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A. V. Goncharenko

EXTREMALITY OF CONTROL AND PREFERENCES DISTRIBUTIONS "GOODNESS"

Department of mechanics, National Aviation University, Kyiv, Ukraine
E-mail: andygoncharenco@yahoo.com

Abstract—*An attempt to check the "Subjective Entropy Extremization Principle" on the basis of the necessary and sufficient conditions is proposed. The maximizing preferences density distribution for a combined continuous alternative is obtained. Mathematical models for the distributions "goodness" are introduced. Calculation experiments are carried out. The necessary diagrams are plotted.*

Index Terms—Subjective entropy maximum principle; operational control; necessary and sufficient conditions; combined continuous alternative; variational problem; distribution "goodness"; controlling function.

I. INTRODUCTION

A transport systems operation is connected with a control of its functioning processes in the situations of multi-alternativeness. For example, it deals with the problems of control in the field of flight safety, navigation and others.

The responsible person (operator/pilot) solves a specific controlling problem of operational (safety, navigational, combined etc.) uncertainty using a distribution of his/her own preferences in regards with the considered by him/her set of reachable for his/her goals set of operational alternatives.

The problem formulation in the general view and its relation to important scientific and practical tasks refers to the necessity of having some "goodness" estimates for the extremal preferences density distributions.

II. ANALYSIS OF THE LATEST RESEARCHES AND PUBLICATIONS

In the following latest researches and publications it was brought forth the solution to the given problem.

A. Criteria

One of the closest to ours attempts was made in works [1], [2] where the important notions of an aggregation of performances [1, P. 264, (1)], preferences for alternatives [1, P. 265, (2)], social temperature [1, P. 266], and Shannon normalized entropy [1, P. 266, (3)] were used to simulate experiments with group decision processes.

Such kinds of criteria are applied in the presented paper.

B. Application of the Subjective Entropy Maximum Principle

In the context of an active system control, we use the postulated in subjective analysis [3] – [6] principle of the maximum of the subjective entropy of individual preferences.

The mentioned principle happened to be a very useful and helpful tool for solving control problems in the variety of applications [3] – [11]. In particular, papers [7], [8] are about control in active systems with respect to psychophysics laws. And the well known laws: Weber-Fechner, Stevens, Zabrodin (for stimuli and sensations/perceptions connections), Jakob Bernoulli (subjective value) are revealed anew on the basis of the postulated variational principle.

Recursive models for active control systems with memory are considered in paper [9]. Quasi-closed (closed for substance, energy, and information, the latter is just in the conserved view) active systems are able to reduce their own entropy.

Entropy measures of certainty/uncertainty compiled on the basis of the traditional entropy of the Shannon's view and their "goodness" and "badness" are discussed in work [10].

Continuous alternatives preferences densities distributions are obtained in paper [11] with the help of the subjective entropy maximum principle.

Methods of calculus of variations [12], [13] are used in this paper.

All the initial ideas of [3] – [13] have been laid down into the basement of the presented problem solution.

III. OUTLINING THE PREVIOUSLY UNSOLVED PARTS OF THE GENERAL PROBLEM

Individual preferences densities distributions obtained for continuous alternatives of common value separately for each discrete alternative got in work [11] do not show themselves the "goodness" of the optimal/best values amongst them.

It is necessary to check the extremality of the obtained distributions.

IV. FORMULATION OF THE PAPER'S MATERIAL OBJECTIVES (PROBLEM SETTING)

This paper is intended to make an attempt to check the extremization on the basis of necessary and sufficient conditions.

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Then, it will be necessary to introduce some measures for continuous individual preferences densities distributions which conveniently indicate their "goodness" as a parameter of the operational control optimality.

V. Consideration of the Research's Main Material with the Complete Substantiation of the Achieved Scientific Results

When there is a set of discrete operational alternatives each of which has a common continuous alternative, like in cases considered in paper [11], we use a combination of described there methods of subjective analysis to find extremums.

A. Specific Case of Two Alternatives

For the presented paper let us consider a set of two discrete navigational alternatives, for example, two equal in width gates that an airplane can fly through. For each of the given two discrete alternatives (gates), there is a common continuous alternative (the width of the gates). The effectiveness (flights through each of the gates) functions are characterized with some values (of possible losses, for instance).

