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DEVELOPMENT OF NON-GAUSSIAN NOISE DETECTION ALGORITHM AND INVESTIGATION OF ITS EFFECTIVENESS

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Abstract*—Considered a method of non-gaussian noise detection algorithm development and investigated its effectiveness.*

Index Terms—Ddetection algorithm; detection probability; effectiveness estimation criterion; non-gaussian noises detection; statistical moments.

I. INTRODUCTION

Limitation of frequency resource on the one side and increasing number of users on the other side, stresses a problem of electro-magnetic compatibility.

Broadcast saturation with telecommunication equipment radiation causes interferences to the radio equipment of flight work, in particular to satellite systems, radar systems, instrumental landing systems, etc.

For radar systems the most essential is impact of pulse noise – a sequence of pulses of arbitrary shape with random amplitude, duration, time of occurrence. This type of interferences, unlike normal noise, has significantly non-gaussian probability distribution.

Detection of this noise allows applying protection methods and reducing its impact on the radar performance characteristics.

II. ANALYSIS OF THE LAST RESEARCHES AND PUBLICATIONS

Analysis of modern researchers and publications showed that problem of difference determination between normal and other kind of distribution is investigated for various purposes.

In article [6] author uses statistical data processing for analysis of converted waves of earthquakes and uses comparison with normal distribution for making conclusions about degree of medium heterogeneity.

In [7] author uses non-gaussiality in independent component analysis method and considers it as a feature of independence of a process.

In [8] authors develop the idea that the modulation of stochastic carrier by the statistical parameters can be used in information transmission systems. In particular, they consider modulation method based on the statistical parameters change by using a special symmetric nonlinear transformation of a random process kurtosis.

III. FORMULATION OF THE PROBLEM

Presented work is investigation of the non-gaussiality character of unknown interferences supplied to the receiving path of radar.

For the elimination or analysis of the noises influence on the performance of radar systems appears the necessity to create an effective algorithm of their detection.

For effective use of information which is obtained during investigation, it's important to detect the difference from normal distribution as accurate as possible.

Thereby, the aim of work is creating an effective algorithm for non-gaussian noise detection even at small deviations from normal distributed noise.

IV. K-CRITERION AND NON-GAUSSIAN NOISE DETECTION ALGORITHM DEVELOPMENT

The detection algorithm must verify a hypothesis $H0$: $f(\bar{x}) \in N(0,\sigma)$, that normalized by the scale and shift parameters sample $\bar{x} = (x_1, x_2, ..x_n)$ belongs to the normal distribution (is a Gaussian noise), against the alternative hypothesis $H1$: $f(\bar{x}) \notin N(0,\sigma)$, that sample $\bar{x} = (x_1, x_2, ..x_n)$ does not belong to normal distribution (is "contaminated" by pulse noise).

Development of algorithm begins from decomposition of one dimensional distribution of sample values in a Gram-Charlier series:

$$f(x) = f_0(x) + \sum_{k=3}^{p} a_k f_0^{(k)}(x). \qquad (1)$$

In formula (1) $f_0(x)$ is normal distribution function (NDF); $f_0^{(k)}(x) = (-1)^k H_k(x) f_0(x)$ is k-th derivative from $f_0(x)$. The derivatives $f_0^{(k)}(x)$ and $H_k(x)$ (Hermite polynomials) have the properties of orthogonality and coefficients a_k are defined using central moments μ_k of the distribution $f(x)$.

Confining by the first members of the series, we have:

$$f(x) = f_0(u) - \frac{1}{6}\sqrt{\frac{\mu_3^2}{\mu_2^3}} f_0^{(3)}(u) + \frac{1}{24}\left(\frac{\mu_4}{\mu_2^2} - 3\right) f_0^{(4)}(u). \qquad (2)$$

In formula (2) $f_0(u)$ is NDF of centered and normalized random value $u=(x-\mu_1)/\mu_2^{0.5}$, $f_0^{(k)}(u)$ is k-th derivative from $f_0(u)$.

For the further work, confine ourselves to the three members of the series (2), using initial moment of first order, second, third and fourth central moments, skewness and kurtosis coefficients. Formulas of the moments and coefficients are taken from [1].

Thus, formula (2) can be presented in such a form:

$$f(x)=f_0(u)-\frac{1}{6}\gamma_1 f_0^{(3)}(u)+\frac{1}{24}\gamma_2 f_0^{(4)}(u). \quad (3)$$

In general, the problem of hypotheses $H0$ and $H1$ testing with respect to the distribution sample $\overline{x}=(x_1,x_2,...,x_n)$ is solved by calculating the logarithm of the likelihood ratio [2] and comparing it with a decision threshold V. Considering the kind of uncomfortable for logarithm density of distribution (3), is selected a different approach for the algorithm synthesis.

It is proposed to express the densities of distribution $f(\overline{x})$ and $f_0(\overline{x})$ by vectors of decomposition coefficients in a Gram-Charlier series $f_0(\overline{x})\to$ $\to[1,0,0]$ and $f(\overline{x})\to\left[1,\frac{1}{6}\gamma^*_1,\frac{1}{24}\gamma^*_2\right]$, where γ_1^* i γ_2^* are estimations of skewness and kurtosis coefficients calculated with sample data.

