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DESIGN OF TWO-DEGREE-OF-FREEDOM SYSTEM FOR CONTROL BY INERTIALLY STABILIZED PLATFORM

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Abstract**—***The problem of design of robust two-degree-of-freedom systems for control by the inertially stabilized platforms functioned at vehicles in the difficult conditions of real operation is considered. The generalized optimization functional taking into consideration the functions of sensitivity by the coordinate disturbances and noise measurements is obtained. The robust controller mathematical description in the state space was determined. The simulation results are represented.*

Index Terms—Inertially stabilized platforms; two-degree-of-freedom system; structural robust synthesis; gyro devices; ground vehicles.

I. INTRODUCTION

Now complexity of control processes, which occur during vehicle operation, constantly increases. In this case is arisen the important problem of stabilization of information-measuring devices ensuring measurement and processing of information necessary for control by vehicle motion, navigation and tracking of reference points. As a rule, the high accuracy requirements are given to such processes. These requirements can not be satisfied without using inertially stabilized platforms, on which information-measuring devices may be mounted.

Choice of a method for such systems design is defined by the following factors. In the first, some parameters of systems may change significantly in the wide range ($\pm 50\,\%$) during operation at vehicles. In the second, functioning of systems is implemented in conditions of various and intensive external disturbances. In the third, parameters measurements are accompanied by influence of the noise.

Taking into consideration these factors, the problem of design of systems for stabilization and control by orientation of the information-measuring devices mounted at the platform is expedient to solve as the problem of structural synthesis of robust systems with combined control [1].

Such approach ensures stability and operation characteristics of a system in the permissible range under action of both structured parametric and coordinate disturbances.

Platform stabilization may be implemented by the error signal, which presents a difference between the command and output signals. Although in this case it is impossible to provide the high accuracy of control by orientation of the information-measuring devices lines-of-sight in the inertial space. To solve this problem it is necessary to use two-degree-of-freedom -systems, which implement control by both error and reference signals [2].

In such systems, the pre-filter controls by the command signal reproduction, and the feedback controller provides robust stability and disturbance attenuation [3], [4].

Now methods for design of stabilization systems based on the modern control theory become widespread. Among them the H_∞-approach which ensures robust performance and stability may be marked. In this case the design problem is formulated as a problem of mathematical optimization directed to search of the optimal robust controller. Advantage of this approach lies in simplicity of application for multi-dimensional systems with cross-connections between channels. Disadvantages are the mathematical complexity and decisive influence of the system mathematical description on successfulness of the problem solution.

II. ANALYSIS OF RESEARCHES AND PUBLICATIONS

The sufficient quantity of papers and textbooks deals with the problems of the robust system design. There are different approaches to robust structural synthesis of such systems. The main principles of H_∞-synthesis are represented, for example, in [3], [4].

For the first time the problem of H_∞-synthesis was formulated in [5]. The most generalized and widespread algorithms of H_∞-synthesis problem solution based on the mathematical descriptions in state space are represented in [6].

One of approaches to design of control systems of the wide class lies in forming transfer functions of the control systems with the given properties. The algorithm represented in the paper [7] combines the statement of the robust stabilization problem with the classical forming of control systems logarithmic frequency characteristics.

It is known, that simultaneous solution of stabilization and tracking problems requires control by both feedback and command signals. In such situations the

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quality of control processes may be ensured by means of the two-degree-of-freedom controller. Improvement of represented in [7] algorithm of two-degree -of-freedom system design was implemented in [1]. Detailed analysis of this algorithm and features of weighting transfer functions choice during forming the control systems transfer functions are given in [4]. The approach to forming the augmented stabilization plant by introducing weight transfer functions (pre-and post- compensators) is suggested in [8] and developed in [4].

III. PROBLEM STATEMENT

The main goal of the paper is creation of a procedure for design of the two-degree-of-freedom controller for the system of stabilizing information-measuring devices mounted at the platform and operated at the vehicles of the wide class.

Statement of the H_∞-synthesis problem [6] may be implemented based on concept of the generalized control system, which may described by the structural scheme represented in Fig. 1 [3], [4].