Let us say, the effectiveness functions and corresponding preferences, for both discrete and continuous alternatives (densities of corresponding preferences distributions) are given in the view shown in Fig. 1.

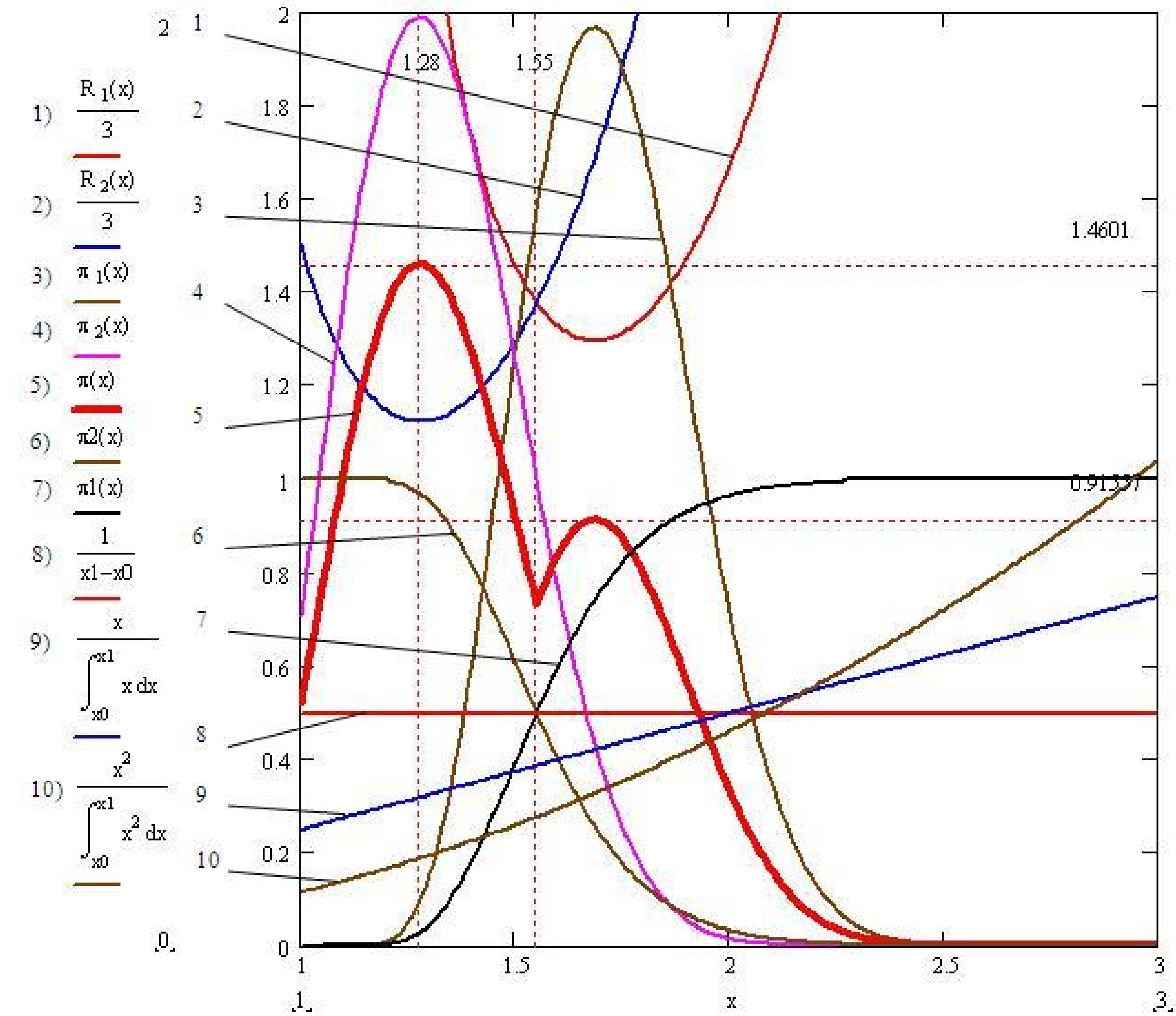

Fig. 1. Effectiveness functions, preferences densities distributions, and corresponding preferences functions for the related alternatives

