COMPUTER-AIDED DESIGN SYSTEMS

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TO THE QUESTION OF DEPENDABILITY CALCULATION FAILURES BASED ON THE EXPONENTIAL MODEL OF DISTRIBUTION OF FAILURES

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Abstract**—***Quantitative estimations of errors of a lambda-method are presented at forecasting of a mean operating time to first failure and the failure rate of technical systems depending on reliability of element base, the complexity of systems and duration of their operation.*

Index Terms—Lambda method error; probabilistic-physical method; mean operating time to failure; failure rate.

I. INTRODUCTION

Scientific and technological progress in microelectronics during the last 50–60 years provided development of such reliable elemental base that on its basis, it is possible to create critical control systems with high security level of applications in aviation and space technology, nuclear power plants, communication systems. In practical applications of the dependability science unscientific situation has happened, the theoretical estimations of quality of techniques for various purposes, obtained by the method of quantitative estimation of reliability, durability and storability based on the exponential distribution model of failures formed in the middle of the 20th century are not confirmed by the maintenance data in most cases [1], [6].

"Methodical" gap between theory and practice, i.e. significant discrepancy of planned and empirical estimates of reliability were not unnoticed. In a situation where for systems with a constant failure rate becomes unacceptable, most methods for selecting the optimal timing of prevention and replacements because they are based on the assumption of increasing failure rate depending on the time. In addition, for systems with increasing failure rate becomes unacceptable, the most of reliability prediction methods based on the assumption of constant failure rate [1]. At the end of the 70s due to the lack of scientific support of maintenance programs of technical devices during their maintenance while research on the dependability theory interest in the study of the distribution functions of the failures of the non – exponential type has increased sharply. The main characteristic of the elements of the technical devices in which the distribution function of time to failure $F(t)$ differs from the exponent it is the failure rate $\lambda(t) = F((t) / [1 - F(t)]$.

"... Modern systems contain hundreds of thousands or more elements connected in series, and the difference in $\sqrt{n}$ time (n is a number of elements in the system) may be three or more orders of magnitude. Such huge discrepancy surprised because methods of calculation based on the two-parameter distributions have been known for long. From the more general arguments, imply that the model, which has more parameters, is more than adequate. However, in practice, continuing to use one-parameter exponential distribution", imposed in all industry standards for dependability calculations, "preferring the simplicity of the calculation of the adequacy of the required parameters of dependability assessment" [7].

This article intended to draw the attention of specialists in dependability to contain the problem, identify the quantitative assessment of the application of the exponential distribution errors in the calculation of reliability and may help supporters EXP-failure model to criticize the obtained on the basis of its results.

II. PROBABILISTIC-PHYSICAL TECHNOLOGY OF DEPENDABILITY RESEARCH

Today approved [2]–[5], [7], [8] that the cause of the discrepancy of predicted value and empirical estimates of reliability is the probability of inadequacy is strictly a one-parameter exponential distribution (known λ-method) degradation processes in the technical elements, leading to in the appearance of failure conditions.

A team of scientists of the National Academy of Sciences of Ukraine during 1985–1993 has developed probabilistic and physical methodology of research of reliability that develops in the dependability department of Institute of mathematical machines and systems (IMMS) NASU in the recent decades. Probabilistic-physical approach is based on using the laws of distribution of failures (models of dependability), which were obtained in the analysis of physical degradation processes in the elements of technical systems and causing their failure. The physical degradation processes are considered as random processes.

In Table 1 gives examples of characteristics of the composite degradation processes in integrated circuits – the type, the activation energy E_{aj}, the coefficient of variation V_j, the inner rate of destruction, participation interest p_j in the generalized process of degradation.

