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G. V. Kit

ALGORITHMS OF SELECTION OF ESSENTIAL FEATURES OF SIGNALS, WHICH ARE INVARIANT TO MOVE AND ROTATION

Department of computer sciences, University "Ukraine", Kyiv, Ukraine
E-mail: nika126@voliacable.com

Abstract*—The mathematical model for invariant processing of pictures in visual system is proposed. The base of this model consists the construction of functional, which reaches it's maximum when the values of transforms $s(i)$ be equal to hided transforms of signal $s_0(i)$.*

Index Terms—Invariant processing; Kruwtchuk functions; ortonormal system; functional of energy; rotation; integral transforms.

I. INTRODUCTION

A problem of automatic recognizing and classification of signals, when processing visual signals deals with selecting of the system of essential features of signal, which are invariant to operator of generalized move. Decision of this problem give us a possibility to carry out task of identification and restoring input signals of visual system. The problem of invariant recognizing deals with the problem of pressing of information for reducing it.

II. THE ANALYSIS OF THE LATEST RESEARCHES AND PUBLICATIONS

As known visual system functions as multi channel Furrier-filter, where every channel is organized on confident space frequency. In [3] as weight function for mathematical modeling was proposed polinoms of Hermit. But these functions by using the computer for modeling such processes lost a property of ortogonality.

Therefore an algorithm of selecting of invariant features of signal, based on ortonormal Kruwtchuk functions, is proposed [4]. Kruwtchuk functions is a full ortonormal system on finite number of discrete points, therefore they are free from defects of functions of continuous argument.

III. TASK STATEMENT

Let on input of system sends a signal $y(t)$, that describe a function of distribution of brightness of picture. Let also function $y(t)$ feels some transforms (moving, rotation) under operators of generalized move, R^s (o. g. m.) [6]: $y[s(i)] = R^s\, y(i), i \in Q$, Q – discrete set. So, on input of system comes a transformed signal $y[s_0(i)]$ with certain transform $s_0(i)$. The task is to determine parameters $s_0(i)$ of transform and select special features of signal.

So, the task is to find such transforms, which gives us

$$W(s, s_0) = \sum_{k \in M} |c_k(s, s_0)|^2 \to \max,$$

where M is the subset of numbers of generalized spectral coefficients. This set M is formed by finding of generalized spectral coefficients of signal, square for which have the greatest values and numbers of these coefficients forms set M.

IV. TASK SOLUTION

Arbitrary linear transform can be presented as superposition of transforms of moving and rotation. For moving we have

$$c_k(s, s_0) = \int_Q R^{s_0} y(i) \varphi_k^{(p)}(i) d\mu(i).$$

For rotation:

$$c_k(s, s_0) = \int_Q y(s_0 t) \varphi_k^{p}(st) d\mu(i).$$

The system of ortonormal functions for rotation transform is formed in polar coordinates.

We choose as the system of ortonormal functions Kruwtchuk functions $\Omega = \left\{F_k^{(p)}, k = \overline{0, N-1}\right\}$.

Polinoms of Kruwtchuk – polinoms, that are ortonormal on points $Q = \{0, 1, \ldots, N-1\}$ relative to binomial disnribution.

$$p(i) = \frac{N!\, p^i (1-p)^{N-i}}{i!(N-i)!}.$$

The polinoms of Kruwtchuk are determined as decision of discrete analogue of hypergeometrical equation

$$\sigma(x) y'' + \tau(x) y' + \kappa y = 0,$$

where $\sigma(x)$ is polinom of the second order; $\tau(x)$ is the polinom of first order.

$$k_n^{(p)}(i,N)=(1-p)^n C_n^i F[-n,i-N;i-n+1;p/(p-1)],$$

$$0\le n\le N;\quad 0<p<1, i=\overline{0,N}.$$

Ortonormal functions of Kruwtchuk are

$$F_n^{(p)}(i,N)=\frac{k_n^{(p)}(i,N)}{k_n^{(p)}(0,N)}.$$

Ortonormal functions of Kruwtchuk are formed as [6]:

$$F_0^{(p)}(i,N)=\sqrt{\frac{N!(1-p)^{N-i}}{i!(N-i)!}};\qquad F_1^{(p)}(i,N)=(i-pN)\sqrt{\frac{p^{i-1}(1-p)^{N-i-1}(N-i)!}{i!(N-i)!}};$$

$$F_{n+1}^{(p)}(i,N)=\frac{i-n-p(N-2n)}{\sqrt{(n+1)(N-n)p(1-p)}}F_n^{(p)}(i,N)-\sqrt{\frac{(N-n+1)n}{(N-n)(n+1)}}F_{n-1}^{(p)}(i,N),$$

where $i=\overline{0,N-1};\quad n=\overline{1,N-1};\quad 0<p<1.$

Parameter p determines degree of unsymmetry of functions of Kruwtchuk $F_n^{(p)}(i,N)=(-1)^n F_n^{(1-p)}\times(N-i,N)$.