In Fig. 1 it is presented the effectiveness functions $R_1(x)$ and $R_2(x)$ for the two navigational alternatives; $\pi_1(x)$, $\pi_2(x)$, and $\pi(x)$ are densities distributions of the preferences for the first, second, and common continuous alternatives of the first and second discrete alternatives, and in common case, when the both continuous alternatives form the common one, in regards; $\pi_1(x)$ and $\pi_2(x)$ are preferences functions for the discrete alternatives; $\frac{1}{x_1 - x_0}$, where x_1 and x_0 are the boundary values of the independent variable, $\frac{x}{\int_{x_0}^{x_1} x dx}$, $\frac{x^2}{\int_{x_0}^{x_1} x^2 dx}$ – illustrative uniform, linear, and squared densities distributions for comparison of the optimal $\pi(x)$ with the others and revealing its "goodness" in this context.

The density distribution of the common preference $\pi(x)$, the common distribution for the common continuous alternative, is a polymodal density distribution of the preference, unlike the separate ones of $\pi_1(x)$ and $\pi_2(x)$ for the continuous alternatives of the two given discrete navigational alternatives with their own preferences functions of $\pi_1(x)$ and $\pi_2(x)$ for the corresponding effectiveness functions of $R_1(x)$ and $R_2(x)$ shown in Fig. 1 in the scale factor of $\frac{1}{3}$ for the expositional ease.

The advantages of the density distribution of the common preference $\pi(x)$, obtained with a special kind of integrand, is that it is visible from the shape of the density distribution what the preference density is the highest; i.e. the mode of the distribution in combination with the discrete preferences $\pi 1(x)$ and $\pi 2(x)$ or the density distributions of the separate navigational alternatives preferences $\pi_1(x)$ and $\pi_2(x)$ shows which discrete alternative is the best, how much it is better (more optimal in case of polymodal density distribution), thus the absolutely optimal value of the common continuous alternative for the considered navigational optional problem with the corresponding effectiveness functions of $R_1(x)$ and $R_2(x)$.

B. Canonical Distributions of Preferences of Alternatives

The density distributions $\pi(x)$, $\pi_1(x)$, and $\pi_2(x)$ are obtained with the help of canonical expressions:

$$\pi_i(x) = \frac{e^{-\beta_i R_i(x)}}{\int_{x_0}^{x_1} e^{-\beta_i R_i(x)} dx}, \qquad (1)$$

where β_i are corresponding endogenous parameters (analogous to the inverse social temperature) of the responsible person's psych.

Expressions (1) are got from the functionals:

$$\Phi_{\pi_i(x)} = \int_{x_0}^{x_1} \left[-\pi_i(x) \ln \pi_i(x) - \beta \pi_i(x) R_i(x) \right] dx + \gamma \left[\int_{x_0}^{x_1} \pi_i(x) dx - 1 \right] - \ln \Delta x, \qquad (2)$$

where γ is a weight coefficient for the normalizing condition,

Δx is a degree of accuracy at the entropy determination,

$$\Phi_{\pi(x)} = \int_{x_0}^{x_1} \left[-\pi(x) \ln \pi(x) - \beta \pi(x) \mathit{Integrand}(x) \right] dx + \gamma \left[\int_{x_0}^{x_1} \pi(x) dx - 1 \right] - \ln \Delta x, \qquad (3)$$

where

$$\mathit{Integrand}(x) = \begin{vmatrix} R_2(x) & \mathit{if} \quad R_2(x) < R_1(x) \\ R_1(x) & \mathit{otherwise} \end{vmatrix}. \qquad (4)$$

For discrete alternatives, preferences functions are

$$\pi_i(x) = \frac{e^{-\beta_i R_i(x)}}{\sum_{j=1}^{N=2} e^{-\beta_j R_j(x)}}. \qquad (5)$$

The preferences of (5) are yielded by functional

$$\Phi_{\pi} = -\sum_{i=1}^{N=2} \pi_i(x) \ln \pi_i(x) - \beta \sum_{i=1}^{N=2} \pi_i(x) R_i(x) + \gamma \left[\sum_{i=1}^{N=2} \pi_i(x) - 1 \right]. \qquad (6)$$

C. Conditions of the Extremum

Both (1) and (5) types of preferences are obtained on the basis of the necessary conditions for an extremum to exist in the form of the well known Euler-Lagrange equation:

$$\frac{\partial F^*}{\partial \pi_i} - \frac{d}{dx} \left(\frac{\partial F^*}{\partial \pi'} \right) = 0, \qquad (7)$$

where F^* is the underintegral function of functionals (2), (3) or the functional itself in the case of (6).