TABLE 1

MAIN CHARACTERISTICS OF THE DEGRADATION PROCESSES OF SILICON BIPOLAR INTEGRATED CIRCUITS

j	The type of degradation process	E_{aj}, eV	V_j	p_j
1	Intermetallic formation at terminals	0.90	0.70	0.15
2	Formation areas of “false” diffusion near the pn-junctions	0.40	0.80	0.14
3	Electromigration in films	0.48	0.80	0.12
4	Localization of current in the areas of micro-defects in the crystal	0.90	0.75	0.12
5	Breakdown of the dielectric	0.30	1.00	0.09
6	Fatigue destruction of the crystal and structural elements	0.20	0.60	0.08
7	Parasitic inner pickup	–	1.00	0.08
8	Electrochemical corrosion	0.45	0.70	0.05
9	Other	–	1.00	0.13

Such approach to research of dependability is called *probabilistic-physical*, because it directly communicates probabilities of reaching the limit level by physical determining parameter, i.e. connects the values of the *probability* of failure and the *physical* parameter that causes failure. Consequently, the parameters of the resulting probability distribution of failures have a specific physical sense.

So, in the probabilistic-physical (PP models of failures which is presented by diffusion non-monotonic (DN) function of density of distribution of mean operating time for the first failure t of the form

$$f(t,\mu,\nu)=\frac{\sqrt{\mu}}{\nu t\sqrt{2\pi t}}\exp\left[-\frac{(\mu-t)^2}{2\nu^2\mu t}\right], \qquad (1)$$

and corresponding to (1) function of reliability (reliability function) of the form

$$R(t,\mu,\nu)=\Phi\left(\frac{\mu-t}{\nu\sqrt{\mu t}}\right)-\exp\left(\frac{2}{\nu^2}\right)\cdot\Phi\left(-\frac{\mu+t}{\nu\sqrt{\mu t}}\right), \qquad (2)$$

where the scale parameter of the distribution (is inversely proportional to the mean rate of change of determining parameter, it makes sense to use the mean operating time to failure, and parameter of form of distribution (coincides with a coefficient of variation of the rate of degradation process and, therefore, a coefficient of variation of mean operating time to failure ($\nu = V$) [2], [5].

Availability of a priori information about the coefficient of variation of time between failures ν, concrete physical meaning of the scale parameter μ. The fact that the indicator of reliability μ = MTTF (where MTTF is a Mean Time to Failure) is included in the analytical structure of function arguments of the standard normal distribution, form the DN-reliability model (2), is objectively undeniable advantage of the PP-technology research reliability.

Neither of known two-parameter distributions of random variables, ever used in dependability theory, do not have such a favorable combination of the above a physical and analytical features. They ensure the implementation of an adequate prediction of the reliability and therefore the detection of errors exponential distribution model failures not taking into account the degradation processes in the elements of technical systems.

In works [3], [8] it is shown that the probability-physical technology of predicting of reliability of elements and of non-redundant systems based on an entire class of PP-models of dependability (alpha distribution, normal parametric, diffusion distributions) indicate the difference in estimation of mean operating time to the first failure of the system by $\sqrt{n}$ times in comparison with the λ-method. The same result was obtained earlier for two-parameter

Weibull and Rayleigh models [7]. While the estimate of reliability for the exponential distribution is opposed to estimates based on all other distributions.

In this case, quantitative evaluation of reliability of the exponential distribution is opposed to estimates based on all two-parameter strictly probabilistic and probabilistic and physical distribution.

This article aims to draw the attention of experts in the field of reliability's measures to the described above problem and may help supporters of the EXP – model to critically attitude dependability estimates based on its results Quantitative error estimates of lambda method obtained based on calculations and visual presentation of results in the information package Mathcad.

III. Error estimates of lambda method of forecasting the mean operating time of the first failure of the element base

Analytical dependences for estimation of mean operating time to the first failure of element base by λ- and PP-methods and errors of the "lambda" method are shown in Table 2.

The results of calculation of estimates $MTTF_{EXP}$ and $MTTF_{DN}$ when changing elements failure rate in the range of $\lambda \in \overline{10^4, 0.1}$ FIT shows in Fig. 1

