UDC 004.94; 004.032.26(045)

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BUILDING A SYSTEM OF SIMULATION MODELING FOR SPATIALLY-DISTRIBUTED PROCESSES

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Abstract*—*The author considered the problem of simulation modeling for solving problems of mathematical physics. To solve these problems the author proposes to use cellular neural networks as means of modeling. As algorithmic software and tool optimization of neural network it is proposed to use locally asynchronous methods that are specifically focused on cellular neural networks.

***Index Terms*—**Spatially distributed processes; simulation modeling; cellular neural networks; locally asynchronous algorithm.

I. INTRODUCTION

Today, in the active development of science and technology we can note a tendency to create a variety of complex high-tech systems. Elements of modern systems are the complex structures, the functioning of which is entrusted by the distinct physical effects. The development of such elements is virtually impossible without solving partial differential equations, which are usually differential equations (DE) in partial derivatives.

Finding of exact analytical solution, unfortunately, is only available for a very limited number of one-dimensional tasks when using a range of assumptions that adversely affects the adequacy of the results. To solve the problems of mathematical physics in the case of multiple measurements is required to use numerical methods to convert differential equations or their systems in systems of algebraic equations. To solve the obtained nonlinear systems of algebraic equations and linear systems of large dimension iterative methods are used. Thus one of the most difficult problems is to ensure the convergence of the iterative process that largely determines the computation time. The accuracy of the solution is determined by the step of the grid, the number of iterations and the digital net of the computer.

II. PROBLEMS OF MATHEMATICAL PHYSICS

Development and research of a large part of the elements of modern VLSI (very large scale integration) and MOEMS (micro-opto-electro-mechanical systems) associated with the solution of the so-called problems of mathematical physics, which include problems of heat conduction, diffusion, electrostatics and electrodynamics, the problem of flow of fluid, the density distribution of the electric current in the conducting medium, the problem of deformation of solids and many others.

Similar problems are described by differential equations in partial derivatives with additional equations expressing the boundary and initial conditions. The author discusses the DE in partial derivatives no higher than second order, since these equations cover a fairly wide range of physical phenomena underlying the elements VLSI and MOEMS and, furthermore, discussed below solution methods applicable to DE in partial derivatives of higher orders. In general, linear differential equations in partial derivatives of second order of n independent variables has the form

$$\sum_{\alpha,\beta=1}^{n} A_{\alpha\beta}(\mathbf{x})\frac{\partial^2 u}{\partial x_\alpha \partial x_\beta} + \sum_{\alpha=1}^{n} B_\alpha(\mathbf{x})\frac{\partial u}{\partial x_\alpha} + C(\mathbf{x})u = f(\mathbf{x}), \quad (1)$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]$ is vector (matrix row) of the independent variables; u is the required function of of the independent variables; $A_{\alpha\beta}(\mathbf{x})$, $B_\alpha(\mathbf{x})$, $C(\mathbf{x})$, $f(\mathbf{x})$ are some real-valued function of the independent variables.

Equation (1) can always be reduced to one of three standard canonical forms. For the ratio of values $A_{\alpha\beta}(\mathbf{x})$ equation belongs to elliptic, parabolic or hyperbolic at the point $\mathbf{x}$. In particular, the differential equations in partial derivatives of second order with two independent variables x, y, which can be represented as

$$A_{xx}(x,y)\frac{\partial^2 u}{\partial x^2} + A_{xy}(x,y)\frac{\partial^2 u}{\partial x \partial y} + A_{yy}(x,y)\frac{\partial^2 u}{\partial y^2} + B_x(x,y)\frac{\partial u}{\partial x} + B_y(x,y)\frac{\partial u}{\partial y} + C(x,y)u = f(x,y), \quad (2)$$

type the DE is determined by the sign expression, which is called the discriminate

$$D(x,y) = A_{xy}^2(x,y) - 4A_{xx}(x,y)A_{yy}(x,y). \quad (3)$$

If $D(x, y) < 0$, differential equation is elliptic at the point *(x, y)*.

