю. Шкварницкая. Особенности синтеза аналоговых согласованных фильтров

Рассмотрены особенности синтеза аналоговых согласованных фильтров. Приведена реализация согласованного фильтра на фазовых контурах с формированием необходимого закона изменения задержки. Даны рекомендации относительно рационального выбора параметров фильтра.

Ключевые слова: нелинейная частотная модуляция; согласованный фильтр; фазовые контуры; добротность полюса; полоса пропускания.

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Образование: Киевский политехнический институт, Киев, Украина (1994).
Направление научной деятельности: теория и методы обработки сигналов.
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