TABLE 2

CALCULATED DEPENDENCES FOR ESTIMATION OF MTTF AND ERROR OF LAMBDA-METHOD

Mean operating time to the first failure		Error of λ-method
$MTTF^{exp}$	$MTTF^{DN} = \mu$	
$t_o^{exp} = (\lambda)^{-1}$	$\frac{\sqrt{\mu}}{\lambda(t_{test})\nu t_{test}\sqrt{2\pi t_{test}}} \exp\left(-\frac{(\mu - t_{test})^2}{2\nu\mu\ t_{test}}\right)$ $= \Phi\left(\frac{\mu - t_{test}}{\nu\sqrt{\mu t_{test}}}\right) - \exp\left(\frac{2}{\nu^2}\right)\Phi\left(-\frac{\mu + t_{test}}{\nu\sqrt{\mu_i t_{test}}}\right)$	$\delta t_o = \frac{t_o^{exp}}{\mu}$

. Assessment $MTTF_{DN}$ was carried out with coefficient of variation of time to failure $\nu_{DN} = 1$, which corresponds to a similar characteristic the exponential distribution ($\nu_{EXP} = 1$).

Analysis of the results clearly shows that the estimates of $MTTF_{\mathbf{EXP}}$, obtained based on the exponential distribution model of failures are significantly (many times) more than the estimates of $MTTF_{\mathbf{DN}}$, obtained by the DN – model of dependability. At linear model dependencies of duration of tests $t_{test.}$ on failure rate exceeding estimates $MTTF_{\mathbf{EXP}}$ is 10 at $\lambda \sim 10^3$ FIT and increases with the reliability of the element base, reaching value of $\sim 10^3$ times at $\lambda \sim 1{,}0$ FIT values, and is $\sim 10^4$ times or 10^6 % at $\lambda \sim 0{,}1$ FIT, which shows the level of "inaccuracy" of EXP-reliability model. Satisfactory convergence of estimates of $MTTF_{\mathbf{EXP}}$ and $MTTF_{\mathbf{DN}}$ is achieved only when the failure rate is (> 10^{-5} hour^{-1}, i.e. at the low dependability of the elemental base.

Because one parametric exponential distribution its coefficient of variation of mean operating time to failure equals to 1, the third and fourth moments are fixed, and therefore the mean operating time to first failure is actually deterministic quantity (failure rate – норма отказа). Specified error Δ_1 overrating MTTF elements should be based on the terminology of [7], referred to methodological errors λ-method of the first kind (Δ_1).

The actual contribution of the coefficient of variation ν, different from 1 in the estimation error $MTTF_{EXP}$ elements illustrates graphs Δ_1 (λ, ν), from which it follows that when (<1 error of the first kind λ-method increases markedly, while ν> 1 – several reduced respect to error Δ_1 (λ, (= 1).

IV. Error estimates for lambda method in predicting of system's mean operating time to first failure

In the work [6] it is conclusively proven that the calculation of reliability measures of systems based on different models is also accompanied by various methodical errors. Given by the author [4] experimental results suggest that the discrepancy between estimates of mean operating time to first failure by $\sqrt{n}$ times – is the methodical error of the second kind of exponential distribution.

As an example for estimation errors lambda method considered hypothetical system in which a number of structure elements n_j varied from 10 to 105.

Our researches of errors of EXP-model based on comparisons of obtaining MTTF ratings of systems of various complexity of lambda- and PP-methods are presented by the graphs in Fig. 2.

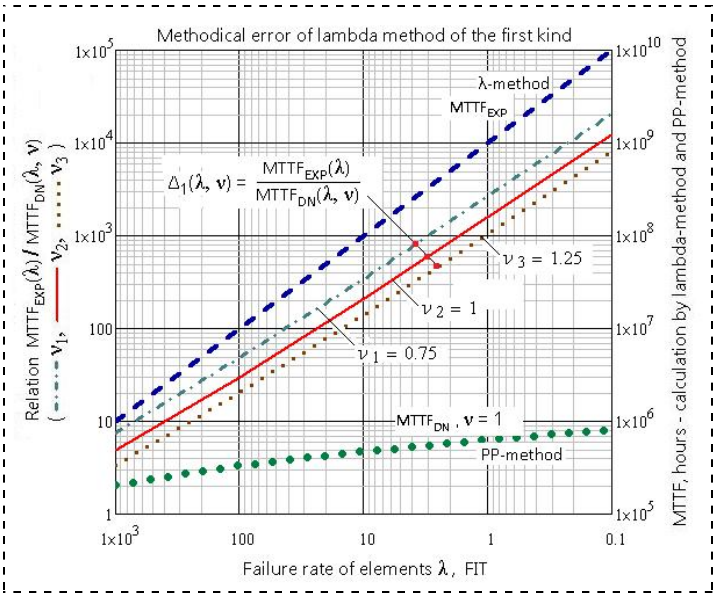

Fig. 1. Estimates of reliability and error $MTTF_{EXP}$ and $MTTF_{DN}$ of λ-method

