AUTOMATIC CONTROL SYSTEMS

UDC 681.5.09 (045)

[1]V. N. Azarskov,
[2] D. D. Gristchak

VIBRATION DAMPING FOR THE PROBLEMS OF AIRCRAFT MOTION

Aircraft Control Systems Department National Aviation University Kyiv, Ukraine
E-mails: [1]azarskov@nau.edu.ua, [2]gristchak@sistd.com

Abstract—*Two-step hybrid asymptotic method based on perturbation methods and phase integrals (method WKB) is used to obtain approximate analytical solutions of the nonlinear problem of the vibrations of the aircraft near the rough surface, which reduces the need for the integration of singular nonlinear differential equation with time-variable periodic coefficients for given initial conditions. This solution is not limited to the value of the dimensionless amplitude of perturbations and the nature of the order non-linear restoring force. The resulting solution has the form of the sum, where each term consists of two functions according to the method of perturbation (in scalar parameter when the nonlinear component of the original equation) and the WKB-approximation, effective in the integration of singular differential equations with variable coefficients. For specific numerical results, a comparison with the data of direct numerical integration of the equations of the original problem.*

Index Terms—Dynamics of the aircraft; a wavy water surface; the hybrid asymptotic approach; approximate analytical solution.

I. INTRODUCTION

It is known [1], due to the influence of regular wavy water surface of an aircraft under certain driving conditions forced vibrations in the plane of the pitch and roll in the area. Of particular interest in terms of the emergence of dynamic effects is the case of joint oscillations in flight at an arbitrary angle to the wave. Mechanical analogy of this dynamic process can serve as a model of oscillations of a mathematical pendulum with a vibrating at a given law point of suspension and the length of which is a function of time. It should be noted that the existing solutions are usually reduced to the solution of the Mathieu equation under the conditions that the time is nonlinear restoring forces (in particular cubic) character [2], and the dimensionless amplitude of the parametric excitation of a small quantity.

In this paper, based on the hybrid asymptotic approach [3] – [5] proposed an approximate analytical solution of the problem of the dynamics of the aircraft near the rough surface when the time restoring forces is nonlinear order, and the dimensionless amplitude of the perturbation can be variable and not small, because in this case may deteriorate the dynamic characteristics of the aircraft.

II. PROBLEM STATEMENT

The basic differential equation. For cruising aircraft type "Duck" design scheme of the dynamics of the process of motion under the assumption that the main influence on the transients in the plane of the roll has a rear carrier wing, represented in Fig. 1.

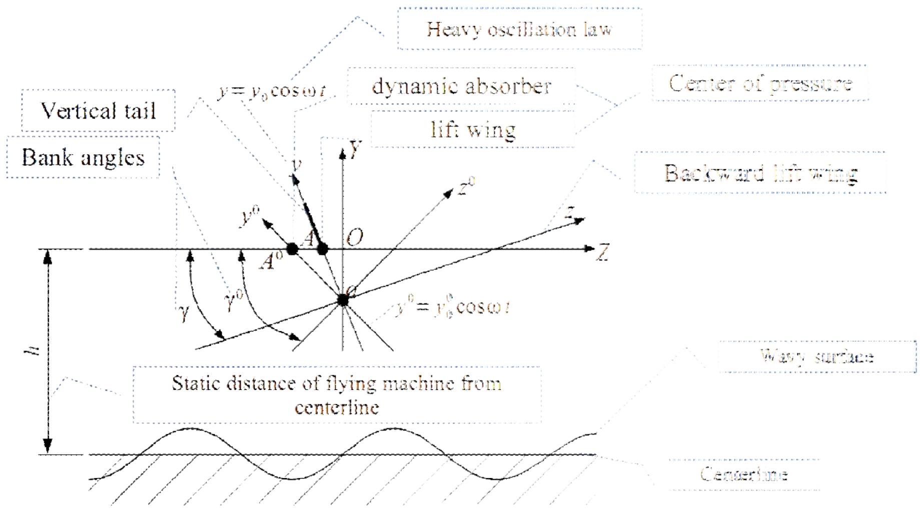

Fig. 1. Design scheme of the dynamics of aircraft near the waved surface

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The diagram of Fig. 1 adopted the following notation: inertial coordinate system moving with the velocity (FV) uniformly in a straight line and progressively. Rigidly connected with the machine coordinate system.