If $D(x, y) = 0$, differential equation is parabolic at the point *(x, y)*.

If $D(x, y) > 0$, differential equation is hyperbolic at the point (*x, y*).

If the coefficients A_{xx}, A_{xy}, A_{yy} are constant and D does not depend on *x, y*, then depending on the sign of D equation is fully elliptic, hyperbolic or parabolic.

Here is a list of the most prominent problems in mathematical physics, leading to the solution of given above three types of equations.

Elliptic equations:

– laplace equation – used for stationary, ie not variable in time, physical processes in the case of a homogeneous medium and the lack of sources;

– poisson's equation – can describe the electrostatic field, a stationary temperature field, pressure field, field capacity and pressure in hydrodynamics.

Parabolic equation:

– heat equation – lots of non-stationary, ie variables in the time physical processes described by equations of parabolic type. For example, the non-stationary heat equation;

– continuity equation – these equations for electrons and holes are the equations of parabolic type, are very important for modeling the processes of charge transport in semiconductors.

Hyperbolic equations:

– wave equation – many physical processes associated with the occurrence of oscillations in a certain environment. For example, the string fluctuations, fluctuations in the membrane of sound vibrations and so on. They are described by the wave equation belonging to hyperbolic equations.

III. Analysis of the Methods

Unfortunately, the analytic solution of partial differential equations is possible only for a very limited number of tasks. In most cases, the solution of differential equations in partial derivatives is only possible by using numerical iterative methods [1].

The essence of these methods lies in the discretization of differential equations, ie the representation of all or part of the derivatives in the form of approximate expressions (finite difference or finite element) that allows you to convert the differential equation in a system of algebraic equations. For this considered area Θ is covered by the grid, and all variables are replaced by grid functions. In other words, the variables investigated for the entire infinite is set of points area Θ, and only for some finite subset G. And in solving nonstationary problems except grid, a grid of time is introduced.

The number of algebraic equations in the resulting system (dimension discrete problem) is determined by the product number of points of the grid on the number of the independent variables in the original differential equations.

The choice of method for solving the resulting system of algebraic equations is determined by the dimension and nature (linear or nonlinear). To solve systems of linear equations (SLE) Gaussian elimination method, the method of LU-decomposition, are widely used [4] For solving systems of nonlinear algebraic equations and linear systems of large dimensions iterative methods Jacobi, Seidel, Newton-Raphson and others are used. [4], [5]. Here is a list of the most popular discretization of differential equations:

– finite difference method – involves the digitization of differential equations for the so-called rectangular grid, that is, nets, elementary cells which are squares two dimensions or three dimensions parallelepipeds. (Finite-difference grid, mesh function difference and finite templates);

– finite element method – involves discretization of differential equations for the so-called triangular grid, ie nets, elementary cells which are the triangles for two dimensions or prism (tetrahedrons) in three dimensions. (Breakdown Dirichlet and Delaunay Triangulation, the method of integral identities).

However, many problems of mathematical physics, in particular boundary value problems can be described with sufficient accuracy using the set of local tasks, each of which describes the dynamics of the field at a fixed point. On use of this approach cellular neural networks are based, in which each neuron combined communication channels with a fixed number of neighboring neurons.

Cellular Neural Networks (CNN), first published in [6], [2], once widely accepted as a paradigm of artificial neural networks focused on asynchronous parallel computing process. Over the last years many algorithms for image processing, pattern recognition, assessment of the dynamics of mechanical systems, and others were invented that successfully use cellular neural networks. A common feature of all these algorithms can be considered a global presence properties decomposition problem for many local problems, each of which can be made of individual cells CNN.

Cellular Neural Network is a homogeneous computing structure of local bonds, which are located at the nodes processor elements (cells). The structure

links the cells may vary depending on the soluble CNN problems. Figure 1 shows examples of possible local connections in the two-dimensional CNN.