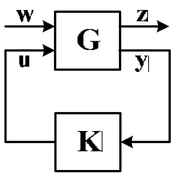

Fig. 1. Structural scheme of the generalized control system

The represented scheme consists of the control plant and controller described by the matrix transfer functions $\mathbf{G}(s)$, $\mathbf{K}(s)$, which are fractionally rational and proper. The generalized control plant has two inputs and two outputs. The vector $\mathbf{w}$ represents the external input, which in the general case consists of disturbances, measurement noise and command signals. The input vector $\mathbf{u}$ represents control signals. The output vector $\mathbf{z}$ defines quality of the control processes, for example, the error of the command signal reproduction, which in the ideal case must be equal to zero. The output vector $\mathbf{y}$ is the vector of the observed signals, which are used for implementation of feedbacks. Choice of the optimal controller is realized on the set of all the controllers, which provide the internal stability of the closed-loop system or on the set of the stabilizing or permissible controllers.

The control system represented in Fig. 1 may be described in the state space in the following way [4]

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{w}(t) + \mathbf{B}_2\mathbf{u}(t);\\
\mathbf{z}(t) &= \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_{11}\mathbf{w}(t) + \mathbf{D}_{12}\mathbf{u}(t);\\
\mathbf{y}(t) &= \mathbf{C}_2\mathbf{x}(t) + \mathbf{D}_{21}\mathbf{w}(t) + \mathbf{D}_{22}\mathbf{u}(t);\\
\mathbf{u}(t) &= \mathbf{K}\mathbf{y}(t).
\end{aligned} \quad (1)$$

Equations of the control plant (1) in the matrix form become [4]

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{z}(t) \\ \mathbf{y}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{C}_1 & \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{C}_2 & \mathbf{D}_{21} & \mathbf{D}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{w}(t) \\ \mathbf{u}(t) \end{bmatrix}. \quad (2)$$

Solving of the H_∞-synthesis problem is based on the Riccati equations. At that the following conditions must be satisfied [3], [9].

1. The pair of matrices $\mathbf{A}, \mathbf{B}_1$ must be stabilized and the pair of matrices $\mathbf{A}, \mathbf{C}_1$ must be detected.
2. The pair of matrices $\mathbf{A}, \mathbf{B}_2$ must be stabilized and the pair of matrices $\mathbf{A}, \mathbf{C}_2$ must be detected.
3. $\mathbf{D}_{12}^{\mathrm{T}}[\mathbf{C}_1 \quad \mathbf{D}_{12}] = [\mathbf{0} \quad \mathbf{I}]$.
4. $\begin{bmatrix} \mathbf{B}_1 \\ \mathbf{D}_{21} \end{bmatrix} \mathbf{D}_{21}^{\mathrm{T}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$.

Conditions 1 and 2 ensure absence of imaginary eigenvalues of the Hamilton matrices, which correspond to the Riccati equations. The condition 3 means orthogonality of components $\mathbf{C}_1\mathbf{x}(t)$ and $\mathbf{D}_{12}\mathbf{u}(t)$ in equations (2). The condition 4 characterizes orthogonality of components $\mathbf{B}_1\mathbf{w}(t)$ and $\mathbf{D}_{21}\mathbf{w}(t)$ in equations (2). Conditions 3, 4 are usual for the H_2-problem and may be spread on the case of the H_∞-synthesis [9].

Notice, that in such problem statements, the control plant is believed to be a set of devices and units which create the real system such as proper control plant, actuator, measuring system and some additional devices [10].

IV. ROBUST TWO-DEGREE-OF-FREEDOM SYSTEM DESIGN

In modern automatic control theory the systems with combined control are called two-degree-of-freedom ones [3], [4]. There are different approaches to robust structural synthesis of such systems. The method proposed by K. Glover and D. Mc Farlan [6] and developed in [4] is based on robust stabilization and representation of parametric disturbances by means of the normalized left coprime factorization. It is based on forming the desired frequency responses of the closed-loop control system using the open-loop system augmentation. Forming the augmented stabilization plant is carried out by introducing weight transfer functions (pre-and post- compensators) [4].

But for many applied applications, primarily for systems of stabilization and attitude control of information-measuring devices operated at vehicles it is very important to take into consideration influence of coordinate external disturbances that act on the plant. The novelty of the presented in this paper algorithm of the robust systems design lies in accounting the influence of coordinate external disturbances and measuring system noise. For this purpose the sensitivity function of the closed-loop system with respect to the coordinate external disturbances and measurement noise is introduced in the optimization functional of the problem. Block diagram of the designed system is shown in Fig. 2.

