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THE OPTIMAL CONTROL MODELS OF INTERURBAN BUS TRANSPORT

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Abstract—*The models for calculating the number of buses of different types depending on the variable passenger time and a specified interval of vehicles, minimizing the expenses of motor company which is considered to be optimal for the carrier and the passenger range of buses on the route. Analysing the influence of the speed of vehicles on the total costs of motor transport enterprises. The proposed model can be used to make effective management decisions regarding the organization of the rolling stock for enterprises passenger transport.*

Index Terms—Mathematical programming; optimization; cost; motor company; buses; intercity transportation; optimal plan.

I. INTRODUCTION. STATEMENT OF THE PROBLEM

For motor transport enterprises (MTE) which are engaged in passenger transportation, the main tasks are to provide quality competitive services and minimize costs for passenger services. Due to the recent rising prices for fuel and lubricants (FL), there is a need in the economy and an efficient use of resources MTE. The objective is to develop mathematical models optimization of rolling stock of different types depending on traffic flow changes in time to minimize maintenance costs MTE vehicles.

II. THE BASIC PRINCIPLES OF OPTIMIZATION OF MTE

The theory of the organization of work of the rolling stock with the use of mathematical modelling has been considered in [1]–[4]. Exploration companies optimize passenger transport and management of rolling stock regarding fluctuations in passenger traffic studied in [5]–[7]. To optimize MTE used multicriteria (vector) methods (e.g., principal criterion for Pareto optimum, methods of successive concessions). Optimization of MTE may be aimed at maximizing profits, the volume of passengers, profitability, productivity, and minimizing the cost, payback period and expenditure of resources.

Motor transport enterprises competitiveness depends on its level of profitability and quality of passenger service. Optimization of MTE is, on the one hand, to meet the demand of consumers of transport services, in reducing the waiting time in the queue (ride comfort for passengers), and the other - in reducing the cost of passenger traffic on the route. In previous studies, the authors found the optimal range of movement of vehicles (MV) on the route for the ith hour of MTE for one type of bus capacity (N):

$$\tau_i = \sqrt[3]{\frac{2NS^2}{F_i^2 H}}, \qquad (1)$$

where N is the capacity of the vehicle; S is the net cost of transport 1 passenger on the route for one hour; F_i is the average passenger flow for the ith hour [8]; H is average cost of cost for one hour the inhabitants of the region. Model (1) allows you to get the best value traffic τ, movement of vehicles (MV) with a mean of passenger traffic F_c on the route.

III. ROLLING STOCK MANAGEMENT MODEL MTE

To satisfy the passenger at the same time to avoid loss of the enterprise, the carrier must make rational use of the rolling stock of different sizes depending on the amplitude-frequency characteristics of passenger transport demand. Consider the problem of a rational choice of four types of rolling stock for 1 hour of work for a private company to transport passengers “Vladis” (city Chernihiv), which serves passengers on the routes “Kyiv-Chernihiv”, “Kyiv-Chernihiv”.

It is known that the costs of any enterprise are divided into constant that cannot “undo” (e.g., land rent, taxes, etc.) and variables related to the production of goods and services. In general, the profit MTE “*P*” can be expressed as the difference between revenues and costs of two types:

$$P = D - V_1 - V_2, \qquad (2)$$

where D is the value of the income associated with the volume of passenger traffic and the cost of tickets; V_1 is vehicle maintenance costs (including salary drivers) that depend on the choice of the structure of the rolling stock; V_2 is production costs (administration, production facilities, social contributions, etc.), which do not depend on the types of buses running on route. Then the task of maximizing profits ($P \to \max$) in the model (2) is reduced to the problem of minimizing expenses (V_1) for maintenance of the rolling stock, which can be represented as:

$$V_1 = \sum_{j=1}^{4}\left(\left(\left(\frac{s_0}{\tau_0}\right)\gamma_j + z\right)K_{1j}\right) \to \min. \qquad (3)$$

and restrictions on the number F_1 of transported passengers per hour and the total number of seats in buses:

$$F_1 \le \sum_{j=1}^{4}\left(K_{1j}N_j\right),$$

as well as the average time interval $\overline{\tau}_1$ of the MV, which shall not be greater than the specified τ_1^+ :

$$\overline{\tau}_1 = \frac{1}{K_1} \le \tau_1^+, ,$$

where γ_j is the direct material costs (UAH/km); z is the drivers salary (UAH/hour) K_{1j} and N_j ($j = 1, 2, 3, 4$) are accordingly, the number of MV and the number of vehicle locations therein for 1 hour; $K_1 = \sum_{j=1}^{4}\left(K_{1j}\right)$ is the total MV; τ_0 and s_0 are respectively MV (hours) and the length of the route in miles.