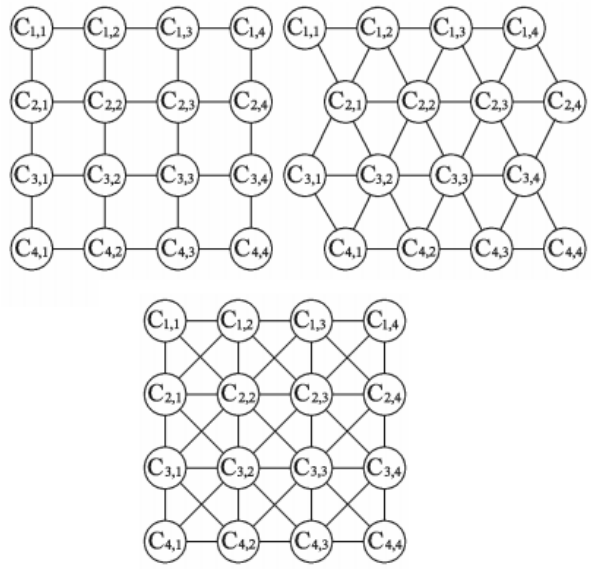

Fig. 1. Heterogeneous structure of the CNN

To determine the number of possible channels of communication with local implementation of parallel algorithms for CNN of each cell $C_{j,i}$ set the set of coordinates of neighboring cells $H_{j,i}$, capacity is called the index of neighborhood $\alpha_{j,i}$. Distinction is made between CNN of homogeneous and heterogeneous structure.

We will call homogeneous such structure dimension $M \times N$, where

$$\alpha_{i,j} = \text{const}; \; i = \overline{1,N}; \; j = \overline{1,M}.$$

Homogeneous structures can be obtained from heterogeneous CNN by addition of missing links. Homogeneous cellular networks reamers of tors characterized by constant index of environment for each cell.

The work of problem-oriented CNN can be described by the differential equation [3]:

$$\frac{dU_{i,j}}{dt} = -a_{i,j}U_{i,j} + \sum_{(k,l)\in H_{i,j}} w_{k,l} f\left(U_{k,l}\right) + I_{i,j}, \tag{4}$$

where $U_{i,j}$ is internal state of $C_{i,j}$; $a_{i,j}$ is weighting local feedback; $w_{k,l}$ is weighting communication $C_{i,j}$ with the cell $C_{k,l}$, included in set of coordinates of neighboring cells $H_{i,j}$; $f(U_{k,l})$ is function of activation of the cell $C_{k,l}$; $I_{i,j}$ is deflection for the cell $C_{i,j}$.

Dynamics of cells is described by the equation (4), can be adapted for solving partial differential equations. Let us show the implementation of this statement as an example of the heat equation, representing the boundary value problem

$$\frac{\partial U(x,t)}{\partial t} = a^2 \frac{\partial^2 U(x,t)}{\partial x^2}, \tag{5}$$

under the following initial and boundary conditions:

$$U(0,t) = 0, \quad U(l,t) = 0, \quad t > 0,$$
$$U(x,0) = f(x), \quad 0 \le x \le l.$$

Let us introduce step discretization in space h and sampling time step τ based on the conditions: $m = l/h$ – totsl; $x_i = ih$ – meaning x in x_i; $t_j = j\tau$ – meaning t in j.

Let us form difference method by presenting approximate partial derivatives in time and space by using the Taylor series:

$$\frac{\partial U(x_i,t_j)}{\partial t} = \frac{U(x_i,t_j) - U(x_i,t_{j-1})}{\tau} + \frac{\tau}{2}\frac{\partial^2 U(x_i,\sigma_j)}{\partial t^2}, \tag{6}$$

$$\frac{\partial^2 U(x_i,t_j)}{\partial x^2} = \frac{U(x_{i+1},t_j) - 2U(x_i,t_j) + U(x_{i-1},t_j)}{h^2} - \frac{h^2}{12}\frac{\partial^4 U(v_i,t_j)}{\partial x^4}, \tag{7}$$

where $\sigma_j \in (t_{j-1}, t_j)$, $v_i \in (x_{i-1}, x_{i+1})$.