.. The obtained data clearly represent areas (zones) of the first methodical errors Δ_1 (n) and the second Δ_2 (n) kind and λ-method are as follows

A. When calculating the reliability of systems whose complexity does not exceed several tens – hundreds of elements (depending on the level of reliability of the latter), there is a methodical error of the first kind, which leads to an overestimation of MTTF system.

B. Overestimation of MTTF system is n = 10 when ~ +2000 %, when n = 50 – ~ +600 % ; when n = 100 – ~ +400 %; with further increase of system complexity methodical error of the first kind $\Delta_1(n)$ reduces to zero.

C. When the complexity of the system, characterized by the value $n > 500$, methodical error of the second kind $\Delta_2(n)$ lambda method (see Fig. 2) appears in the calculation of system reliability, which underestimates MTTF.

D. For this example understatement of MTTF system is $n = 10^3$ will be –150 %, when $n = 10^4$ – ~ –1500 %, when $n = 10^5$ – ~ -10^4 % and continues to increase with further increase of system complexity.

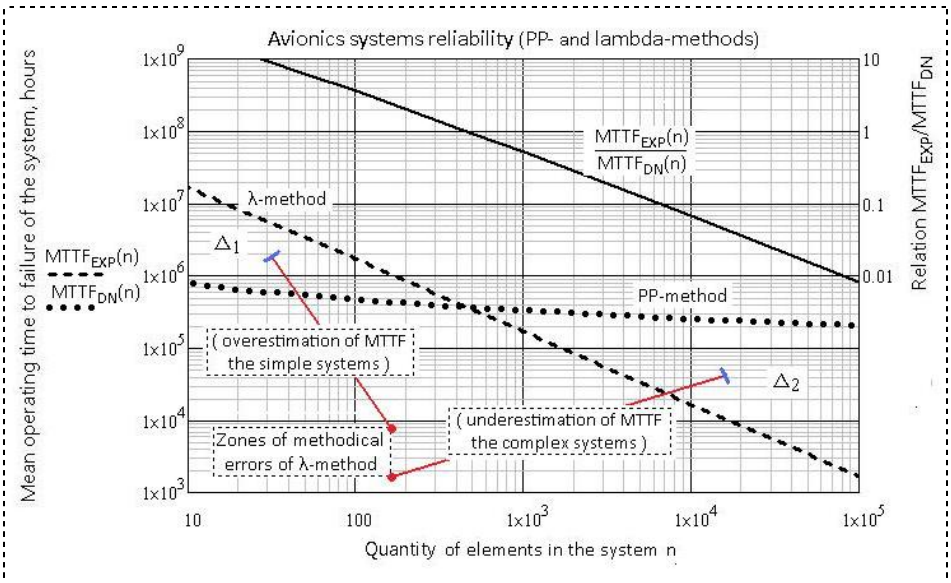

Fig. 2. Dependence of methodological errors λ-method from the complexity of the technical system

V. Error estimates for the exponential distribution in predicting the failure rate of the system

Failure rate (together with MTTF is calculating the indicators of reliability systems for various applications in the range their up state and is based on the fundamental relation of dependability theory

$$\Lambda(t) = \frac{f(t)}{R(t)} \ . \qquad (3)$$

In estimating of indicator $\Lambda(t)$ that based on the exponential failure distribution ratio (3) becomes an identity $\Lambda_{\text{EXP}}(t) \equiv \Lambda_{\text{EXP}} = \text{const}$, because Λ is the singular parameter of the distribution. However, at the formation stage of dependability theory as a science, the researchers have favorably received simplicity of the model.