Entering symbols $a_j = \frac{s_0}{\tau_0} + z,\ x_j = K_{1j},\ c_j = N_j$ the task can formulate as a linear programming problem with the target function:

$$V_1 = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \to \min \qquad (4)$$

the limitations:

$$\begin{cases} c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \ge F_1; \\ x_1 + x_2 + x_3 + x_4 \ge \frac{1}{\tau_1^+}; \\ 0 \le x_j \le \mu_j, \quad j = 1, 2, 3, 4, \end{cases} \qquad (5)$$

where μ_j is restriction on the availability of the relevant type of vehicle.

The average fill factor of MV $\overline{\varepsilon}_1$ defines ratios:

$$F_1 = \sum_{j=1}^{4}\left(\varepsilon_{1j}K_{1j}N_j\right);$$

$$\overline{\varepsilon}_1 = \frac{F_1}{\sum_{j=1}^{4}\left(K_{1j}N_j\right)}, \quad (0 < \overline{\varepsilon}_1 \le 1),$$

where ε_{1j} is fill factor for jth MV.

The simulation results have shown an adequate assessment to support decision-making, which is confirmed by the actual performance of the rolling stock of the MTE in Chernigov. According to the study, the authors of the proposed model, when using an optimization cost MTE decreases ranging from 2 to 15 %, depending on the interval of MV. The results of modelling the structure of the rolling stock in comparison to the actual choice of the carrier have shown that when using the proposed models, for example, for an interval of 20 minutes, costs reduced by 12.5 %, for 15 minutes, at 12 % for 5 minutes at 9.7 %.

Using the features “Solver” in MS Excel, we obtain the optimal production plan on the route for the four types of buses for a given passenger flow F_1 and set interval movement of the vehicle τ_1^+. Model (4-5) allows the carrier to optimize efficient use of buses of different capacity and at the same time take into consideration the interests of passengers due to the restriction on the optimal time interval (1).

V. INFLUENCE OF TECHNOLOGICAL FEATURES OF ROLLING STOCK IN THE OPTIMAL MANAGEMENT OF MTE

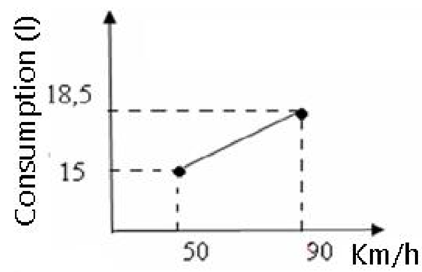
The different types of rolling stock have different economic and technical characteristics. For example, table 1 shows the cost of fuel at the rate of four types of rolling stock from 60 km/h to 90 km/h.

TABLE 1

Fuel costs depending on the speed of the bus four types

Types of bus	“Etalon” (30 seats)		“Bogdan” (19 seats)		“Mercedes” (18 seats)		“Volkswa-gen” (19 seats)	
	Speed, km/h							
	60	90	60	90	60	90	60	90
The cost of fuel, l	15	18,5	15	16,5	14	11,5	13	11

Due to the rising cost of petrol, oil and lubricants need to find not only the best types of vehicle on the route, but the speed of their movement. The dependence of fuel consumption from the speed of MV could be close to linear (Figure).



Linear approximation of fuel consumption on the speed of the bus “Etalon” (30 seats)

We get a system of equations with two unknowns for each type of bus stop:

$$\begin{cases} \alpha_1 + \beta_1 \cdot 60 = 15; \\ \alpha_1 + \beta_1 \cdot 90 = 18.5, \end{cases} \Rightarrow \quad \alpha_1 = 8, \quad \beta_1 = 117;$$

$$\begin{cases} \alpha_2 + \beta_2 \cdot 60 = 15; \\ \alpha_2 + \beta_2 \cdot 90 = 16.5, \end{cases} \Rightarrow \alpha_2 = 12, \quad \beta_2 = 0.05;$$

$$\begin{cases} \alpha_3 + \beta_3 \cdot 60 = 14; \\ \alpha_3 + \beta_3 \cdot 90 = 11.5, \end{cases} \Rightarrow \alpha_3 = 19, \quad \beta_3 = -0.083;$$

$$\begin{cases} \alpha_4 + \beta_4 \cdot 60 = 13; \\ \alpha_4 + \beta_4 \cdot 90 = 11, \end{cases} \Rightarrow \alpha_4 = 17, \quad \beta_4 = -0.67.$$