Accordingly to the methods of calculus of variations [12], [13], for the sufficient conditions of having the extremum, we ought to add two more conditions to (7). Namely, for the weak maximum of (2), (3), (6) along with the extremal, [12, p. 115, § 24]:

$$F^*_{\pi'\pi'} < 0 \qquad (8)$$

– the strengthened condition by Legendre. This condition replaces by itself the condition of nonpositiveness of the Weierstrass function, [13, p. 375].

Also, the segment of $[x_0, x_1]$ does not contain points adjoint to the point of $x = x_0$ (the strengthened Jacobi's condition). This condition replaces by itself the requirement of the possibility for the given extremal to be included into the extremals field.

But for (2), (3), (6) the condition of (8) turns into

$$F^*_{\pi'\pi'} \equiv 0, \tag{9}$$

as the functionals do not depend upon the derivative of π' explicitly.

Let us analyze the necessary sign of the second variation:

$$\delta^2\Phi_\pi[h_\pi] = \frac{1}{2}\int_{x_0}^{x_1}\left(F^*_{\pi_i\pi_i}h_\pi^2 + 2F^*_{\pi_i\pi_i'}h_\pi h_\pi' + F^*_{\pi_i'\pi_i'}h_\pi'^2\right)dx, \tag{10}$$

where $h_\pi = h_\pi(x)$ is an increment of the function of $\pi_i = \pi_i(x)$.

Hence, it will be necessary, instead of (8) and because of (9) and through (10),

$$F^*_{\pi_i\pi_i} < 0. \tag{11}$$

Indeed,

$$F^*_{\pi_i} = -\ln\pi_i - 1 - \beta_i R_i + \gamma, \quad F^*_{\pi_i\pi_i} = -\frac{1}{\pi_i} < 0. \tag{12}$$

Which, at the absence at the segment of $[x_0, x_1]$ of the adjoint to the point of $x = x_0$ points, will ensure the maximum to the functional.

Concerning the latter condition the Jacobi's equation:

$$\left(F^*_{\pi_i\pi_i} - \frac{d}{dx}F^*_{\pi_i\pi_i'}\right)u - \frac{d}{dx}\left(F^*_{\pi_i'\pi_i'}u'\right) = 0, \tag{13}$$

where $u = \dfrac{\partial\pi_i(x, C)}{\partial C}$ is a certain function, along with each fixed from the family of the curves, where C is a parameter of the bunch of the extremals with the center in the initial boundary point of $A(x_0, \pi_i(x_0))$.

Although, for the specific case, the equation of (13) does not help much, since

$$F^*_{\pi_i\pi_i}u = 0, \quad \Rightarrow \quad u = 0, \tag{14}$$

because of (9) – (12).

D. Experiments

Calculation experiments for functional (3) with the data $x_0 = 1$, $x_1 = 3$, $\beta = 0.9$, $\Delta x = 0.01$,

$$R_1(x) = b_1(x - d_1)^{c_1} + \frac{1}{a_1(x - d_1)}, \tag{15}$$

$b_1 = 3$, $d_1 = 0.999$, $c_1 = 3$, $a_1 = 0.5$,

$$R_2(x) = 0.5 + b_2(x - d_2)^{c_2} + \frac{1}{a_2(x - d_2)}, \tag{16}$$

$b_2 = 3$, $d_2 = 0.657$, $c_2 = 3$, $a_2 = 0.75$, give the results illustrated in Fig. 1.

In order to check the existence of the maximum, let us consider the functional of (3) with the densities of preferences of $\dfrac{1}{x_1 - x_0}$, $\pi_1(x)$, and $\pi_2(x)$ as some variations of the extremal preference density of $\pi(x)$. Then substituting $\pi(x)$ in turn with $\dfrac{1}{x_1 - x_0}$, $\pi_1(x)$ and $\pi_2(x)$ in functional (3), we can make sure in the maximum existence provided with the density distribution of $\pi(x)$.

It is represented in Fig. 2.

