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DESIGN OF DIGITAL LONGITUDINAL AUTOPILOTS BASED ON L$_1$-OPTIMIZATION APPROACH

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Abstract—*This paper deals with the l_1-optimal control to be implemented via the digital longitudinal autopilot capable to ensure a desired pitch attitude of aircraft in the presence of an arbitrary external unmeasured disturbance. The optimization is achieved by determining the two parameters of the digital PI controller needed to stabilize the pitch rate and also the one parameter of P controller required for the stabilization of the pitch attitude. An illustrative numerical example and simulation results are given to demonstrate the effectiveness of this approach.*

Index Terms—Aircraft; longitudinal dynamics; digital autopilot; discrete time; PI controller; l$_1$-optimization.

I. INTRODUCTION

The problem of designing the control system for an aircraft, especially for unmanned aerial vehicle, capable to ensure its high performance index remains actual up to now [1]. To this end, the different approaches based on achievements of the modern control theory have been proposed by many researches (see, in particular, [2] – [5]). Nevertheless, most of these works dealt with an ideal case when there are no disturbances, where they are always present in reality.

To reject an arbitrary disturbance that is not measurable, the so-called l$_1$-optimization methods have been reported in modern control literature including [6] – [8], etc. Last important results in this research area can be found in [9].

In order to implement approaches advanced in modern control theory, digital technique seems to be appropriate, because digital control has become a highly developed technology in control applications by the end of the twentieth century [10]. This technique has some features associated with sampling [10], [11]. Namely, it allows giving the accurate discrete-time model of any (linear or nonlinear) system to be controlled [10]. Unfortunately, in some of the older transport aircraft, the displacement-type autopilot with the usual continuous-time proportional controller is employed to hold the aircraft in straight and level flight with little or no maneuvering capability [12, sect. 2-1]. To improve aircraft dynamics, the additional feedback exploiting the pitch rate signals has traditionally been introduced in the continuous-time closed-loop control system, see [12, p.68]. Meanwhile, a new digital autopilot utilizing l$_1$-optimization approach was recently devised in [13] to maintain a given roll orientation with a desired accuracy and to cope with an arbitrary external disturbance (a gust) for an unmanned aerial vehicle.

This paper continues and extends certain ideas of [13]. Its main contribution consists in the application of the l$_1$-optimization methodology in deriving the optimal discrete-time PI and P control algorithms exploiting in digital longitudinal autopilot systems. As is [13], they are designed as two-circuit closed-loop control system with the inner and external feedback loops.

I. FOUNDATIONS OF L1-OPTIMIZATION METHODOLOGY

Let a plant to be controlled be described in discrete time $t = nT_0$ $(n = 0, 1, 2, ...)$ by the linear difference equation

$$D(z^{-1})y_n = C(z^{-1})u_n + v_n, \qquad (1)$$

where $C(z^{-1})$ and $D(z^{-1})$ are the polynomials of the inverse shift operator $z^{-1} = e^{-sT_0}$ $(z^{-1}x_n = x_{n-1})$ with the properties $C(0) = 0$, $D(0) = 1$. The variables $y_n := y(nT_0)$, $u_n := u(nT_0)$ and $v_n := v(nT_0)$ denote the measured output, control input and unmeasurable disturbance, respectively. This equation leads directly to the transfer function from u to y of the form

$$W_0(z^{-1}) = C(z^{-1}) / D(z^{-1}). \qquad (2)$$

Suppose $\{w_n\} = w_0, w_1, w_2, ...$ is some sequence of arbitrary variables satisfying the constraint

$$|\Delta w_n| \le \varepsilon < \infty, \quad n = 1, 2, ... \tag{3}$$

with an ε where $\Delta w_n := w_n - w_{n-1}$.

Consider the closed-loop control system containing the plant (1) and the modified PI controller described by the transfer function

$$W_c(z^{-1}, k_P, k_I) = k_P + \frac{k_I(1+z^{-1})}{2(1-z^{-1})}, \tag{4}$$

with fixed k_P and k_I. Due to (4), the feedback control law is

$$u_n = u_{n-1} - (k_P + k_I/2)y_n - (k_I/2 - k_P)y_{n-1}. \tag{5}$$

Using (4), from (1) we get

$$y_n = \frac{1}{Q(z^{-1}, k_P, k_I)}\Delta w_n, \tag{6}$$

where

$$Q(z^{-1}, k_P, k_I) = [(k_P + k_I/2) - (k_I/2 - k_P)z^{-1}]C(z^{-1}) + (1 - z^{-1})D(z^{-1}).$$