Then the cost of fuel at 1 km r_1, r_2, r_3, r_4 depending on the speed of movement for each of the four types of buses "Bogdan", "Etalon", "Mercedes", "Volkswagen" will be:

$$r_1 = 0{,}08 + 0{,}001 \cdot v_1;$$
$$r_2 = 0{,}12 + 0{,}0005 \cdot v_2;$$
$$r_3 = 0{,}19 - 0{,}0001 \cdot v_3;$$
$$r_4 = 0{,}17 - 0{,}0001 \cdot v_4.$$

Taking into account other direct material expenses on maintenance of the rolling stock of each of the four types of buses on 1 km (replacement of oils, tires and the like), we obtain the following dependencies:

$$\gamma_1 = 1{,}3 + 0{,}0175 \cdot v_1;$$
$$\gamma_2 = 1{,}9 + 0{,}0075 \cdot v_2;$$
$$\gamma_3 = 2{,}95 - 0{,}0125 \cdot v_3;$$
$$\gamma_4 = 2{,}65 - 0{,}01 \cdot v_4,$$

where $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are the total cost of ownership of the "Etalon" buses "Bogdan", "Mercedes", "Volkswagen" for 1 km.

Let us denote the desired speed for the four types of buses through x_5, x_7. Then the model to minimize the cost of maintenance of the rolling stock (3) can be thought of as a problem of nonlinear programming in the form of:

$$V_1 = s_0\Bigg(\left(1{,}3 + 0{,}0175 \cdot x_5 + \frac{z}{x_5}\right)x_1 + \left(1{,}9 + 0{,}0075 \cdot x_6 + \frac{z}{x_6}\right)x_2 + \left(2{,}95 - 0{,}0125 \cdot x_7 + \frac{z}{x_7}\right)x_3 + \left(2{,}65 - 0{,}01 \cdot x_8 + \frac{z}{x_8}\right)x_4\Bigg) \to \min, \quad (6)$$

the limitations

$$\begin{cases} c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \geq F_1; \\ x_1 + x_2 + x_3 + x_4 \geq \dfrac{1}{\tau_1^+}; \\ 0 \leq x_j \leq \mu_j, \ j = 1, 2, 3, 4; \\ 60 \leq x_j \leq 90, \ j = 5, 6, 7, 8, \end{cases} \quad (7)$$

where $\mu_1 = 18, \ \mu_2 = 3, \ \mu_3 = 23, \ \mu_4 = 6.$

In the model (6) $s_0 \dfrac{z}{x_5}$ displays the driver's salary from time to time, for any driver performs the carriage of passengers. For MTE "Vladis, salary of driver for decorated flight is 100 UAH. And therefore does not depend on the speed of the bus on the route. Then the cost of MTE "Vladis" for servicing rolling stock would be:

$$V_1 = (150(1.3 + 0.0175x_5) + 100)x_1 + (150(1.9 + 0.0075x_6) + 100)x_2 + (150(2.95 - 0.0125x_7) + 100)x_3 + (150(2.65 - 0.01x_8) + 100)x_4) \to \min, \quad (8)$$

at the constraints (7).

Tables 2, 3 shows the optimal decoupling linear programming problem (4), (5), which corresponds to Plan 2 and nonlinear programming (6), (7), corresponds to plan 3 under different restrictions on passenger traffic, the interval of motion, the speed of their vehicle movement.

TABLE 2

The optimal speed for "Etalon" buses "Bogdan", "Mercedes", "Volkswagen"

Speed, km/h	v_1	v_2	v_3	v_4
Condition 1	60	-	90	90
Condition 2	60	-	-	90
Condition 3	60	-	-	90
Condition 4	60	-	-	90

TABLE 3

Optimal structure of rolling stock plans for the route "Chernihiv-Kyiv"