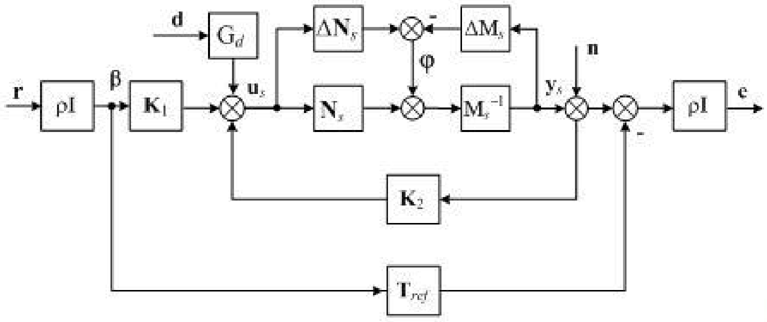

Fig. 2. Block diagram of the robust two-degree-of-freedom system: **r** is the command signal; ρ is the scalar design parameter; **I** is the identity matrix; β is the input signal of the pre-filter; **d** is the coordinate disturbance; $\mathbf{u_s}$ is the control signal; φ is the parametric disturbance; $\mathbf{y_s}$ is the output signal; **e** is the error of command signal reproduction; **n** is the measurement noise

For implementation of the H_∞-synthesis design procedure, it is reasonable to represent coordinate external disturbances acting on the studied system in the form of moments which applied to the input of the stabilization plant. In this case the transfer function by disturbance $\mathbf{G}_d$ may be introduced.

The feedback controller $\mathbf{K_2}$ is designed to ensure the robust stability and the attenuation of parametric structured disturbances taking into account the influence of coordinate external disturbances and measurement noise.

Pre-filter $\mathbf{K_1}$ ensures correspondence of closed-loop system responses to the command signal in accordance with a predetermined reference model $\mathbf{T}_{ref}$ [3], [4].

Thus, the main task of the structural synthesis of the robust two-degree-of-freedom system is to find a stabilizing controller

$$\mathbf{K} = [\mathbf{K}_1 \quad \mathbf{K}_2],$$

for the stabilization plant augmented by means of the weighting transfer functions to the form $\mathbf{G}_s = \mathbf{W}_2\mathbf{G}\mathbf{W}_1$ and represented as a result of the normalized left coprime factorization [4], [7]

$$\mathbf{G}_s = \mathbf{M}_s^{-1}\mathbf{N}_s.$$

In this case, the controller $\mathbf{K_1}$ is feedforward, and the controller $\mathbf{K_2}$ – feedback.

In accordance with Fig. 2 the connection between the vectors of input and output signals of the researched system can be represented as follows.

$$\begin{bmatrix} \mathbf{u}_s \\ \mathbf{y}_s \\ \mathbf{e} \end{bmatrix} = \begin{bmatrix} \rho(\mathbf{I}+\mathbf{K}_2\mathbf{G}_s)^{-1}\mathbf{K}_1 & \mathbf{K}_2(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}\mathbf{G}_d\mathbf{G}_s & \mathbf{K}_2(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1} & \mathbf{K}_2(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}\mathbf{M}_s^{-1} \\ \rho(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}\mathbf{G}_s\mathbf{K}_1 & (\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}\mathbf{G}_d\mathbf{G}_s & (\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1} & (\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}\mathbf{M}_s^{-1} \\ \rho^2[(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}\mathbf{G}_s\mathbf{K}_1 - \mathbf{T}_{ref}] & \rho(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}\mathbf{G}_d\mathbf{G}_s & \rho(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1} & \rho(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}\mathbf{M}_s^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{d} \\ \mathbf{n} \\ \varphi \end{bmatrix}, \qquad (3)$$

or $\mathbf{z} = \mathbf{\Phi}\mathbf{w}$, where $\mathbf{z}^{\mathrm{T}} = [\mathbf{u}_s \quad \mathbf{y}_s \quad \mathbf{e}]$ is the output vector allowing to estimate quality of system and including control, output signals and error respectively, $\mathbf{w}^{\mathrm{T}} = [\mathbf{r} \quad \mathbf{d} \quad \mathbf{n} \quad \varphi]$ is the input vector which includes command signals, as well as coordinate external disturbances, measurement noise and parametric internal disturbances, respectively; $\mathbf{\Phi}$ is a matrix transfer function of the closed-loop system.