By virtue of (6), the transfer function from Δw to y in the closed-loop system (1), (5) is defined as

$$W(z^{-1}, k_P, k_I) = \frac{1}{Q(z^{-1}, k_P, k_I)}. \tag{7}$$

It is well known [11] that this control system will be asymptotically stable if and only if

$$Q(z^{-1}, k_P, k_I) \ne 0 \quad \text{for all} \quad |z| \ge 1. \tag{8}$$

Let $\{w_n\} = w_0, w_1, w_2, ...$ be the sequence describing the impulse response from Δw to y at each discrete time n which, according to (7), is determined by the expansion

$$\frac{1}{Q(z^{-1}, k_P, k_I)} = w_0 + w_1 z^{-1} + w_2 z^{-2} + ...$$

Then, due to (6), it can be written

$$y_n = \sum_{i=0}^{n} w_i \Delta v_{n-i}, \tag{9}$$

if, of course, the initial conditions are zero.

Equation (9) yields

$$|y_n| \le \sum_{i=0}^{n} |w_i| \max_{0 \le i \le n} |\Delta v_i|. \tag{10}$$

If the condition (8) is satisfied, then the series

$$|w_0| + |w_1| + |w_2| + ...$$

converges so that (10) leads to

$$|y_n| \le \sum_{i=0}^{\infty} |w_i| \sup_{0 \le i < \infty} |\Delta v_i| \tag{11}$$

for each fixed $n \in [0, +\infty)$.

The upper bound on $|y_n|$ given by (11) implies that

$$\begin{aligned} \|y_n\|_\infty &\le \|\Delta v\|_\infty \|W(k_P, k_I)\|_1 \\ &\le \varepsilon \|W(k_P, k_I)\|_1 < \infty, \end{aligned} \tag{12}$$

where the notations

$$\|W\|_1 := \sum_{n=0}^{\infty} |w_n| < \infty, \ \|x\|_\infty := \sup_{0 \le n < \infty} |x_n| < \infty$$

of the so-called l_1- and l_∞-norms of $W(z^{-1})$ and of an arbitrary bounded sequence $\{x_n\} = x_0, x_1, ...,$ respectively, are introduced.

For any initial conditions, instead of (12), we have

$$\begin{aligned} \limsup_{n \to \infty} |y_n| &\le \|W(k_P, k_I)\|_1 \|\Delta v\|_\infty \\ &\le \varepsilon \|W(k_P, k_I)\|_1 . \end{aligned} \tag{13}$$

Expression (13) shows that the l_1-optimization of the closed-loop system (1), (5), allowing to minimize $\sup|y_n|$ as n tends to ∞, reduces directly to minimizing the l_1-norm of $W(z^{-1}, k_P, k_I)$ in the two controller parameters k_P and k_I. Namely, the desired parameter vector $k_c^* = [k_P^*, k_I^*]^T$ of l_1-optimal PI controller is given as

$$k_c^* = \arg\min_{k_P, k_I} \|W(k_P, k_I)\|_1, \tag{14}$$

assuming that k_P and k_I belong to the stability region defined by (8). This choice of controller parameters leads to certain rejection of arbitrary disturbances v_n whose first difference, Δv_n, is upper bounded in modulus (according to (3)).

Since $\|W(k_P, k_I)\|_1$ is a non-differentiable function in k_P and k_I, which cannot be exactly calculated, numerical methods are needed to calculate this l_1-norm as proposed in [14] and to determine k_c^* by using techniques taken from [15] and [16].

III. Digital Autopilot Design by Using L_1-Approach

A. Basic Assumptions and Optimal Control Task

Denote the pitch rate and the elevator deflection by $\dot{\vartheta}$ and δ_e, respectively. It is known (see, for example [12, sect 1.9]) that longitudinal dynamics of an aircraft may be described (after linearization) by the transfer function

$$W_0^{(1)}(s) := \frac{\dot{\vartheta}(s)}{\delta_e(s)} = \frac{K(s+a_1)}{s^2 + b_1 s + b_2} \quad (15)$$

from the elevator deflection, δ_e, to the pitch rate, $\dot{\vartheta}$, under the ideal conditions where there are no disturbances. The transfer function $W_0^{(1)}(s)$ of the form (15) can be rewritten as

$$W_0^{(1)}(s) = \frac{k(\tau_1 s + 1)}{\tau_2^2 s^2 + 2\tau_2 \xi s + 1}$$

whose four parameters are

$$k = Ka_1 / b_2,\ \tau_1 = 1/a_1, \quad \tau_2 = \sqrt{1/b_2},\ \xi = b_1 / 2b_2\tau_2. \quad (16)$$

Equation (15) gives that if any disturbances are absent then

$$\dot{\vartheta}(s) = W_0^{(1)}(s)\delta_e(s).$$

However, in the presence of some disturbance, $d(t)$, this equation has to be replaced by

$$\dot{\vartheta}(s) = W_0^{(1)}(s)\delta_e(s) + W_d(s)d(s), \quad (17)$$

where $W_d(s)$ represents the transfer function from d to $\dot{\vartheta}$.