Valid argument in favor of the EXP-distribution as a model then was served not only the empirical distribution density failures $f^*(t)$ described by an falling exponential function, but also a result of the conversion function of the relationship $R(t) = \exp\left(-\int_0^t \Lambda(t)\,dt\right)$ between indicators $R(t)$ and $\Lambda(t)$ that after the substitution $\Lambda_{\text{EXP}} = \text{const}$ becomes an exponential model of reliability $R(t) = \exp(-\Lambda_{\text{EXP}}\, t)$; the same expression is obtained from the fundamental relation $R(t) = \int_t^{\infty} f(t)\,dt$.

In PP-reliability, prediction method for the expression of the failure rate of electronic components and systems after substituting dependency (1) and (2) in relation (3) has the form

$$\Lambda_{DN}(t) = \frac{\dfrac{\sqrt{\mu}}{\nu \cdot t \cdot \sqrt{2\pi \cdot t}} \cdot \exp\left[-\dfrac{(\mu - t)^2}{2\nu^2 \cdot \mu \cdot t}\right]}{\Phi\left(\dfrac{\mu - t}{\nu\sqrt{\mu t}}\right) - \exp\left(\dfrac{2}{\nu^2}\right) \cdot \Phi\left(-\dfrac{\mu + t}{\nu\sqrt{\mu t}}\right)} . \qquad (4)$$

$$\sqrt{\frac{2}{\pi X}} \exp\left[-\frac{(1 - X)^2}{2\nu^2 X}\right] = \left(3 - \frac{1 - X^2}{\nu^2 X}\right) \Phi\left(\frac{1 - X}{\nu\sqrt{X}}\right) - \exp\left(\frac{2}{\nu^2}\right) \Phi\left(-\frac{1 + X}{\nu\sqrt{X}}\right),$$

where $X = \dfrac{t_{mo}}{\mu}$;

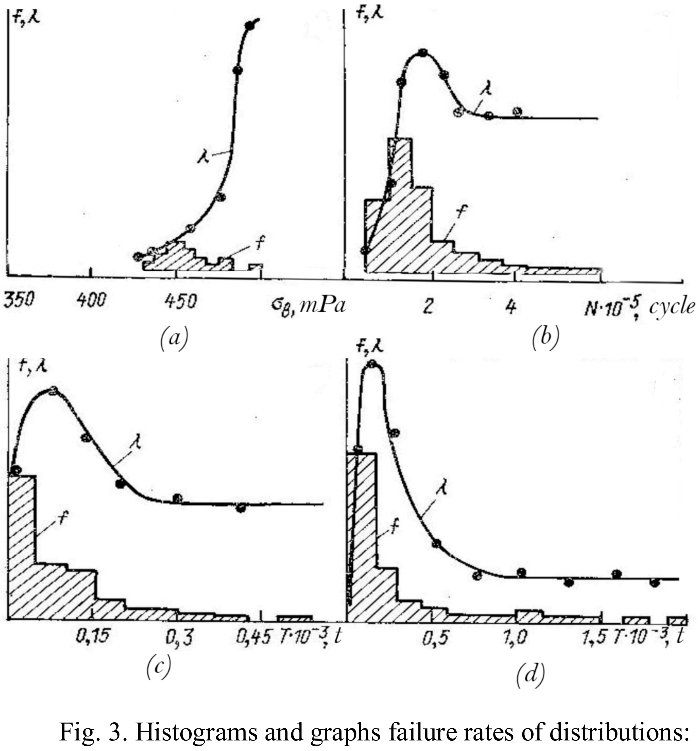

Fig. 3. Histograms and graphs failure rates of distributions:
(a) is the tensile strength of steel (ν = 0,06); (b) is the fatigue degradation (ν = 0,56); (c) is the time between failures of radio equipment (ν = 1,1); (d) is a life of rolling bearings (ν = 1,49)

By now, in scientific work [2]–[5] was performed detailed analytical and empirical research function $\Lambda_{\text{DN}}(t)$, the conclusions of which can be formulated as follows:

1) The function $\Lambda_{\text{DN}}(t)$ starts with zero values, i.e., $\Lambda_{\text{DN}}(t=0+)=0$.