H_∞-norm of the matrix transfer function $\mathbf{\Phi}$ described by equation (3) is a generalized functional of the design system optimization problem, since the elements of the matrix in accordance with [4] define the system accuracy, robustness and control costs. The components $(\mathbf{I}+\mathbf{G}_s\mathbf{K}_2)^{-1}$, $(\mathbf{I}+\mathbf{K}_2\mathbf{G}_s)^{-1}$ represent the input and output sensitivity functions of the system [4].

The problem of the robust combined system design is reasonable to solve in terms of γ-optimal control. With this approach, it is searched the optimal regulator, which ensures execution of the condition $\|\Phi\|_\infty < \gamma$, where γ is a predetermined number $\gamma > \gamma_{\min}$ [4]. The optimal solution is provided by using an algorithm D. Doyle, that allows to perform the iterative reduction of γ [11].

The procedure of the synthesis of the robust two-degree-of-freedom system is completed by replacing the pre-filter $\mathbf{K}_1$ on the weighting filter $\mathbf{K}_1\mathbf{W}_i$ that is ensured by the choice of the system parameters in the stable state [4].

In order to use the H_∞-synthesis it is necessary to define the generalized plant P, which provides connection between the system input and output signals in the form [4]

$$\begin{bmatrix} \mathbf{z} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{u} \end{bmatrix}. \quad (4)$$

For the considered system the expression (4) may be represented in the form

$$\begin{bmatrix} \mathbf{u}_s \\ \mathbf{y}_s \\ \mathbf{e} \\ \boldsymbol{\beta} \\ \mathbf{y}_s \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{d} \\ \mathbf{n} \\ \varphi \\ \mathbf{u}_s \end{bmatrix}. \quad (5)$$

where $\mathbf{y}^{\mathrm{T}} = [\boldsymbol{\beta} \quad \mathbf{y}_s]$; $\mathbf{u} = \mathbf{u}_s$.

To determine the matrix of the generalized plant it is possible to use relationships between the external input signals $\mathbf{w}$, control signals $\mathbf{u}$, measured output signals $\mathbf{y}$ and output signals $\mathbf{z}$ that are used to estimate the system quality

$$\begin{aligned} &\mathbf{u}_s = \mathbf{I}\mathbf{u}_s; \\ &\mathbf{y}_s = \mathbf{G}_d\mathbf{G}_s\mathbf{d} + \mathbf{M}_s^{-1}\varphi + \mathbf{G}_s\mathbf{u}_s + \mathbf{n}; \\ &\mathbf{e} = -\rho^2\mathbf{T}_{ref}\mathbf{r} + \rho\mathbf{G}_d\mathbf{G}_s\mathbf{d} + \rho\mathbf{M}_s^{-1}\varphi + \rho\mathbf{G}_s\mathbf{u}_s; \\ &\beta = \rho\mathbf{I}\mathbf{r}. \end{aligned} \quad (6)$$

On the basis of the relations (5) and (6) the generalized plant may be described by the following matrix

$$\mathbf{P} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{G}_d\mathbf{G}_s & \mathbf{M}_s^{-1} & \mathbf{I} & \mathbf{G}_s \\ -\rho^2\mathbf{T}_{ref} & \rho\mathbf{G}_d\mathbf{G}_s & \rho\mathbf{M}_s^{-1} & \mathbf{0} & \rho\mathbf{G}_s \\ \rho\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_d\mathbf{G}_s & \mathbf{M}_s^{-1} & \mathbf{I} & \mathbf{G}_s \end{bmatrix}. \quad (7)$$

In the state space the matrix (7) becomes

$$\mathbf{P} = \begin{bmatrix} \mathbf{A}_s & \mathbf{0} & \mathbf{0} & \mathbf{B}_d & \mathbf{0} & (\mathbf{B}_s\mathbf{D}_s^{\mathrm{T}} + \mathbf{Z}_s\mathbf{C}_s^{\mathrm{T}})\mathbf{D}_{M_s} & \mathbf{B}_s \\ \mathbf{0} & \mathbf{A}_r & \mathbf{B}_r & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{C}_s & \mathbf{0} & \mathbf{0} & \mathbf{D}_d & \mathbf{0} & \mathbf{D}_{M_s^{-1}} & \mathbf{D}_s \\ \rho\mathbf{C}_s & -\rho^2\mathbf{C}_r & -\rho^2\mathbf{D}_r & \rho\mathbf{D}_d & \mathbf{0} & \rho\mathbf{D}_{M_s^{-1}} & \rho\mathbf{D}_s \\ \mathbf{0} & \mathbf{0} & \rho\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_s & \mathbf{0} & \mathbf{0} & \mathbf{D}_d & \mathbf{I} & \mathbf{D}_{M_s^{-1}} & \mathbf{D}_s \end{bmatrix}. \quad (8)$$