Suppose $d(t)$ is an arbitrary variable which depends on the continuous time t and satisfies

$$|\dot{d}(t)| \le \Lambda < \infty.$$

Let $\vartheta_r(t)$ be the desired pitch attitude at the time instant t (the reference signal). Denoting the current variation $\vartheta(t)$ from $\vartheta_r(t)$ by

$$e(t) := \vartheta_r(t) - \vartheta(t),$$

introduce the performance index of the aircraft control system as

$$J := \lim_{t\to\infty} \sup |e(t)|.$$

The task is to minimize the performance index J via suitable choice of the controller parameters.

B. Control Strategy

To implement the approach proposed in this paper, two samplers are incorporated in the feedback loops, as shown in Fig. 1. These samplers are needed in order to convert analogue signals $\dot{\vartheta}(t)$ and $\vartheta(t)$ to the digital form at each nth time instant $t = nT_0$ $(n = 0, 1, 2, \ldots)$ to produce the discrete-time signals $\dot{\vartheta}(nT_0)$ and $\vartheta(nT_0)$, respectively, with the sampling period T_0. On the other hand, the signal $\delta_e(nT_0)$ formed by digital controller at the same time instant converts to the analogue form $\delta_e(t)$ using the so-called zero-order hold (ZOH) [10], [11]. This allows representing the control input as follows:

$$\delta_e(t) = \delta_e(nT_0) \quad \text{for} \quad nT_0 \le t < (n+1)T_0. \quad (18)$$

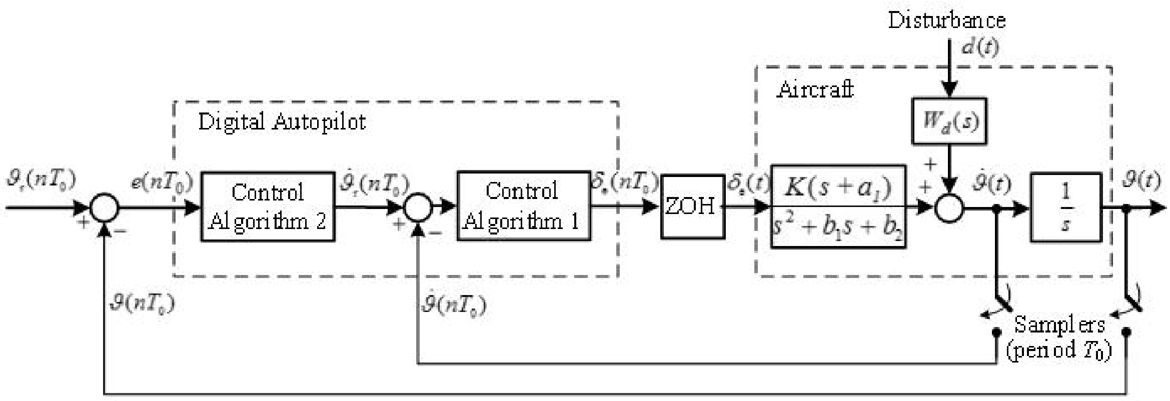

Fig. 1. Digital autopilot system

The aim of the inner control loop is to stabilize the pitch rate $\dot{\vartheta}(nT_0)$ at a given value, $\dot{\vartheta}_r(nT_0)$, which is the output of the external control loop (see Fig. 1). The control algorithm 1 should be designed so that the inner control circuit became optimal. Then, the control algorithm 2 should optimize the external control circuit.

C. Design of Control Algorithm 1

Using the Z-transform technique [10], [11] and taking into account (18), in the absence of $v(t)$ it can be written

$$W_0^{(1)}(z^{-1}) = (1 - z^{-1}) Z\left\{\frac{W_0^{(1)}(s)}{s}\right\}, \quad (19)$$

Substituting (1) into (19) we get

$$W_0^{(1)}(z^{-1}) = \frac{C_1(z^{-1})}{D_1(z^{-1})}, \tag{20}$$

where

$$C_1(z^{-1}) = c_1^{(1)} z^{-1} + c_2^{(1)} z^{-2}, \quad D_1(z^{-1}) = 1 + d_1^{(1)} z^{-1} + d_2^{(1)} z^{-2} \tag{21}$$

are the polynomials whose coefficients depend on K, a_1, b_1, b_2 and also on T_0.