Let us write equation (5), using finite differences with (6), (7) and replacing the continuous function $U(x,t)$ with the difference function r, defined only at the points of sampling and responding condition $U(x_i,t_j) = r_{i,j}$.

$$\frac{r_{i,j} - r_{i,j-1}}{\tau} - a^2 \frac{r_{i+1,j} - 2r_{i,j} + r_{i-1,j}}{h^2} = 0. \tag{8}$$

Local error (8) can be written as

$$\Delta_{ij} = -\frac{\tau}{2}\frac{\partial^2 U(x_i,\sigma_j)}{\partial t^2} - a^2 \frac{h^2}{12}\frac{\partial^4 U(v_i,t_j)}{\partial x^4}.$$

Entering a replacement $\lambda = \tau a^2/h^2$ and simplifying similar terms, the equation (8) represented as

$$r_{i,j-1} + \lambda r_{i+1,j} + \lambda r_{i-1,j} - (1+2\lambda) r_{i,j} = 0, \tag{9}$$

where $i = 1, 2..., m - 1$; $j = 1, 2...$.

Based on the initial and boundary conditions of the boundary value problem

$$r_{i,0} = f(x_i) \forall i = \overline{1, m-1}, \; r_{0,j} = r_{m,j} = 0 \forall j = 1, 2, \ldots. \tag{10}$$

Thus, the boundary value problem (5) with the error Δ_{ij} can be represented by difference boundary value problems, including a system of algebraic equations (9). If we compare each of the equations of the system with the corresponding cells of the neural network it is easy to see that unknown appearing in these equations can be represented by cells from a variety of neighborhood $H_{i,j}$. This fact shows the principal possibility to organize the computation of boundary value problems for homogeneous computing structures with local ties. However, an unambiguous answer to the question of the effectiveness of CNN for these purposes must also consider the order of interaction between cells in the computational process.

If you are using analog computing devices that provide algorithm, a process characterized by natural parallel computation of the entire system, since the interaction between cells is continuous in nature. Recently, to emulate the work of CNN are increasingly using computers and computer systems. This approach involves working with discrete data, which requires the determination of rules of interaction between objects that describe the cells.

Cellular neural networks are characterized by the absence of a single control center, so communication between neighboring cells is done locally, which corresponds to the asynchronous mode of functioning of discrete systems. To organize computing for solving boundary value problems (9) method for locally asynchronous computation is proposed [3], which ensures the convergence of the iterative process to solve in terms of asynchronous data exchange between the cells.

Iterative methods for solving boundary value problems for neural networks is a relatively young area of artificial ANN. The main obstacle to their use for a long time been the technological problems associated with high computational complexity in each neuron. Recent advances in technology have allowed high build neurocomputers that goes back thousands of neurons, each of which can provide complex calculations and a local memory. This led to the development of new computational methods, including iterative.

IV. Developed Software

Been developed software that allows you to solve the problem of simulation modeling spatially distributed processes using cellular neural networks using locally asynchronous methods. On Fig. 2 provided the structure of the software.

As can be seen from the scheme (Fig. 2) developed software can be divided into three parts:

– work locally asynchronous algorithm for parallelization of solution of problems of mathematical physics;

– implementation (structural and parametric synthesis) neural network to solve the problem of simulation modeling;

– graphical representation of the results of the neural network for user.