where $\mathbf{A}_s, \mathbf{B}_s, \mathbf{C}_s, \mathbf{D}_s$ are state, control, observation and disturbance matrices, that define the description of the augmented stabilization plant in the state space; $\mathbf{A}_r, \mathbf{B}_r$ are the state and control matrices that define the reference model $\mathbf{T}_{ref}$ in the state space. In accordance with represented in [7, 8] approaches to state-space representation of mathematical models obtained by means of the normalized left coprime factorization, the following relations take place

$$\mathbf{D}_{M_s} = \mathbf{R}^{-1/2}; \qquad \mathbf{D}_{M_s^{-1}} = \mathbf{R}^{1/2}.$$

where $\mathbf{R} = \mathbf{I} + \mathbf{D}_s\mathbf{D}_s^{\mathrm{T}}$.

Component $\mathbf{Z}_s$ that is the part of the matrix (8) is a positive definite solution of the algebraic Riccati equation [3], [4]

$$(\mathbf{A}_s - \mathbf{B}_s\mathbf{S}^{-1}\mathbf{D}_s^{\mathrm{T}}\mathbf{C}_s)Z + \mathbf{Z}(\mathbf{A}_s - \mathbf{B}_s\mathbf{S}^{-1}\mathbf{D}_s^{\mathrm{T}}\mathbf{C}_s)^{\mathrm{T}} - \mathbf{Z}\mathbf{C}_s^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{C}_s\mathbf{Z} + \mathbf{B}_s\mathbf{S}^{-1}\mathbf{B}_s^{\mathrm{T}} = \mathbf{0},$$

where $\mathbf{S} = \mathbf{I} + \mathbf{D}_s^{\mathrm{T}}\mathbf{D}_s$.

Algorithm of the two-degree-of-freedom controller design includes the following stages [1], [4].

1. Choice of the desired transfer function $\mathbf{T}_{ref}$ from the command signals to the control outputs.
2. Determination of the scalar parameter ρ, which usually lies in the range 1…3.
3. Choice of weighting transfer functions for the plant with the bounded amplitude-frequency characteristics [4]

$$\mathbf{G}_s = \mathbf{W}_2\mathbf{G}\mathbf{W}_1,$$

where $\mathbf{W}_1 = \mathbf{W}_p\mathbf{W}_a\mathbf{W}_g$ [4].

Choice of the weighting transfer functions requires heuristic approaches. Choosing $\mathbf{W}_p$ it is possible to ensure desired eigenvalues and the high gain at the low frequencies and slope of the amplitude-frequency characteristic near 20 dB per decade at the desired bandwidth. So, the weighting transfer function $\mathbf{W}_p$ influences on the dynamic characteristic of the system. Choice of the integrator provides dynamic characteristics at the low frequencies. Advance by a phase decreases the slope of the amplitude-frequency characteristic at the crossover frequency. Delay by the phase increases the slope of the amplitude-frequency characteristic at the high frequencies. Choice of eigenvalues for ensuring of the desired bandwidth may be implemented by the weighting transfer function $\mathbf{W}_a$. The weighting transfer function $\mathbf{W}_g$ defines characteristics of the actuator. Based on this function choice the actuator signals must not proceeded values determined for the given command signals and typical disturbances. Choice of the transfer functions must keep the system stability.

4. Solving the generalized H_∞-synthesis problem for the augmented stabilization plant $\mathbf{G}_s = \mathbf{W}_2\mathbf{G}\mathbf{W}_1$, desired reaction of the system, which is formed by the transfer function $\mathbf{T}_{ref}$, and the scalar parameter ρ. As result of synthesis, the controller $\mathbf{K} = [\mathbf{K}_1\ \mathbf{K}_2]$ is determined.

5. Substitution of the pre-filter $\mathbf{K}_1$ by the weighting pre-filter $\mathbf{K}_1\mathbf{W}_i$ to ensure the dynamic characteristics of the system in the stable state.