If $d(t) \equiv 0$ then (20) causes

$$D_1(z^{-1})\dot{\vartheta}(nT_0) = C_1(z^{-1})\delta_e(nT_0). \tag{22}$$

In the presence of $d(t)$ acting on the pitch attitude (according to (17)), such an action may be taken into account by adding an equivalent disturbance $v(t)$ at $t = nT_0$ to the input $\delta_e(nT_0)$. In this case,

$$D_1(z^{-1})\dot{\vartheta}(nT_0) = C_1(z^{-1})\left[\delta_e(nT_0) + v(nT_0)\right], \tag{23}$$

where $v(nT_0)$ obeys the boundedness condition

$$|\,\Delta v(nT_0)\,| \le \varepsilon < \infty. \tag{24}$$

Thereby, instead of (22), we will exploit equation (23).

Let

$$e_1(nT_0) = \dot{\vartheta}_r(nT_0) - \dot{\vartheta}(nT_0) \tag{25}$$

be the deflection of true pitch rate, $\dot{\vartheta}(nT_0)$, from a desired pitch rate, $\dot{\vartheta}_r(nT_0)$ (see Fig. 1). Since the inner control circuit is designed as the digital PI controller, whose transfer function, $W_c^{(1)}(z^{-1})$, is defined by (4), the control algorithm 1 can be shown to take the following form:

$$\delta_e(nT_0) = \delta_e((n-1)T_0) + (k_P^{(1)} + k_I^{(1)}/2)e_1(nT_0) + (k_I^{(1)}/2 - k_P^{(1)})e_1((n-1)T_0). \tag{26}$$

Equations (23), (25) and (26) together with the expressions (21) of $C_1(z^{-1})$ and $D_1(z^{-1})$ give finally the transfer function $W^{(1)}(z^{-1}, k_P^{(1)}, k_I^{(1)})$ from Δv to e_1 for the inner control circuit as

$$W^{(1)}(z^{-1}, k_P^{(1)}, k_I^{(1)}) = -\frac{c_1^{(1)} z^{-1} + c_2^{(1)} z^{-2}}{1 + q_1^{(1)} z^{-1} + q_2^{(1)} z^{-2} + q_3^{(1)} z^{-3}}, \tag{27}$$

where

$$\left.\begin{aligned} q_1^{(1)} &= c_1^{(1)}(k_P^{(1)} + k_I^{(1)}/2) + d_1 - 1, \\ q_2^{(1)} &= c_1^{(1)}(k_I^{(1)}/2 - k_P^{(1)}) + c_2^{(1)}(k_P^{(1)} + k_I^{(1)}/2) - d_1 + d_2, \\ q_3^{(1)} &= c_2^{(1)}(k_I^{(1)}/2 - k_P^{(1)}) - d_2. \end{aligned}\right\} \tag{28}$$

According to [11, p.55] the necessary and sufficient conditions for asymptotical stability of

$$Q^{(1)}(z^{-1}, k_P^{(1)}, k_I^{(1)}) = 1 + q_1^{(1)} z^{-1} + q_2^{(1)} z^{-2} + q_3^{(1)} z^{-3}$$

which represents the denominator of (27) are

$$\left.\begin{aligned} &1 + q_1^{(1)} + q_2^{(1)} + q_3^{(1)} > 0; \\ &1 - q_1^{(1)} + q_2^{(1)} - q_3^{(1)} > 0; \\ &3(1 - q_3^{(1)}) + q_1^{(1)} - q_2^{(1)} > 0; \\ &3(1 + q_3^{(1)}) - q_1^{(1)} - q_2^{(1)} > 0; \\ &1 - \left(q_3^{(1)}\right)^2 - q_2^{(1)} + q_1^{(1)} q_3^{(1)} > 0. \end{aligned}\right\} \tag{29}$$

This gives that if (29) are satisfied then

$$\| W^{(1)}(k_P^{(1)}, k_I^{(1)}) \|_1 < \infty.$$

Following to (14), the optimal parameters of the inner PI controller specified by the vector $k_c^{*(1)} = [k_P^{*(1)}, k_I^{*(1)}]$ are

$$k_c^{*(1)} = \arg\min_{k_P^{(1)},\, k_I^{(1)}} \| W^{(1)}(k_P^{(1)}, k_I^{(1)}) \|_1 . \tag{30}$$

This completes the design of the l_1-optimal control algorithm 1 given by (26).