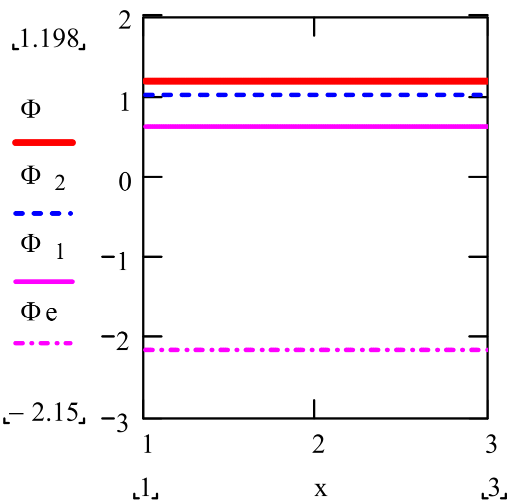

Fig. 2. The values of the optimized functional

Thus, the responsible person controls the operational process, through the optimal preferences densities distributions as the optimal process controlling functions, with respect to the minimization of the corresponding negative effects of the considered operational alternatives related with the functions of possible risks (losses, harmfulness etc.) $R_i(x)$ and uncertainty of the choice.

In Fig. 2 it is depicted: Φ is for the value of integral (3); Φ_2 as the value of integral (3) variated with preferences distribution $\pi_2(x)$; Φ_1 – the same with $\pi_1(x)$; Φ_e – with $\dfrac{1}{x_1 - x_0}$.

We have to say that unconditional entropy does not give the same result. Using the equation for the entropy of the individual preferences densities distributions:

$$H_{\Delta x}^{(i)} = -\int_{x_0}^{x_1} \pi_i(x) \ln \pi_i(x) dx - \ln \Delta x\,, \tag{17}$$

we get the following picture shown in Fig. 3.

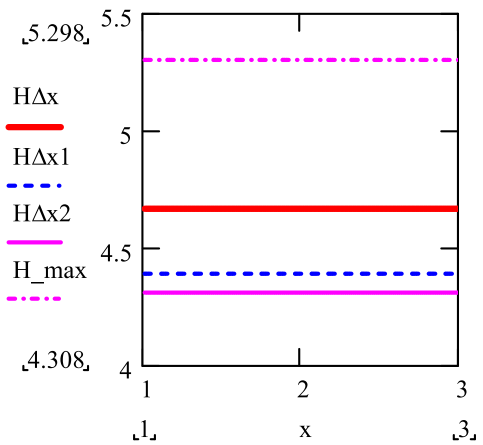

Fig. 3. Entropies of preferences densities distriburions

In Fig. 3 it is depicted: HΔx is for the value of integral (17) $H_{\Delta x}$ with $\pi(x)$; HΔx1 – $H_{\Delta x}^{(1)}$ with $\pi_1(x)$; HΔx2 – $H_{\Delta x}^{(2)}$ with $\pi_2(x)$; H_max – $H_{\Delta x}^{(e)}$ with

$$\pi_e(x) = \frac{1}{x_1 - x_0}\,. \tag{18}$$

It is quite naturally that the uniform preferences density of (18) delivers the maximal value to the subjective entropy of (17). This situation radically differs from that one with the optimized functional (3). Which is visible from the comparison of the correspondingly considered values in Fig. 2 and Fig. 3.

E. The "Goodness"

In order to discover the "goodness" of the optimal preference's density distribution $\pi(x)$, let us introduce the following measures:

$$|\pi(x) - \pi_i(x)|\,, \qquad \Phi_{\pi(x)} - \Phi_{\pi_i(x)}\,. \tag{19}$$

Here, in measures (19), integrals $\Phi_{\pi(x)}$ and $\Phi_{\pi_i(x)}$ are taken from the form of (3) with the corresponding preferences densities of: $\pi(x)$ and $\pi_1(x)$, $\pi_2(x)$, in the form of (1), obtained from the related (3) and (2) functionals; and $\pi_e(x)$ in the form of (18).

As far as we can see the measures of (19) take into consideration the absolute differences between the compared preferences densities distributions and increment values of the optimized functionals with the integrand of (4).

It is logically to choose $\pi_e(x)$ of the expression of (18) for the basis comparison distribution since it delivers the maximal value to the subjective entropy of $H_{\Delta x}^{(e)}$ of the functional of (17) and means total uncertainty of controlling functions (individual preferences densities) for the according alternatives in an operational situation.