V_1 UAH / h	"Volkswagen" (19 seats)	"Mercedes" (18 seats)	"Bogdan" (19 seats)	"Etalon" (30 seats)	Optimal structure of the necessary rolling stock	Number of transported passengers per hour and optimal range of movement of vehicles
3235	**1**	**6**	**3**	**4**	Plan 1	Condition 1
2765,63	**6**	**1**	**0**	**5**	Plan 2	F = 282 pass./h
2409,79	**5**	**0**	**1**	**6**	Plan 3	$\tau_1^+ = 0.08$ h
1820	**0**	**0**	**1**	**6**	Plan 1	Condition 2
1797,5	**1**	**0**	**0**	**6**	Plan 2	F = 198 pass./h
1605,42	**6**	**0**	**0**	**1**	Plan 3	$\tau_1^+ = 0.16$ h
1136,88	**1**	**1**	**2**	**1**	Plan 1	Condition 3
1000,63	**1**	**0**	**0**	**3**	Plan 2	F = 102 pass./h
889,17	**3**	**0**	**0**	**1**	Plan 3	$\tau_1^+ = 0.25$ h
684,38	**1**	**1**	**0**	**1**	Plan 1	Condition 4
673,13	**2**	**0**	**0**	**1**	Plan 2	F = 66 pass./h
584,58	**1**	**0**	**0**	**2**	Plan 3	$\tau_1^+ = 0.5$ h

In Table 4 percent decrease is designed for linear (4), (5) and non-linear (6), (7) model. Plan 4 corresponds to the model (8). The table shows that the optimum speed reduces cost of MTE for servicing rolling stock approximately 10–15 %. It is essentially a constant growth in fuel prices.

TABLE 4

Comparison of the effectiveness of the proposed models for the carrier's actual costs in percentage terms

Plans / Conditions	Plan 2	Plan 3	Plan 4
Condition 1	14,51	25,51	31,45
Condition 2	1,24	11,79	15
Condition 3	11,98	21,79	24
Condition 4	1,64	14,58	14

As you can see, using the proposed model (3) MTE decrease costs in the range of 1.2 – 14.5 %, and the use of the model (6), which complements the model (3) restrictions on the rational speed - in the range of 11.8 – 25.5 %. Consequently, the model (6) allows the carrier by almost a quarter to reduce maintenance costs of rolling stock in MTE constantly rising prices for fuel and lubricants. According to the authors, the use of model (8) is optimal for the problem of optimal control intercity passenger services.

V. CONCLUSIONS

Mathematical model determine the optimal structure of the necessary rolling stock for ensure-demand for passenger transport, taking into account the conditions to minimize the cost of MTE. The model takes into account the economic interests of the carrier and the passenger that is expressed by the optimal interval movement of the vehicle, which was found by the authors in previous studies. The purpose of the follow-up studies of authors is to create software for the introduction of developed models in automated system management decision-making with respect to the rational choice of rolling stock MTE with statistical epicycles of passengers on long-distance routes in the region. Alsotasks, connected with creating it new lycoined MTE to transport passengers with optimum structure of the rolling stock in limiting the starting capital of the entrepreneur, as well as the challenge of planning for optimal acquisition of new vehicles by entering related financial restrictions, given their payback.

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М. Г. Медведєв, Л. М. Олещенко. Моделі оптимального управління міжміськими автобусними пасажирськими перевезеннями

Описано моделі для розрахунку кількості автобусів різних типів залежно від змінного пасажиропотоку в часі і заданого інтервалу руху транспортних засобів. У моделі мінімізації витрат автотранспортного підприємства враховується оптимальний для перевізника і пасажира інтервал руху автобусів на маршруті. Проаналізовано вплив швидкості руху транспортних засобів на загальні витрати автотранспортного підприємства. Запропоновані моделі можуть використовуватися для ухвалення ефективних управлінських рішень щодо організації роботи рухомого складу для підприємств пасажирського транспорту.

Ключові слова: математичне програмування; оптимізація; вартість; автотранспортне підприємство; автобуси; міжміський транспорт; оптимальний план.

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Н. Г. Медведев, Л. М. Олещенко. Модели оптимального управления междугородными автобусными пассажирскими перевозками

Описаны модели для расчета количества автобусов разных типов в зависимости от переменного пассажиропотока во времени и заданного интервала движения транспортных средств. В модели минимизации расходов ав-

тотранспортного предприятия учитывается оптимальный для перевозчика и пассажира интервал движения автобусов на маршруте. Проанализировано влияния скорости движения транспортных средств на общие издержки автотранспортного предприятия. Предложенные модели могут использоваться для принятия эффективных управленческих решений относительно организации работы подвижного состава для предприятий пассажирского транспорта.
Ключевые слова: математическое программирование; оптимизация; стоимость; автотранспортное предприятие; автобусы; междугородний транспорт; оптимальный план.

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