In contrast to [1], where nonlinearity moment cubic restoring forces takes place, in this case the order nonlinearity will be assumed. In this case, the differential equation describing the rolling takes the form

$$I_x\ddot{\gamma}(t)+n\dot{\gamma}(t)+c_{11}\gamma(t)+c_{22}\gamma^m(t)=0, \quad (1)$$

where c_{11}, c_{22} are determined by the expressions of [1]:

$$c_{11}=c_1+c_2;\quad c_{22}=3c_1l_1^2/\left(7h_1^2\right)+c_2\,3l_2^2/\left(7h_2^2\right);$$
$$c_1=2\alpha_1 ql_1^3/\left[5\bar{h}_1^2\left(1+6/\lambda_1^2\right)\right]; \quad (2)$$
$$c_2=2\alpha_2 ql_2^2/\left[5\bar{h}_2^2\left(1+6/\lambda_2^2\right)\right],$$

where q is the velocity head, l_1, l_2 denote the semispan wings; $\bar{h}_1, \bar{h}_2$ are the relative distances between the wings and the screen; λ_1, λ_2 are the elongations; α_1, α_2 are the numerical coefficients, n is the total damping coefficient.

Set

$$h_1=h_2=h^*=h+I_0\cos\omega t, \quad (3)$$

taking into account the periodic addition of vertical oscillations in the pitch plane and introducing the dimensionless time

$$2\tau=\omega t, \quad (4)$$

the basic equations of the problem in the form:

$$\frac{d^2\gamma}{d\tau^2}+\nu\frac{d\gamma}{d\tau}+(a-2\mu\cos 2\tau)\gamma+\eta\gamma^m=0, \quad (5)$$

where

$$\nu=2n/(I_x\omega);\quad a=4c_{11}/\left(I_x\omega^2\right);$$

$$\eta=4c_{22}/\left(I_x\omega^2\right);\quad \mu=aI_0/h=ka.$$

In the general case, equation (5) can be represented as:

$$\varepsilon^2\gamma''(\tau)+\bar{\nu}\gamma'(\tau)+b(\tau)\gamma(t)+\bar{\eta}\gamma^m(\tau)=0. \quad (6)$$

In this equation

$$(\ )'=\frac{d(\ )}{d\tau};\quad b(\tau)=\frac{a}{\mu}-2\cos 2\tau;$$
$$\varepsilon^2=\frac{1}{\mu};\quad \bar{\nu}=\frac{\nu}{\mu}, \quad (7)$$

where ε and $\bar{\eta}$ are the scalar parameters.

Equation (6) describes the process is singular (in the case of smallness) nonlinear differential equations with variable time factor. The solution of the problem reduces generally [2] – [5], to the determination of the stability regions using Ince-Strutt diagram with known analytical solutions. In the general case, the equation (6) has no analytical exact solution. Usually in these cases, using direct numerical integration methods or asymptotic approaches.

This paper uses a hybrid approach based on the asymptotic perturbation method in the parameter and the method of phase integrals (WKB method) in the parameter [6] – [8].

III. HIBRID ASYMPTOTIC SOLUTION

In accordance with the decisions of the equation (6) based on a hybrid approach [5], we have

$$\gamma(\tau)=E(\tau)\left\{\sin I(\tau)\left[c_1+\bar{c}_1\left(G(\gamma_0)\right)\right]+\cos I(\tau)\left[c_2+\bar{c}_2\left(G(\gamma_0)\right)\right]\right\}, \quad (8)$$

where

$$E(\tau)=\exp\int-\frac{\bar{\nu}}{2}d\tau;\quad I(\tau)=\int\varepsilon^{-1}Q(\tau)^{1/2}d\tau;$$
$$Q(\tau)=\left[-\frac{\bar{\nu}^2}{4}+b(\tau)\right];$$
$$\bar{c}_1=-\bar{\eta}\int\frac{\cos I(\tau)G(\gamma_0)}{\exp\int-\frac{\bar{\nu}}{2}d\tau I'(\tau)};$$
$$\bar{c}_2=+\bar{\eta}\int\frac{\sin I(\tau)G(\gamma_0)}{\exp\int-\frac{\bar{\nu}}{2}d\tau I'(\tau)}; \quad (9)$$
$$G(\gamma_0)=[\gamma_0]^m;$$
$$\gamma_0=E(\tau)\left[c_1\sin I(\tau)+c_2\cos I(\tau)\right].$$