When $p=1/2$ functions of Kruwtchuk are symmetric.

Generalized spectral coefficients of signal $y(t)$ obtained for every $p=0,1;0,2;\ldots;0,9$:

$$c_k^{(p)}(j)=\sum_{i=0}^{N-1}y(i)F_k^{(p)}((j-i)\bmod N,N);\ j,k=\overline{0,N-1}.$$

When a maximum of functional of energy is determined, $W^{(p)}(s,s_0)$, that is values $s(i)$ became equal $s_0(i)$, a signal $y(i)$ is restored by

$$\tilde{y}(i)=\sum_{k\in M}c_k(s,s_0)R^{s_0}F_k^{p_0}(i,N).$$

Algorithm of determination of spectral coefficients for rotating transforms is similar, but it is necessary to determine functions of Kruwtchuk relative to rotation.

We consider generalized spherical functions and there relation with functions of Kruwtchuk.

Any representation of group of rotation $g\to T(g)$ in linear space R can be written as

$$T(g)=e^{-i\varphi\mathbf{nJ}},$$

where $\mathbf{J}=(J_x,J_y,J_z)$ are infinitesimal operators.

Basis $\{\psi_{jm}\}$ under action of operator $T(g)$ transforms to a new basis $\{\psi'_{jm}\}$:

$$\psi'_{jm'}=T(g)\psi_{jm},\quad \psi'_{jm'}=\sum_m D_{mm'}^j(g)\psi_{jm}.$$

$$D_{mm'}^j(g)=\left(\psi_{jm},T(g)\psi_{jm'}\right).$$

It important to know matrices elements $D_{mm'}^j(g)$ for application, when rotation g is described by Euler angles: $g=g(\alpha,\beta,\gamma)$. $T(g)=e^{-i\gamma J_{z'}}e^{-i\beta J_{y''}}e^{-i\alpha J_z}$, Where $J_z,J_{y''},J_{z'}$ are infinitesimal operators for rotation around axis z,y'',z' corresponding. Dependence of matrices elements $D_{mm'}^j(\alpha,\beta,\gamma)=\left(\psi_{jm},T(g)\psi_{jm'}\right)$ from angles Euler α,β,γ

$$D_{mm'}^j(\alpha,\beta,\gamma)=e^{-im\alpha}d_{mm'}^j(\beta)e^{-im'\gamma};$$

$$d_{mm'}^j(\beta)=\left(\psi_{jm},e^{-i\beta J_y}\psi_{jm'}\right).$$

(a,b) is a scalar product in space L_G^2.

Functions $D_{mm'}^j(\alpha,\beta,\gamma)$ are known as generalized spherical functions or D-functions Vigner. The matrix $D_{mm'}^j(\alpha,\beta,\gamma)$ is

$$\sum_{m''}D_{mm''}^j(\alpha,\beta,\gamma)D_{m'm''}^{*j}(\alpha,\beta,\gamma)=\delta_{mm'},$$

$$d_{mm'}^j(\beta)=(-1)^{m-m'}d_{m'm}^j(\beta).$$

$$d_{mm'}^j(s)=\frac{(-1)^{j-m'}}{2^j}\sqrt{\frac{(j+m)!}{(j-m)!(j-m')!(j+m')!}}(1-s)^{-\frac{m-m'}{2}}(1+s)^{-\frac{m+m'}{2}}\frac{d^{j-m}}{ds^{j-m}}\left[(1-s)^{j-m'}(1+s)^{j+m'}\right].$$

$$s=\cos\beta.$$

The functions $d_{mm'}^j(\beta)$ expressed through functions of Kruwtchuk as:

$$(-1)^{m-m'}d_{mm'}^j(\beta)=F_k^{(p)}(n,N).$$

here $p=\sin^2(\beta/2)$.

So, functions $d_{mm'}^j(\beta)$ – full ortonormal system of base functions concerning to rotation.

$$d_{m,m'+1}^j=\frac{2\left(m'\cos\beta-m\right)/\sin\beta}{\sqrt{\left(j-m'\right)\left(j+m'+1\right)}}d_{m,m'}^j-\sqrt{\frac{\left(j+m'\right)\left(j-m'+1\right)}{\left(j-m'\right)\left(j+m'+1\right)}}d_{m,m'-1}^j.$$

The mistake of restore of signal is determined as $\tilde{\varepsilon} = \|\tilde{y} - y\|_{R^M}$. This stage modeling one of stages of recognizing of picture, when the information about picture not enough for its recognizing.

Functions of Kruwtchuk for $p = 0.2$ and $p = 0.4$ are shown on Figs 1 and 2.

The distribution of spectral coefficients are shown on Figs 3 and 4.