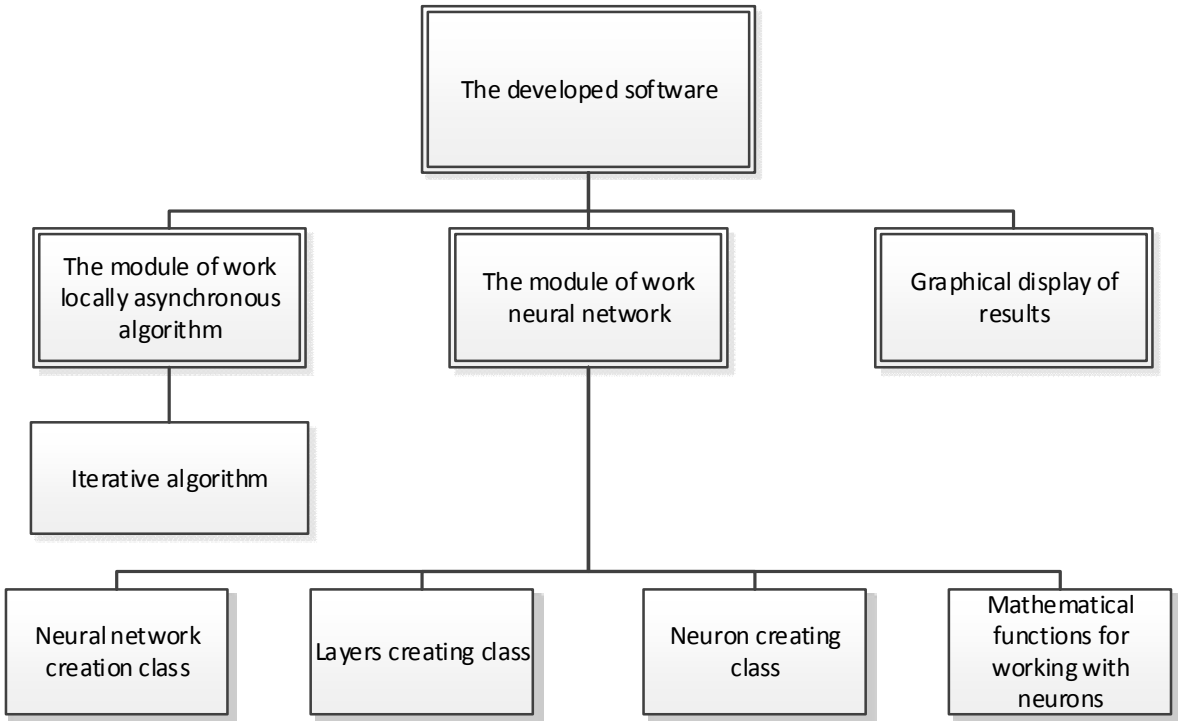

Fig. 2. Overall block diagram of the software

V. Conclusions

In this article we have shown the possibility of using cellular neural networks to implement simulation modeling of spatially distributed processes. As the algorithm software, locally asynchronous methods were used. This approach to solving the boundary value problem provides preliminary parallelization problem, and then solution by using cellular neural networks. This article describes the structure of software that was developed as part of the research.

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Received 19 February 2014.

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О. І. Чумаченко, А. Ю. Лужецький. Побудова системи імітаційного моделювання для просторово-розподілених процесів

Розглянуто задачу імітаційного моделювання для розв'язання задач математичної фізики. Для вирішення задачі запропоновано використовувати клітинні нейромережі, як засіб моделювання. Як алгоритмічне забезпечення та засіб оптимізації нейромережі запропоновано використовувати локально-асинхронні методи, які є спеціально орієнтованими на клітинні нейронні мережі.

Ключові слова: просторово-розподілені процеси; імітаційне моделювання; клітинні нейронні мережі; локально-асинхронний алгоритм.

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Е. И. Чумаченко, А. Ю. Лужецкий. Построение системы имитационного моделирования для пространственно-распределенных процессов

Рассмотрена задача имитационного моделирования для решения задач математической физики. Для решения задачи предлагается использовать клеточные нейросети, как средство моделирования. Как алгоритмическое обеспечение и средство оптимизации нейросети предлагается использовать локально-асинхронные методы, которые являются специально ориентированными на клеточные нейронные сети.
Ключевые слова: пространственно-распределенные процессы; имитационное моделирование; клеточные нейронные сети; локально-асинхронный алгоритм.

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