During the investigation and to further development the non-gaussian noises detection algorithm, a sequence of random numbers, distributed by poligaussian law, was modeled:

$$f(x)=(1-p)N(0,1)+pN(m,\sigma_1) \quad (4)$$

where p is a distribution parameter which takes values (0…1); $N(0,1)$ is NDF; $N(m,\sigma_1)$ is NDF with different mathematical expectations and variance.

It should be noted, that when p = 0, random sequence is distributed according to the normal law, and by changing p > 0, coefficients of skewness and kurtosis also change and we can investigate the effectiveness of deviation from normal law detection with the help of K-criterion.

With reference to Fig. 1a and b can be hypothesized that distributions of sample coefficients can be approximated by Gaussian law. Checking by Chi-square test showed that the hypothesis about Gaussian distribution can be accepted.

The problem of hypotheses testing, relatively to sample $\overline{x}=(x_1,x_2,...,x_n)$, can be formulated as a problem of hypothesis check with respect to decomposition coefficients of sample distribution density in a Gram-Charlier series (3).

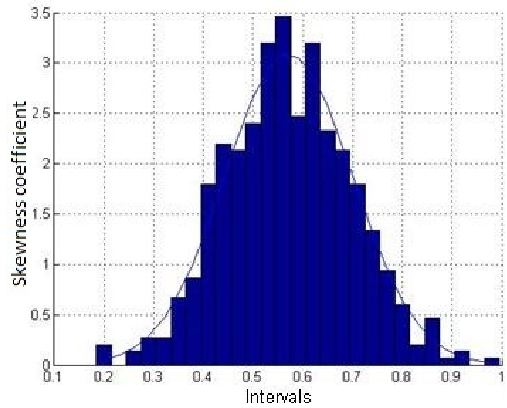
$$H0\colon f(\gamma_1^*,\gamma_2^*)=N(0,\sigma_1)\times N(0,\sigma_2). \quad (5)$$

$$H1\colon f(\gamma_1^*,\gamma_2^*)=N(b_1\neq 0,\sigma_{\gamma_1^*})\times N(b_2\neq 0,\sigma_{\gamma_2^*}). \quad (6)$$

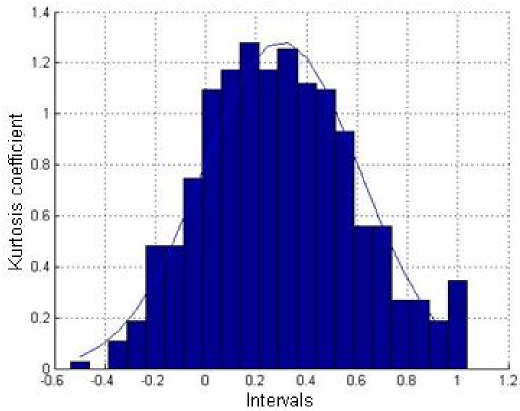
In formulas (5), (6) b_1 and b_2 are average values of decomposition coefficients (3), different from zero.

In this case, a locally optimal two-side decision rule takes the form:

$$K(x_1,x_2,...,x_n)=\left|\frac{\gamma^*_1}{\sigma_{\gamma_1^*}}\right|+\left|\frac{\gamma^*_2}{\sigma_{\gamma_2^*}}\right|. \quad (7)$$



a



b

Fig. 1. Histogram of selective coefficients of decomposition at p = 0.16: (a) of γ_1^*; (b) of γ_2^*

The principle of detection algorithm operation (Fig. 2) is following:

– noise X with unknown distribution is applied to the input;

– estimations of kurtosis γ_1^* and skewness γ_2^* coefficients are calculated and the value of K-criteria is defined;

– determined threshold value V and the value of criteria compared and decision is made: “1” – if the noise distribution considered to be non-gaussian or “0” – otherwise.

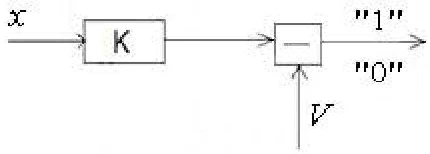


Fig. 2. General structural scheme of detection algorithm

The effectiveness of the algorithm on *K*-criterion basis was proved by comparison of its detection characteristics with classical Kolmogorov criterion (Fig 3).

In Fig. 4 a realization of simulated noise according to formula (4) is shown at $p = 0.16$. In Figs 5 and 6 are plotted non-gaussian density of probability distribution and cumulative functions of normal and polygaussian distributions respectively at $p = 0.16$.