7. Analysis of obtained results and repetition of the controller design procedure in the case of the necessity. For this it is necessary to introduce new values ρ, weighting transfer functions $\mathbf{W}_1$, $\mathbf{W}_2$ and desired transfer function $\mathbf{T}_{ref}$.

The structure scheme of the stabilization system with the designed to-degree-of-freedom controller is represented in Fig. 3.

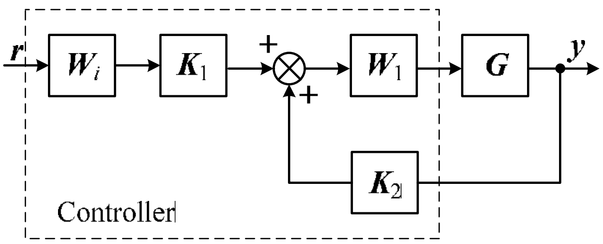

Fig. 3. Structured scheme of the designed two-degree-of-freedom stabilization system

The weighting transfer function $\mathbf{W}_2$ in Fig. 3 is absent as for the systems of the studied type it may be chosen unit.

Notice that now the discrete controllers are actual for practical applications. They may be obtained on the basis of continuous ones by means of the linear transformation.

V. Applied Realization of Robust Two-degree-of-freedom System

The proposed procedure of the robust structural synthesis can be considered on the applied example of the system that is used for stabilization and control by orientation of lines-of-sight of information-measuring devices operated at the ground vehicles [1].

The researched system includes observation-measuring equipment, direct current motor and rate gyro.

Control by the motor is carried by means of the pulse-width-modulator which generates a sequence of impulses with the predetermined amplitude and a width determined by the controller signal.

The most critical parameters of the researched system are the moment of inertia of the stabilization plant and the rigidity of the elastic connection between the actuator and the base on which the stabilization plant is set. During operation these parameters may be changed within ± 50 %.

In accordance with the recommendations [4] it is advisable to carry out stabilization and control by information-measuring devices attitude using integration of the difference between the command signal and the feedback signal entering from the rate gyro that measures the absolute angular rate of the stabilization plant.

Feedback, such as the current in the motor armature circuit, which is proportional to the torque of the external disturbance, is used in order to improve the quality of control by systems of the studied type [12].

With regard to external disturbances, among them there are a moment of unbalance and the moments caused by the angular motion of the vehicle due to irregularities in the relief of roads and terrain.

Creation of design procedure of the robust two-degree-of-freedom system for stabilization and control by attitude of information measuring devices operated at vehicles includes development of the system mathematical description and choice of the reference model, design scalar parameter ρ, as well as pre- and post- compensators [4].

The main features and assumptions used for creating mathematical description of the control system of considered type include the necessity to take into consideration the elastic connection between the actuator and the base on which the stabilization plant is mounted, the neglect of the dynamics of the rate gyro sensor, the linearization of the system, including usage of a linear pulse-width modulator [1], [12].

The linearized model of one channel of the studied stabilization system looks like [12]

$$\dot{x}_1 = r - k_{rg} x_5; \quad \dot{x}_2 = x_5; \quad \dot{x}_3 = x_6; \quad \dot{x}_4 = -\frac{1}{T_a} x_4 - \frac{c_e}{T_a} x_6 + k_{pwd} \frac{U_{pwd}}{T_a};$$
$$\dot{x}_5 = -\frac{c_r}{J_{os}} x_2 + \frac{c_r}{n_r J_{os}} x_3 - \frac{f_{os}}{J_{os}} x_5 - \frac{M_{dist}}{J_{os}}; \quad \dot{x}_6 = \frac{c_r}{n_r J_{mot}} x_2 - \frac{c_r}{n_r^2 J_{mot}} x_3 + \frac{c_m}{R_w J_{mot}} x_4 - \frac{f_{mot}}{J_{mot}} x_6. \tag{9}$$

where r is the command signal; k_{rg} is the transfer coefficient of the angular rate sensor; T_a is the time constant of the motor armature circuit; c_e is the coefficient of proportionality between the angular rate of the motor and the electromotive force; k_{pwd} is the transfer coefficient of the linearized PWM; U_{pwm} is the input voltage of PWM; J_{os} is the moment of inertia of the stabilization plant; c_r is the rigidity of the elastic connection between the actuator and the base on which the stabilization plant is mounted; n_r is the reducer ratio; f_{os} is the linearized coefficient of a friction torque of the stabilization plant; J_{mot} is the moment of inertia of the motor; f_{mot} is the linearized coefficient of the motor friction torque; c_m is the constant of loading torque at the motor shaft; R_w is the resistance of the motor armature circuit windings.