As an example we take the initial conditions in the form:

$$\gamma(0)=1,\quad \gamma'(0)=0. \quad (10)$$

We compute the principal components of the solution (8):

$$E(\tau)=\exp\left[-\frac{\bar{\nu}}{2}\tau\right];$$

$$Q(\tau)=\exp\left[-\frac{\bar{\nu}^2}{4}+\frac{a}{\mu}-2\cos 2\tau\right];$$

$$I(\tau)=\varepsilon^{-1}\int\left[-\frac{\nu^2}{4}+\frac{a}{\mu}-2\cos 2\tau\right]^{1/2}d\tau.$$

IV. THE NUMERICAL RESULTS

For the analysis corresponds to the assumption of an approximate analytical solution of direct numerical integration of the original equation we take the following parameters:

$$\mu = 10;\ \ \varepsilon = \left(10^{-1}\right)^{0,5} = \left(\frac{1}{10}\right)^{0,5} = 0,316;\ \ \nu = 1;\ \ a = 1;\ \ \bar{\nu} = \frac{\nu}{\mu} = 0,1;\ \ E(\tau) = \exp\left[-\frac{\bar{\nu}}{2}\tau\right] = \exp[-0,05\tau];$$

$$Q(\tau) = \exp\left[-\frac{\bar{\nu}^2}{4} + \frac{a}{\mu} - 2\cos(2\tau)\right] = \left[-0,25\cdot10^{-2} + 0,1 - 2\cos(2\tau)\right],$$

$$Q(\tau) = [0,0975 - 2\cos 2\tau] = 0,0975[1 - 20,5188\cos 2\tau];$$

$$Q(\tau)^{1/2} = 0,0975^{0,5}[1 - 20,5188\cos 2\tau]^{0,5} = 0,312[1 - 20,52\cos 2\tau]^{0,5}$$
$$= 0,312\left[1 - 10,26\cos 2\tau - 52,634\cos^2 2\tau\right] = 0,312 - 3,201\cos 2\tau - 16,422\cos^2 2\tau.$$

$$I = 0,316\int Q(\tau)^{1/2}d\tau = 0,316\int\left(0,312 - 3,201\cos 2\tau - 16,422\cos^2 2\tau\right)d\tau.$$

$$I(\tau) = -2,5t - 0,506\sin 2t - 0,65\sin 4t,\quad I'(\tau) = -2,5 - 1,012\cos 2t - 2,6\cos 4t;;$$

$$\gamma(t) = \exp[-0,05\tau]\left[c_1\sin I(\tau) + c_2\cos I(\tau)\right],$$

$$\gamma''(t) = (-0,05)\exp[-0,05\tau]\left[c_1\sin I(\tau) + c_2\cos I(\tau)\right] + +\exp[-0,05\tau]\left[c_1 I'(\tau)\cos I(\tau) - c_2 I'(t)\sin I(\tau)\right];$$

$$I(0) = 0;\quad I'(0) = -2,5 - 1,012 - 2,6 = -6,112;\quad \gamma(0) = 1[c_1\cdot 0 + c_2] = 1,\quad c_2 = 1;$$

$$\gamma'(0) = (-0,05)\cdot 1[c_2] + 1\cdot\left[c_1(-6,112)\right] = 0;\quad -0,05 - 6,112c_1 = 0,\quad c_1 = -122,24;$$

$$\gamma(t) = \exp[-0,05\tau]\{(-122,24)\sin[-2,5t - 0,506\sin 2t - 0,65\sin 4\tau]$$
$$+1\cdot\cos[-2,5t - 0,506\sin 2t - 0,65\sin 4\tau]\}.$$

Original equation has the form:

$$(0,316)^2 y''(t) + 0,1y'(t) + (0,1 - 2\cos 2t)y(t) = 0;$$

$$\gamma'(0) = (-0,05)\left[-I + 1\left[c_1\cos I(-2 - 1,012\cos 2t - 2,60\cos 4\tau)\right]\right] = 0;$$

$$c_1 = 0;$$

$$\gamma(t) = \exp[-0,05\tau]\cos[-2,5\tau - 0,506\sin 2\tau - 0,65\sin 4\tau].$$

V. RESULTS OF NUMERICAL IMPLEMENTATION

Results of numerical analysis of obtained an approximate solution are given below on Figs 2 – 7.