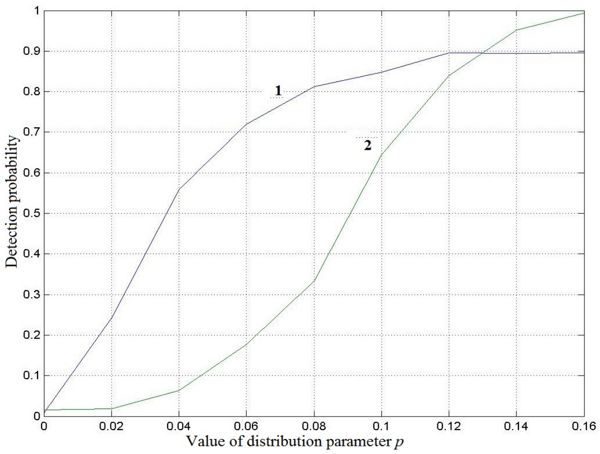


Fig. 3. Detection characteristics of *K*-criterion (1) and Kolmogorov criterion (2)

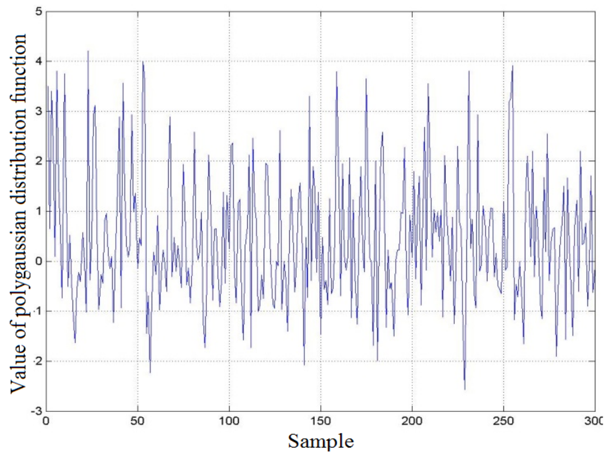


Fig. 4. Noise realization at $p = 0.16$

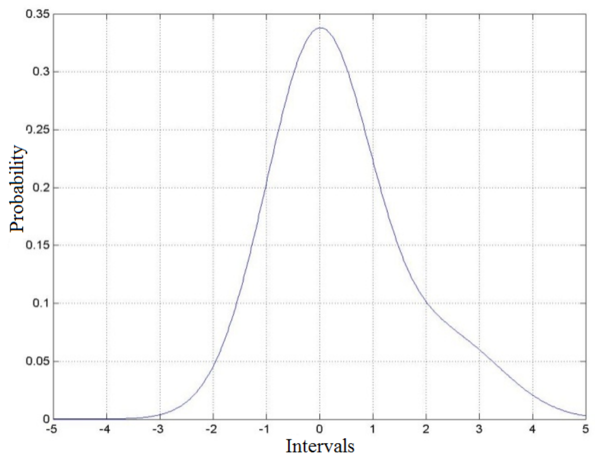


Fig. 5. Non-gaussian density of probability distribution at $p = 0.16$

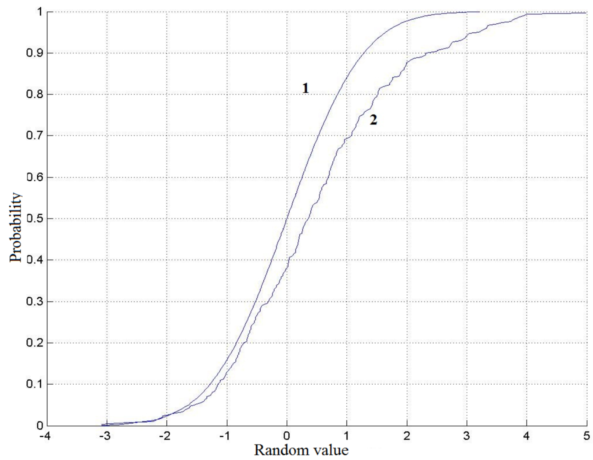


Fig. 6. Cumulative functions of normal (1) and polygaussian (2) distributions (at $p = 0.16$)

Conclusion

A non-gaussian noises detection algorithm was proposed on base of developed K-test and its effectiveness proved by comparison its detection characteristics with detection characteristic of classical Kolmogorov criterion.

Figure 3 shows, that algorithm is sensitive enough and even at small deviation from normal distribution, the probability of non-gaussian noise detection is high.

Thus, *K*-criterion use is advisable for small deviations from the gaussian distribution of the sample, i.e., with minor gaussian noise "pollution" by impulse noise.

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І. Г. Прокопенко, Н. С. Бабанська. Розробка алгоритму виявлення негаусівських завад та дослідження його ефективності

Розглянуто метод розробки алгоритму виявлення негаусівських завад та досліджена його ефективність.

Ключові слова: алгоритм виявлення, виявлення негаусівських завад, ймовірність виявлення, критерій ефективності оцінки, статистичні моменти.

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И. Г. Прокопенко, Н. С. Бабанская. Разработка алгоритма обнаружения негауссовских помех и исследование его эффективности

Рассмотрен метод разработки алгоритма обнаружения негауссовских помех и исследована его эффективность.

Ключевые слова: алгоритм обнаружения, вероятность обнаружения, критерий эффективности оценки, обнаружение негауссовких помех, статистические моменты.

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