For the vector of state variables of the set of equations (9) $\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}$ the matrices of state, control and observation in the space of states look like

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & k_{rg} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{T_a} & 0 & -\frac{c_e}{T_a} \\ 0 & -\frac{c_r}{J_{os}} & \frac{c_r}{n_r J_{os}} & 0 & -\frac{f_{os}}{J_{os}} & 0 \\ 0 & 0 & -\frac{c_r}{n_r^2 J_{mot}} & \frac{c_m}{R_w J_{mot}} & 0 & -\frac{f_{mot}}{J_{mot}} \end{bmatrix}; \quad \mathbf{B}^{\mathrm{T}} = \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{pwd} U_{pwd} / T_a & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{dist}/T_{os} & 0 \end{bmatrix};$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/R_w & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathrm{k}_{rg} & 0 \end{bmatrix},$$

where state variables are the integral of the difference between the command signal and the voltage corresponding to the absolute angular rate of the stabilization plant; angles of the plant and motor shaft turns; angular rates of the plant and motor shaft turns; voltage in the motor armature circuit.

The choice of the reference model is determined by the requirements to the transients of the command signal reproduction. In this case, the model may be represented in the form $T_{ref} = \dfrac{k}{T^2 p + 2\xi T p + 1}$, where $k = 1$; $T = 0{,}2$; $\xi = 0{,}7$.

The value of constructional parameter ρ was assumed to be equal to 1 [4].

The choice of the transfer functions of pre- and post- compensators W_2, W_1 is based mainly on the method of trial and errors. Based on the recommendations [4], it is advisable to take W_2=1. As to the pre-compensator, based on results of the researches, it may be represented by the following transfer function $W_1 = 10 \dfrac{0{,}15}{0{,}1s + 1} \dfrac{0{,}4s + 25}{s + 25}$.

The state space representation of the designed controller after the maximum possible reduction of the order takes the form

$$\mathbf{A}_p = \begin{bmatrix} 0.06 & 0.4 & 0.72 & 0.53 & 0.11 \\ 0.05 & -0.95 & 0.01 & -0.03 & 0.02 \\ -0.55 & -0.09 & 1.28 & 0.29 & 0.09 \\ 0.47 & 0.27 & -0.18 & 0.61 & -0.11 \\ -0.8 & 0.15 & 0.47 & 0.49 & 1.07 \end{bmatrix};$$

$$\mathbf{B}_p = \begin{bmatrix} 0,56 & -173,4 & -52,48 \\ 0,07 & -20,41 & -6,18 \\ -0,55 & -17,22 & -3,04 \\ 1 & -106,4 & -25,5 \\ -6,54 & -34,61 & -12,39 \end{bmatrix}; \qquad \mathbf{C}_p = \begin{bmatrix} -0,004 & 0,007 & 0,006 & 0,001 & -0,0004 \end{bmatrix}; \qquad \mathbf{D}_p = \begin{bmatrix} 0,02 & -3,64 & -1,09 \end{bmatrix}.$$

The simulation results of the designed system are shown in Fig. 4. Here the nominal values of inertia and unbalance moment and their changes in limits $\pm 50\,\%$ were considered.