D. Design of Control Algorithm 2

Now, consider the external circuit of the feedback control system. To design the control algorithm 2, we first need the transfer function from δ_e to ϑ which is

$$W_0^{(2)}(s) = \frac{1}{s} W_0^{(1)}(s) \tag{31}$$

(see Fig. 1). Taking (15) into account and using (19), from (31) we have

$$W_0^{(2)}(z^{-1}) = \frac{C_2(z^{-1})}{D_2(z^{-1})} = \frac{C_2(z^{-1})}{(1 - z^{-1})D_1(z^{-1})}, \tag{32}$$

where

$$C_2(z^{-1}) = c_1^{(2)} z^{-1} + c_2^{(2)} z^{-2} + c_3^{(2)} z^{-3}. \tag{33}$$

(Recall that $D_1(z^{-1})$ is given by the second expression of (21).) Similarly to (23) it can be written

$$(1 - z^{-1})D_1(z^{-1})\vartheta(nT_0) = C_2(z^{-1})\left[\delta_e(nT_0) + v(nT_0)\right]$$

to obtain

$$\vartheta(nT_0) = W_0^{(2)}(z^{-1})\left[\delta_e(nT_0) + v(nT_0)\right]. \tag{34}$$

Introduce the transfer function $W_c^{(1)}(z^{-1}, k_P^{*(1)}, k_I^{*(1)})$ that is the transfer function of the l_1-optimal inner controller. Then, due to (4), equation (26) together with (25) causes

$$\delta_e(nT_0) = W_c^{(1)}(z^{-1}, k_P^{*(1)}, k_I^{*(1)})\left[\dot\vartheta_r(nT_0) - \dot\vartheta(nT_0)\right]. \quad (35)$$

Since the plant contains the integrator unit (see (31)), the control algorithm 2 forming the signal $\dot\vartheta_r(nT_0)$ is designed as

$$\dot\vartheta_r(nT_0) = W_c^{(2)}(z^{-1}, k_P^{(2)})\left[\vartheta_r(nT_0) - \vartheta(nT_0)\right], \quad (36)$$

where

$$W_c^{(2)}(z^{-1}, k_P^{(2)}) = k_P^{(2)} \quad (37)$$

represents the transfer function of the external controller that is P controller.

Further, using expression (20), rewrite (23) as follows:

$$\dot\vartheta(nT_0) = W_0^{(1)}(z^{-1})\left[\delta_e(nT_0) + v(nT_0)\right]. \quad (38)$$

Equations (34) – (38) together with (21), (32), (33), (37) and (4) for $k_P = k_P^{*(1)},\ k_I = k_I^{*(1)}$ yield the transfer function from Δv to e of the form

$$W^{(2)}(z^{-1}, k_P^{(1)}, k_I^{(1)}, k_P^{(2)}) = -\frac{c_1^{(2)}z^{-1} + c_2^{(2)}z^{-2} + c_3^{(2)}z^{-3}}{1 + q_1^{(2)}z^{-1} + q_2^{(2)}z^{-2} + q_3^{(2)}z^{-3} + q_4^{(2)}z^{-4}} \quad (39)$$

whose coefficients are

$$\left.\begin{aligned}
q_1^{(2)} &= c_1^{(1)}(k_P^{(1)} + k_I^{(1)}/2) + c_1^{(2)}(k_P^{(2)}k_P^{(1)} + k_P^{(2)}k_I^{(1)}/2) + d_1^{(1)} - 1,\\
q_2^{(2)} &= -2c_1^{(1)}k_P^{(1)} + c_2^{(1)}(k_P^{(1)} + k_I^{(1)}/2) + c_1^{(2)}(k_P^{(2)}k_I^{(1)}/2 - k_P^{(2)}k_P^{(1)})\\
&\quad + c_2^{(2)}(k_P^{(2)}k_I^{(1)}/2 + k_P^{(2)}k_P^{(1)}) - 2d_1^{(1)} + d_2^{(1)} + 1,\\
q_3^{(2)} &= c_1^{(1)}(k_P^{(1)} - k_I^{(1)}/2) - 2c_2^{(1)}k_P^{(1)} + c_2^{(2)}(k_P^{(2)}k_I^{(1)}/2 - k_P^{(2)}k_P^{(1)})\\
&\quad + c_3^{(2)}(k_P^{(2)}k_I^{(1)}/2 + k_P^{(2)}k_P^{(1)}) + d_1^{(1)} - 2d_2^{(1)},\\
q_4^{(2)} &= c_2^{(1)}(k_P^{(1)} - k_I^{(1)}/2) + c_3^{(2)}(k_P^{(2)}k_I^{(1)}/2 - k_P^{(2)}k_P^{(1)}) + d_2^{(1)}.
\end{aligned}\right\} \quad (40)$$