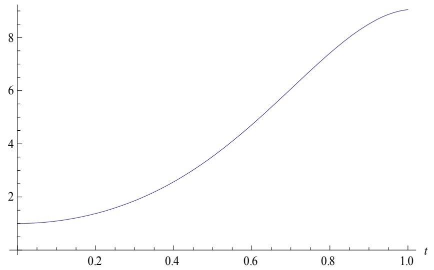

Fig. 2. Numerical solution of the linear problem

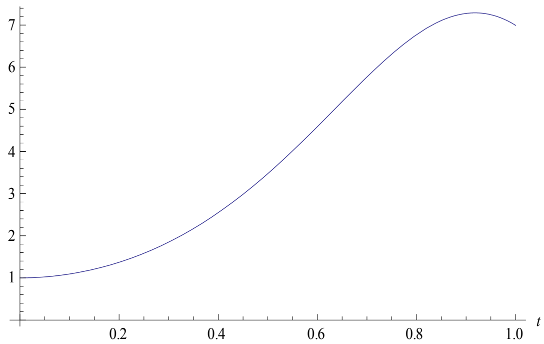

Fig. 3. Numerical solution of the nonlinear problem ($m = 3$)

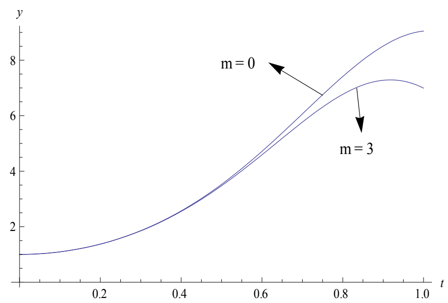

Fig. 4. Comparison of numerical solutions of the nonlinear problem ($m = 0, \ m = 3$)

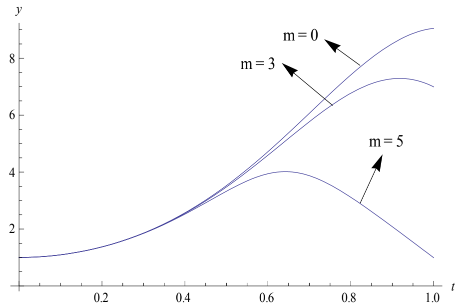

Fig. 5. Comparison of the numerical solution of linear and nonlinear problems

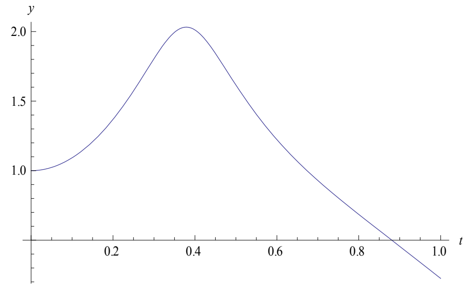

Fig. 6. The numerical solution of the nonlinear problem ($m = 100$)

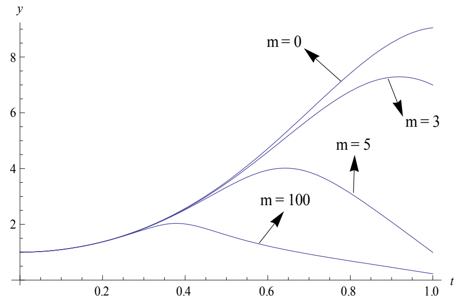

Fig. 7. Comparison of numerical solutions for varying degrees of nonlinearity

CONCLUSION

On the base of the hybrid asymptotic approach suggested approximate analytical solution of the nonlinear problem of the dynamics of the aircraft near the wave surface. The comparison of the results on the base of the proposed analytical solution and direct numerical integration of the original equation, the solution of the problem essentially depends on the degree m of its nonlinearity and the magnitude of the dimensionless perturbation amplitude. At a certain range of the time behavior of the studied system is not sensitive to the nonlinearity exponent *m*. To expand the range of variation of the scalar parameters of the asymptotic expansion is promising to use a hybrid WKB-Galerkin method (internal asymptotics) based on the perturbation method (external asymptotics).