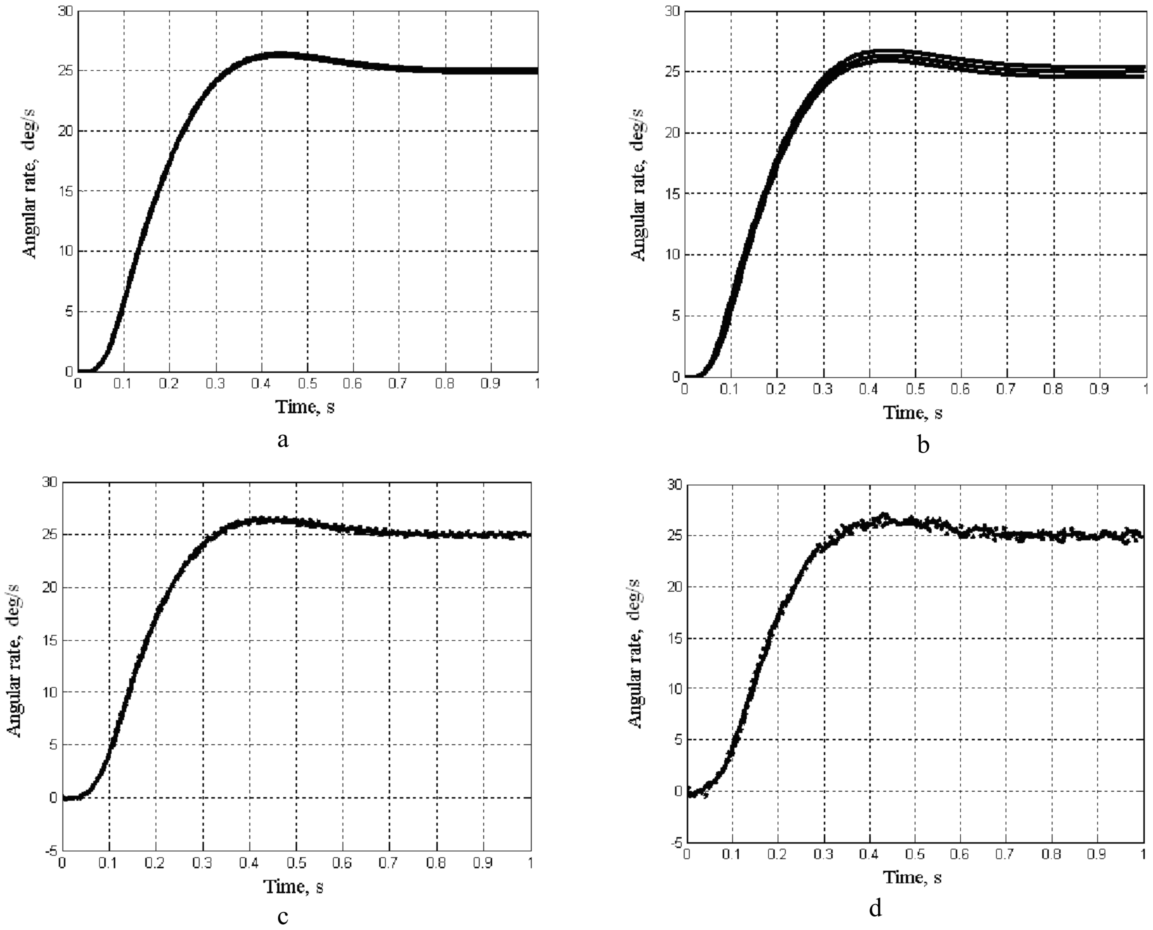

Fig. 4. The results of robust two-degree-of-freedom-system simulation: reproduction of the command signal for change of the moment of inertia (a) and the moment of unbalance (b); the influence of the road with long irregularities (c) and cross-country with clumps (d)

CONCLUSIONS

The problem of H_∞-synthesis of the robust two-degree-of-freedom system taking into conside-ra-tion the coordinate external disturbances and measurement noise is solved. The effectiveness of the suggested design procedure is proved by its application to the system for stabilization and attitude control of information-measuring devices operating at the ground vehicles.

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О. А. Сущенко. Проектування системи з двома ступенями свободи для інерціальної стабілізованої платформи

Розглянуто проблему проектування робастної системи з двома ступенями свободи для керування інерціальними стабілізованими платформами, що функціонують на рухомих об’єктах в складних умовах реальної експлуатації. Запропоновано узагальнений оптимізаційний функціонал з урахуванням функцій чутливості за координатними збуреннями та перешкодами вимірювань. Визначено математичний опис робастного регулятора у просторі станів. Представлено результати моделювання.

Ключові слова: інерціальні стабілізовані платформи; система з двома ступенями свободи; структурний робастний синтез; гіроскопічні прилади; наземні рухомі об’єкти.

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О. А. Сущенко. Проектирование системы с двумя степенями свободы для управления инерциальной стабилизированной платформой

Рассмотрена проблема проектирования робастной системы с двумя степенями свободы для управления инерциальными стабилизированными платформами, которые функционируют на подвижных объектах в сложных условиях реальной эксплуатации. Определен обобщенный оптимизационный функционал с учетом координатных возмущений и помех измерений. Создано математическое описание робастного контроллера в пространстве состояний. Представлены результаты моделирования.

Ключевые слова: информационно-измерительные устройства; робастные системы; структурный синтез; гироскопические устройства; метод смешанной чувствительности.

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