Exploiting the result related to the stability of

$$Q^{(2)}(z^{-1}, k_P^{(1)}, k_I^{(1)}, k_P^{(2)}) = 1 + q_1^{(2)}z^{-1} + q_2^{(2)}z^{-2} + q_3^{(2)}z^{-3} + q_4^{(2)}z^{-4},$$

which can be found in [11, p.55], we establish the necessary and sufficient conditions for its asymptotical stability as the following inequalities:

$$\left.\begin{aligned}
&\beta_j > 0 \quad j = 0, 1, 2, 3, 4,\\
&g > 0,\\
&\beta_3 g - \beta_1^2\beta_4 > 0
\end{aligned}\right\} \quad (41)$$

with

$$\left.\begin{aligned}
\beta_0 &= 1 + q_1^{(2)} + q_2^{(2)} + q_3^{(2)} + q_4^{(2)},\\
\beta_1 &= 4(1 - q_4^{(2)}) + 2(q_1^{(2)} - q_3^{(2)}),\\
\beta_2 &= 6(1 + q_4^{(2)}) - 2q_2^{(2)},\\
\beta_3 &= 4(1 - q_4^{(2)}) + 2(q_3^{(2)} - q_1^{(2)}),\\
\beta_4 &= 1 - q_1^{(2)} + q_2^{(2)} - q_3^{(2)} + q_4^{(2)},\\
g &= \beta_1\beta_2 - \beta_0\beta_3.
\end{aligned}\right\} \quad (42)$$

It can be observed from (41) that, after fixing $k_P^{(1)} = k_P^{*(1)}$ and $k_I^{(1)} = k_I^{*(1)}$, the stability of the external circuit depends on $k_P^{(2)}$ only.

Now design of the l_1-optimal algorithm 2 is competed by choosing

$$k_P^{(2)*} = \arg\min_{k_P^{(2)}} \| W^{(2)}(k_P^{*(1)}, k_I^{*(1)}, k_P^{(2)}) \|_1 . \quad (43)$$

and assuming the conditions (41) to be satisfied.

IV. A Numerical Example and Simulation

Assume that the transfer function, $W_0(s)$, from δ_e to $\dot\vartheta$ derived by the linearization of the longitudinal equations has the form

$$W_0^{(1)}(s) = \frac{-1.39(s + 0.306)}{s^2 + 0.805s + 1.325} \quad (44)$$

corresponding to (15) with the numerical coefficients K, a_1, b_1 and b_2 taken from [12, p.66].

Let $T_0 = 0.01$ s (as in [13]). Now, applying formula (19) to (44) and using Tabl. 1–1 of [11] we obtain

$$W_0^{(1)}(z^{-1}) = \frac{-0.01387z^{-1} + 0.01382z^{-2}}{1 - 1.992z^{-1} + 0.992z^{-2}}.$$

By virtue of (28) together with the numerical expression of $W_0^{(1)}(z^{-1})$, it can be written

$$\left.\begin{aligned}
c_1^{(1)} &= -0.01387 \text{ (independently of } k_P^{(1)}, k_I^{(1)}),\\
c_2^{(1)} &= 0.01382 \text{ (independently of } k_P^{(1)}, k_I^{(1)}),\\
q_1^{(1)} &= -0.0139k_P^{(1)} - 0.00693k_I^{(1)} - 2.99,\\
q_2^{(1)} &= 0.0277k_P^{(1)} - 0.000021k_I^{(1)} + 2.98,\\
q_3^{(1)} &= -0.0138k_P^{(1)} + 0.00691k_I^{(1)} - 0.992.
\end{aligned}\right\} \quad (45)$$

Utilizing the inequalities (29) together with (45) we find the stability region of the inner circuit depicted in Fig. 2.

To obtain l_1-norm of $W^{(1)}(z^{-1}, k_P^{(1)}, k_I^{(1)})$ depending on $k_P^{(1)}, k_I^{(1)}$, the numerical technique taken from [13] was used. Next, applying the simplest version of Powel's method [14] to calculate $k_c^{*(1)} = [k_P^{*(1)}, k_I^{*(1)}]^T$ by formula (30), the optimal values $k_{\mathrm{P}}^{*(1)} = -107.8$ and $k_{\mathrm{I}}^{*(1)} = -72.1$ were derived as shown in Fig. 3. The initial estiates weretaken as $k_{\mathrm{P}}^{(1)} = -34$ and $k_{\mathrm{I}}^{(1)} = -0.75$.