Also, let us introduce the following combined measures:

$$|\pi(x) - \pi_i(x)| \cdot \left(\Phi_{\pi(x)} - \Phi_{\pi_i(x)}\right), \qquad \frac{|\pi(x) - \pi_i(x)|}{\Phi_{\pi(x)} - \Phi_{\pi_i(x)}}\,,$$

$$|\pi_j(x) - \pi_i(x)| \cdot \left(\Phi_{\pi_j(x)} - \Phi_{\pi_i(x)}\right), \qquad \frac{|\pi_j(x) - \pi_i(x)|}{\Phi_{\pi_j(x)} - \Phi_{\pi_i(x)}}\,,$$

$$|\pi_j(x) - \pi_i(x)| \cdot \frac{\Phi_{\pi_j(x)} - \Phi_{\pi_i(x)}}{\Phi_{\pi_k(x)} - \Phi_{\pi_q(x)}}\,,$$

$$\frac{|\pi_j(x) - \pi_i(x)|}{\pi_j(x) - \pi_i(x)} \cdot \left(\Phi_{\pi_j(x)} - \Phi_{\pi_i(x)}\right),$$

$$\frac{|\pi_j(x) - \pi_i(x)|}{\pi_k(x) - \pi_q(x)} \cdot \left(\Phi_{\pi_j(x)} - \Phi_{\pi_i(x)}\right),$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots,$$

$$\frac{|\pi_j(x) - \pi_i(x)|}{|\pi_k(x) - \pi_q(x)|} \cdot \frac{\Phi_{\pi_k(x)} - \Phi_{\pi_q(x)}}{\Phi_{\pi_j(x)} - \Phi_{\pi_i(x)}}\,, \tag{20}$$

and many all other possible combinations.

These models of (20) have a meaning of a comparison of the second order. They consider first differences and increments, the varieties of their absolute and relative values.

One of the models of (20), in particular

$$\frac{|\pi(x) - \pi_i(x)|}{|\pi(x) - \pi_e(x)|} \cdot \frac{\Phi_{\pi(x)} - \Phi_{\pi_i(x)}}{\Phi_{\pi(x)} - \Phi_{\pi_e(x)}}\,, \tag{21}$$

which considers the basis of $\pi_e(x)$, gives a result illustrated in Fig. 4.

F. The Researches Results

The presented researches, described with the formulas of (1) – (21), portrayed in Fig. 1 – Fig. 4, give the following results. The obtained optimal controlling functions are (1) and (5) for operational controlled functionals (2), (3) and (6) in case of the continuous and discrete operational alternatives (see Fig. 1).

Sufficient conditions of maximum in the given case are (7) and (11) instead of (7) and (8).

The maximal value of functional (3) (see Fig. 2) corresponds with the minimal one of losses (15) and (16) (see Fig. 1). It does not coincide with the maximal value of subjective entropy (17) (see Fig. 3).

For the evaluation of the extremal control "goodness" there is a few models (19) – (21).

The variant of (21) is an interesting one because it shows all climaxes and corner points (see Fig. 4) of the functions depicted in Fig. 1 in the relative interpretation which is rather convenient.