2) $\Lambda(t)$ is a non-monotonic function of duration t unimodal and operating parameters μ and ν distribution system failures

3) $\max\{\Lambda_{\text{DN}}(t_{mo})\}$ achieved at the point of thickest value and is determined from the transcend dental equation

4) Asymptote is value $\lim_{t\to\infty}\Lambda(t)=(2\nu^2\mu)^{-1}$, to which the curve of the failure rate is approaching the top

The results of an analytical study of the expression (4) are fully supported by the scientific work [3]–[5] about the behavior of the empirical failure rates (Fig. 3) The so-called "classic" failure rate curve, which we crossed, inasmuch as the proper construction of the empirical failure rate of any of the above unimodal distributions can not have regularity "bath-shaped" curve showed in the Fig. 4 [3].

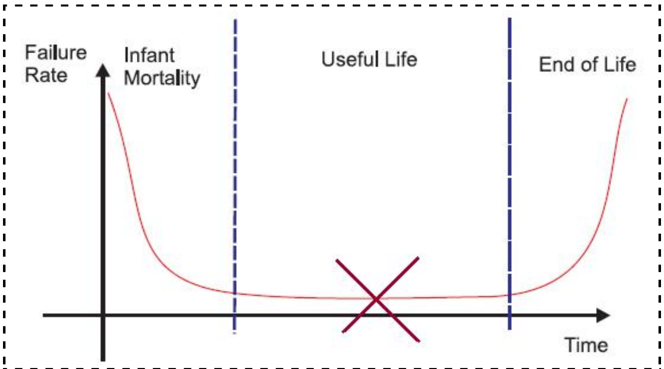

Fig. 4. The common representation of empirical failure rate

Thus, the PP - technology provides a dependability research to practical use is actually a new science-based function of reliability - failure intensity $\Lambda(t)$, in contrast to the known parameter Λ of exponential model, leaving it terminology "failure rate" indicating clearly and regardless of the duration of operation on the expected occurrence of failure after developments $T_{0\text{EXP}}=(\Lambda_{\text{EXP}})^{-1}$.

Prediction based on exponential distribution indicators of measure of reliability is also accompanied by the first methodological errors $\Delta_1(t)$ and second $\Delta_2(t)$ kind of quantitative evaluation (Fig. 5) as an example of the system with the specified parameters μ_C and ν_C of failure distribution.

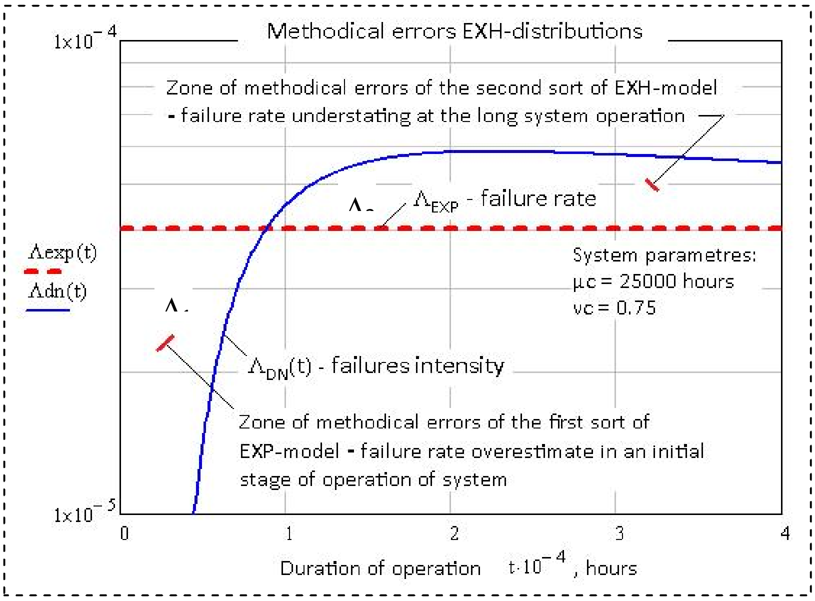

Fig. 5. The methodical error of the first and second kind in the assessment of the failure rate in the calculations based on the exponential distribution