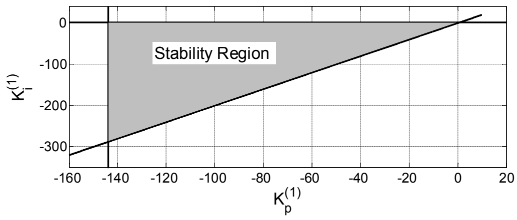

Fig. 2. Stability region of the inner circuit

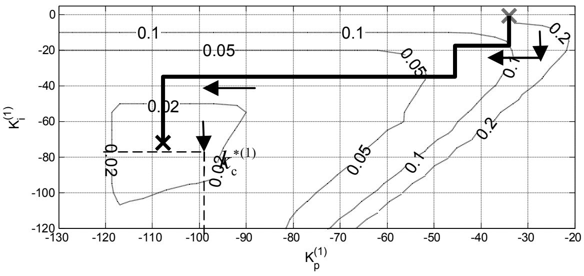

Fig. 3. Two-dimensional optimization of inner circuit

It turned out that the stability condition (41), (42) of external circuit is defined by $0 < k_{\mathrm{P}}^{(2)} < 116$.

Using the well-known golden-section method [16] for one-dimensional optimization of $\| W^{(2)}(k_{\mathrm{P}}^{(2)}) \|_1$ over (0, 116), the optimal $k_{\mathrm{P}}^{*(2)} = 65.2$ defined by (43) was derived after substituting $k_{\mathrm{P}}^{(1)} = k_{\mathrm{P}}^{*(1)} = -107.8$ and $k_{\mathrm{I}}^{(1)} = k_{\mathrm{I}}^{*(1)} = -72.1$ into (40) and calculating the l_1-norm of (27) based on the approximation technique of [14].

To illustrate the features of the digital autopilot, a simulation experiment was done by using SIMULINK. As in [13], $\{v(nT_0)\}$ was simulated as a random sequence whose first difference, $\{\Delta v(nT_0)\}$, was bounded within the interval [-1, 1] to satisfy (24) with $\varepsilon = 1$.

Results of a simulation experiment are presented in Fig. 4. From these figures we can observe that the control system is able to achieve good tracking performance.

To analysize the robustness features of the l_1-optimal autopilot, the coefficients $q_i^{(2)}$ of $W^{(2)}(z^{-1}, k_{\mathrm{P}}^{*(1)}, k_{\mathrm{I}}^{*(1)}, k_{\mathrm{P}}^{*(2)})$ given by (40) for fixed $k_{\mathrm{P}}^{(1)} = k_{\mathrm{P}}^{*(1)}$, $k_{\mathrm{I}}^{(1)} = k_{\mathrm{I}}^{*(1)}$ and $k_{\mathrm{P}}^{(2)} = k_{\mathrm{P}}^{*(2)}$ as some functions $q_i^{(2)} = q_i^{(2)}(k, \tau_1, \tau_2, \xi)$ $(i = 1, 2, 3, 4)$ depending on k, τ_1, τ_2 and ξ have been considered. To this end, we assumed that expression (44) corresponds to the transfer function of the nominal model whose parameters equal to $k_{\mathrm{nom}} = k = -0.321$, $\tau_{1,\mathrm{nom}} = \tau_1 = 3.27\,\mathrm{s}$, $\tau_{2,\mathrm{nom}} = \tau_2 = 0.87\,\mathrm{s}$, $\xi_{\mathrm{nom}} = \xi = 0.3497$ were calculated by formulas (16). Next, we supposed that the two parameter pairs $\{k, \tau_1\}$ and $\{\tau_2, \xi\}$ describing true but possibly unknown model belong to the bounded domains Q' and Q'', respectively, given as

$$Q' := [\underline{k}, \overline{k}] \times [\underline{\tau}_1, \overline{\tau}_1], \qquad Q'' := [\underline{\xi}, \overline{\xi}] \times [\underline{\tau}_2, \overline{\tau}_2].$$

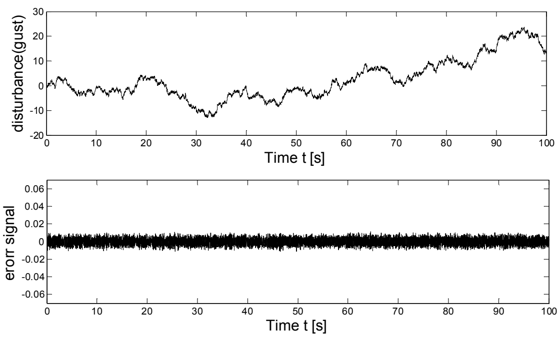

Fig. 4. Simulation results

Further, based on the so-called probability approach to the robustness analysis which becomes now popular [17], we generated 1000 random sets $\{k, \tau_1, \xi, \tau_2\}$ from $Q' \times Q''$ satisfying $\{k, \tau_1\} \in Q'$ and $\{\xi, \tau_2\} \in Q''$ with independently identically distributed (i.i.d.) components and calculated $q_i^{(2)}$s. Next, we verified inequalities (41) together with (42), results of which are presented in Fig. 5.