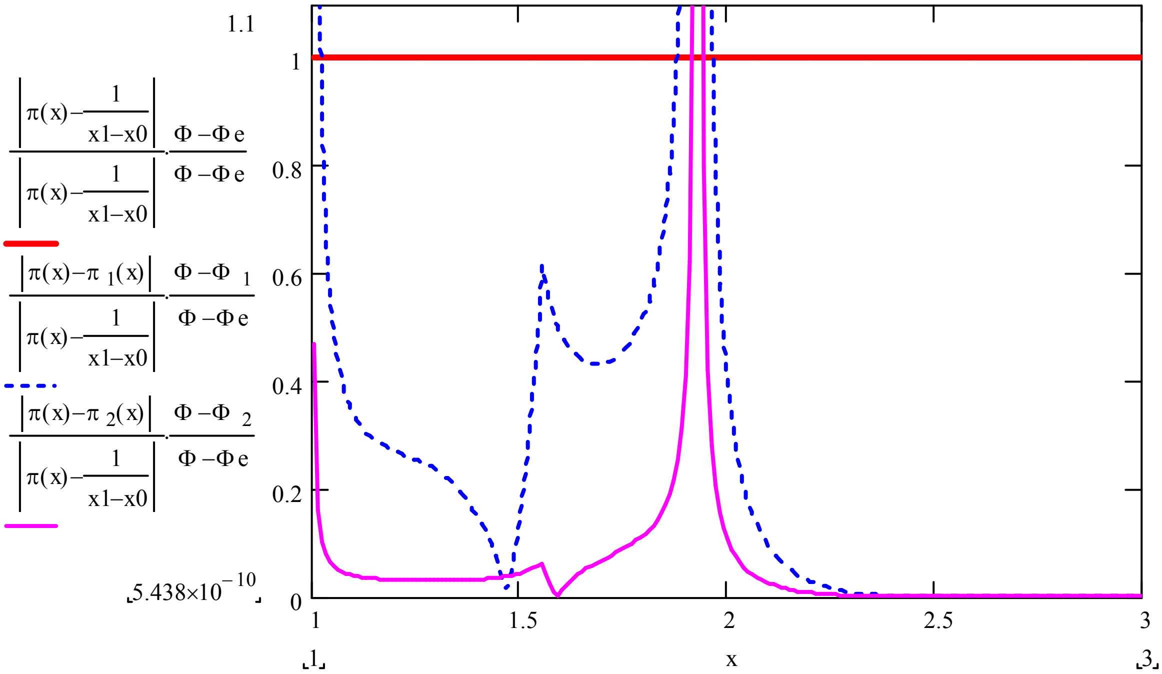

Fig. 4. Relative "goodness" of the optimal preferences densities distriburions compared with its "goodness" to the uniform (maximal subjective entropy/uncertainty) one

CONCLUSIONS ON THE PRESENTED RESEARCH

The proposed approach allows finding optimal control functions as extremals. And describe their "goodness" in some convenient manner. It is impossible to see the advantage of the optimal preference density distribution of $\pi(x)$ on the basis of the subjective entropy itself in the traditional view of (17).

PROSPECTS OF FURTHER STUDYING IN THE SPECIFIED DIRECTION

The further researches are worth of investigating some other models of the optimal control "goodness".

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Goncharenko Andriy. PhD. Graduate for a doctor's degree.
Department of Mechanics, National Aviation University, Kyiv, Ukraine.
Education: Odessa Institute of Marine Engineers, Odesa, Ukraine (1984).
Research area: Control in active systems, Flight safety, Variational principles.
Publications: 74.
E-mail: andygoncharenco@yahoo.com

А. В. Гончаренко. Екстремальність керування та «якість» розподілів переваг

Запропоновано спробу перевірки «Принципу екстремізації суб'єктивної ентропії» на основі необхідних та достатніх умов. Для комбінованої неперервної альтернативи отримано розподіл щільності переваг, що максимізує. Введено математичні моделі для «якості» цих розподілів. Виконано розрахункові експерименти. Побудовано необхідні діаграми.

Ключові слова: принцип максимуму суб'єктивної ентропії; експлуатаційне керування; необхідні та достатні умови; комбінована неперервна альтернатива; варіаційна задача; «якість» розподілу; керуюча функція.

Андрій Вікторович Гончаренко. Кандидат технічних наук. Докторант.
Кафедра механіки, Національний авіаційний університет, Київ, Україна.
Освіта: Одеський інститут інженерів морського флоту, Одеса, Україна (1984).
Напрям наукової діяльності: керування в активних системах, безпека польотів, варіаційні принципи.
Кількість публікацій: 74.
E-mail: andygoncharenco@yahoo.com

А. В. Гончаренко. Экстремальность управления и «качество» распределений предпочтений

Предложена попытка проверки «Принципа экстремизации субъективной энтропии» на основе необходимых и достаточных условий. Для комбинированной непрерывной альтернативы получено максимизирующее распределение плотности предпочтений. Введены математические модели для «качества» этих распределений. Выполнены расчетные эксперименты. Построены необходимые диаграммы.

Ключевые слова: принцип максимума субъективной энтропии; эксплуатационное управление; необходимые и достаточные условия; комбинированная непрерывная альтернатива; вариационная задача; «качество» распределения; управляющая функция.

Андрей Викторович Гончаренко. Кандидат технических наук. Докторант.
Кафедра механики, Национальный авиационный университет, Киев, Украина.
Образование: Одесский институт инженеров морского флота, Одесса, Украина (1984).
Направление научной деятельности: управление в активных системах, безопасность полетов, вариационные принципы.
Количество публикаций: 74.
E-mail: andygoncharenco@yahoo.com