Proposed hybrid asymptotic approach can be effective in cases where the distance h of the aircraft from the centerline of the wave is a function of time (the presence of the AC component of the coefficient of nonlinear) and a variable damping coefficient n, which is promising for further research of dynamic stability of the aircraft near the wave surface

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Received 04 October 2014

Azarskov Valeriy. Doct. of Sci, prof.
Education: National Aviation University, Kyiv, Ukraine.
Research interests: system identification, aircraft control systems.
Publications: more than 200 papers.
E-mail: azarskov@nau.edu.ua

Gristchak Dmitry. Student.
Education: National Aviation University, Kyiv, Ukraine.
Research interests: aircraft control systems.
Publications: 7.
E-mail: gristchak@sistd.com

В. М. Азарсков, Д. Д. Грищак. Демпфування коливань у завданнях руху літака

Використано двокроковий гібридний асимптотичний метод, заснований на методах теорії збурень і фазових інтегралів (метод Вентцеля–Крамерса–Бріллюена (ВКБ)) для отримання наближених аналітичних рішень нелінійної задачі коливання літака біля схвильованої поверхні, що знижує потребу в інтеграції особливих нелінійних диференціальних рівнянь зі змінними в часі періодичними коефіцієнтами при заданих початкових умовах. Це рішення не обмежується величиною безрозмірної амплітуди збурень і природи порядку нелінійності відновлювальної сили. Отримано результат, що має вигляд суми, де кожен член складається з двох функцій відповідно до методу збурень (в скалярному параметрі з нелінійним компонентом вихідного рівняння) і ВКБ-наближення, ефективних в інтеграції сингулярних диференціальних рівнянь зі змінними коефіцієнтами. Надано порівняння з даними прямого чисельного інтегрування рівнянь вихідної задачі для конкретних чисельних результатів.

Ключові слова: динаміка повітряних суден; схвильована поверхня води; гібридне асимптотичне наближення; наближене аналітичне рішення.

Азарсков Валерій Миколайович. д.т.н, професор.
Освіта: Національний авіаційний університет, Київ, Україна.
Напрямок наукової діяльності: ідентифікація систем управління, системи управління літальними апаратами
Кількість публікацій: більше 200 наукових робіт.
E-mail: azarskov@nau.edu.ua

Грищак Дмитро Дмитрович. Студент.
Освіта: Національний авіаційний університет, Київ, Україна.
Напрямок наукової діяльності:, системи управління літальними апаратами
Кількість публікацій: 7.
E-mail: gristchak@sistd.com

В. М. Азарсков, Д. Д. Грищак. Демпфирования колебаний в задачах движения самолета

Использован двухшаговый гибридный асимптотический метод, основанный на методах теории возмущений и фазовых интегралов (метод Вентцеля—Крамерса— Бриллюэна (ВКБ)) для получения приближенных аналитических решений нелинейной задачи колебания самолета у взволнованной поверхности снижает потребность в интеграции нелинейных дифференциальных уравнений с переменными во времени периодическими коэффициентами при заданных начальных условиях. Это решение не ограничивается величиной безразмерной амплитуды возмущений и природы порядка нелинейности восстанавливающей силы. Получен результат, который имеет вид суммы, где каждый член состоит из двух функций в соответствии с методом возмущений (в скалярном параметре с нелинейным компонентом исходного уравнения) и ВКБ-приближения, эффективного при интегрировании сингулярных дифференциальных уравнений с переменными коэффициентами. Дано сравнение с данными прямого численного интегрирования уравнений исходной задачи для конкретных численных результатов.

Ключевые слова: динамика воздушных судов; взволнованная поверхность воды; гибридное асимптотическое приближение; приближенное аналитическое решение.

Азарсков Валерий Николаевич. д.т.н., профессор.
Образование: Национальный авиационный университет, Киев, Украина.
Направление научной деятельности: идентификация систем управления, системы управления летательными апаратами.
Количество публикаций: более 200 научных работ.
E-mail: azarskov@nau.edu.ua

Грищак Дмитрий Дмитриевич. Студент.
Образование: Национальный авиационный университет, Киев, Украина.
Направление научной деятельности :, системы управления летательными аппаратами
Количество публикаций: 7.
E-mail: gristchak@sistd.com