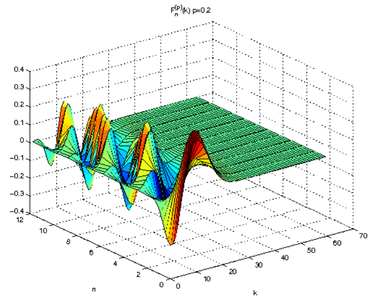

Fig. 1. Functions of Kruwtchuk for $p = 0.2$

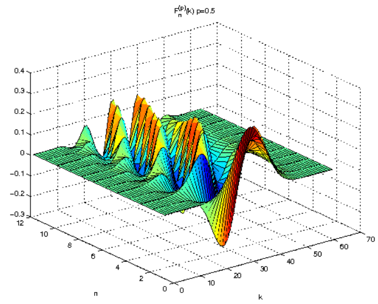

Fig. 2. Functions of Kruwtchuk for $p = 0.4$

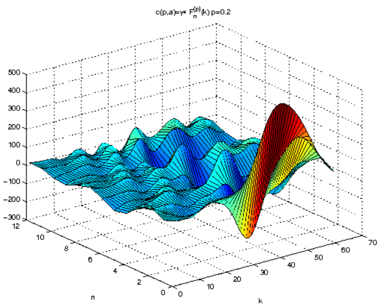

Fig. 3. The distribution of spectral coefficients for $p = 0.2$

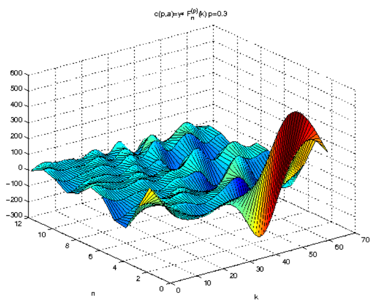

Fig. 4. The distribution of spectral coefficients for $p = 0.5$

Restored signal is represented on Fig. 5.

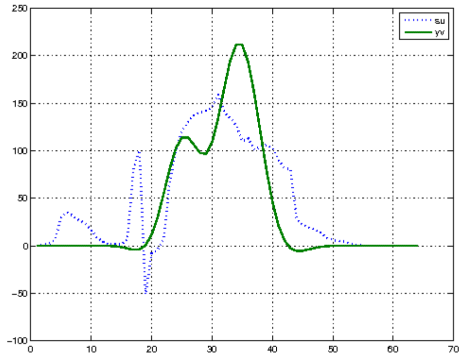

Fig. 5. Restored signal: accuracy $\tilde{\varepsilon} = 0{,}1$

CONCLUSIONS

The mathematical model for invariant processing of pictures in visual system is proposed. The base of this model consists the construction of functional, which reaches it's maximum when the values of transforms $s(i)$ be equal to hided transforms of signal $s_0(i)$. A program providing for processing of signals is developed, which can de used for deciding of problems to press and restore of electro cardiograms and encephalograms and so on.

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Kit Grigorj. Associate Professor.
Department of computer sciences, University "Ukraine", Kyiv, Ukraine.
Education: University "Ukraine", Kyiv, Ukraine (2000).
Research area: information technology, mathematical modeling.
Publications: 8.
E-mail: nika126@voliacable.com

Г. В. Кіт Алгоритми виділення суттєвих ознак сигналів зорових систем, інваріантних відносно зсуву та обертання

Запропоновано математичні моделі побудови систем розпізнавання і відновлення сигналів зорових систем, інваріантних відносно перетворень зсуву та обертання. Підґрунтя для побудови інваріантних систем є пошук максимуму функціоналу енергії, яке досягається у разі збігу значення перетворень сигналу $s(i)$ із значеннями прихованих ознак $s_0(i)$.

Ключові слова: інтегральні перетворення; ітераційна схема; інваріантні перетворення; ортонормовані базисні функції; функції Кравчука.

Кіт Григорій Васильович. Доцент.
Кафедра комп'ютерних наук університету «Україна», Київ, Україна.
Освіта: Університет «Україна», Київ, Україна, (2000).
Напрям наукової діяльності: інформаційні технології, математичне моделювання систем.
Кількість публікацій: 8.
E-mail: nika126@voliacable.com

Г. В. Кит. Алгоритмы выделения существенных признаков сигналов зрительных систем, инвариантных относительно сдвига и вращения

Предложены математические модели построения систем распознавания и восстановления сигналов зрительных систем, инвариантных относительно сдвига и вращения. Основанием для построения инвариантных систем является поиск максимума функционала энергии, которое достигается при совпадении значения преобразований сигнала $s(i)$ со значениями скрытых признаков $s_0(i)$.

Ключевые слова: интегральные преобразования, итерационная схема, инвариантные преобразования, ортонормированные функции, функции Кравчука.

Кит Григорий Васильевич. Доцент.
Кафедра компьютерных наук Университета «Украина», Киев, Украина.
Образование: Университет «Украина», Киев, Украина, (2000).
Научные интересы:
Информационные технологии, математическое моделирование систем.
Количество публикаций 8.
E-mail: nika126@voliacable.com