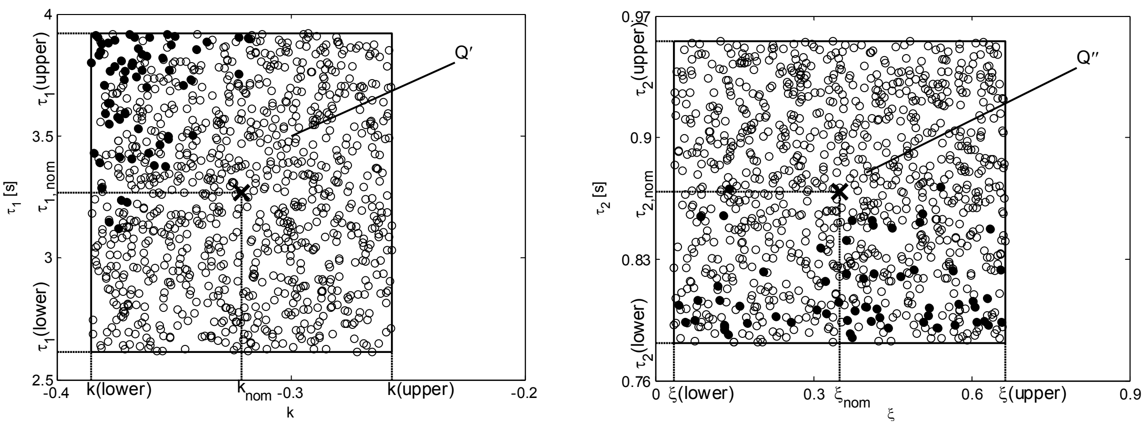

Fig. 5. Robustness evaluation of l_1-optimal autopilot (∘ corresponds to a stable case, • corresponds to an unstable case)

It turned out that the probability for the l_1-optimal autopilot to be robustly stable may be evaluated as 94 %.

CONCLUSION

The optimization of the discrete-time longitudinal autopilot was addressed in this paper. The control objective consisted in minimizing the upper bound on the absolute value of the difference between the desired and true pitch attitude. The control system has been designed as the two-circuit discrete-time closed-loop system containing the PI- and P-type optimal digital feedback controllers. To determine their parameters, l_1-optimization approach has been utilized. The simulation results showed that the application of this approach is appropriate for designing the digital longitudinal autopilot to be able to cope with the wind gust. From robustness point of view, the performance of l_1-optimal autopilot seems to be satisfactory.

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Л. С. Житецький, І. Р. Кваша, А. Ю. Пільчевський, К. В. Мельник. Побудова цифрових автопілотів для поздовжнього каналу на основі l_1-оптимізаційного підходу

Ця стаття стосується **l_1**-оптимального управління, що має бути реалізована цифровим автопілотом поздовжнього каналу, здатного забезпечити бажаний кут тангажа літального апарата за наявності довільного зовнішнього невимірювального збурення. Оптимізація досягається визначенням двох параметрів цифрового ПІ-регулятора, необхідного для стабілізації кутової швидкості тангажа, а також одного параметра П-регулятора, який потрібний для стабілізації кута тангажа. Для демонстрації ефективності цього підходу у статті наведений один ілюстративний числовий приклад та результати моделювання.

Ключові слова: літальний апарат; динаміка поздовжнього руху; цифровий автопілот; дискретний час; ПІ-регулятор; **l_1**-оптимізація.

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Л. С. Житецкий, И. Р. Кваша, А. Ю. Пильчевский, К. В. Мельник. Построение цифровых автопилотов для продольного канала на основе l_1-оптимизационного подхода

Статья касается l_1-оптимального управления, осуществляемого посредствам цифрого авпопилота продольного канала, который способен обеспечить желаемый угол тангажа летательного аппарата при наличии произвольного внешнего неизмеряемого возмущения. Оптимизация достигается определением двух параметров цифрового

ПИ-регулятора, необходимого для стабилизации угловой скорости тангажа, и одного параметра П-регулятора, требуемый для стабилизации угла тангажа. Для демонстрации эффективности этого подхода в статье приведен один пример иллюстративный числовой пример и результаты моделирования.
Ключевые слова: летательный аппарат; динамика продольного движения; цифровой автопилот; дискретное время; ПИ-регулятор; **l_1-**оптимизация.

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