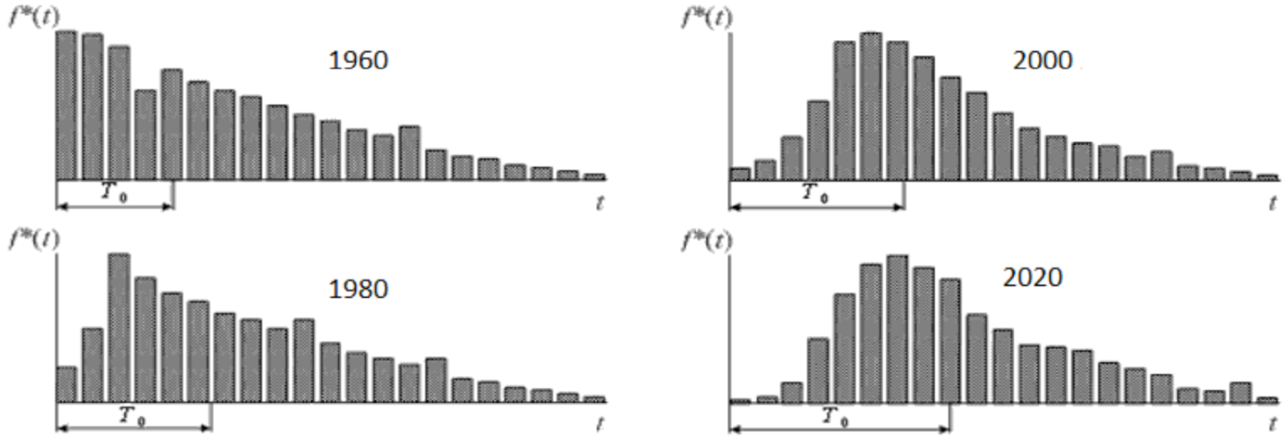

Fig. 6. Deformation of the failure distribution with an increase in the reliability of the element base

As follows from Fig. 5, at the beginning of operation of technical systems, taking into account the properties of the first property of function $\Lambda_{DN}(t)$, **Ошибка! Закладка не определена.** measure of inaccuracy of the exponential distribution in the assessment of the failure rate is several orders of magnitude. With increasing operating time error of the first kind $\Delta_1(t)$ decreases with subsequent transition from overestimation to underestimation of reliability. During long-term operation due to error of the 2nd kind $\Delta_2(t)$ the actual value of the failure rate underestimated by 2–3 times.

Transformation of model of density distribution of failures from the exponential to unimodal, which is shown schematically in Fig. 6, is due to scientific and technical progress in the development of microelectronic components, each next generation of which is more reliable than previous one. "There is no reason to believe that the projected statistics will be different" [5].

CONCLUSION

In connection with the above it obvious that receiving of adequate assessments of the quality of technical systems on the basis of the exponential distribution is impossible, because the density distribution of time to failure of different physical nature of the components and systems are subject to a two-parameter diffusion model.

Methodical errors of the first and second kind of estimating of MTTFs – mean operating time to failure of complex systems have different signs (overestimation- underestimation), and while consistently applying *the EXP - model* (solution to the first problem - forecasting the MTTF elements with the overestimate of result and solving the second problem is the forecasting of MTTF complex system with low results) seem partially to be mutually compensated. However, under conditions of complete uncertainty about the extent of error compensation it is hardly possible to speak about the adequacy of solving problems of dependability based on the exponential distribution.

One-parameter exponential distribution takes place in all industry standards for dependability calculations, and researchers continue to use it, while:

– preferring simple calculation based on the EXP model of the adequacy of the resulting estimates of dependability measures;

– without paying attention to the fact that an increase in of reliability components and system complexity unreliability of grade for reliability produced on the basis of λ-method increases significantly;

– without realizing the fact that the EXP model of failures that was accepted at the beginning of increase of dependability as a science, corresponding at that time level of dependability of existing elemental base, has exhausted its possibilities and it leads to false estimates of reliability in application to modern a highly reliable elemental base of MTTF and MTBF, therefore, durability systems is inadequately estimated.

Appearance of probabilistic and physical methodology – a natural step further formation of the science of reliability engineering. At the present level of development of the theory of PP-forecasting is the most "reliable" tool to obtain objective assessments of adequate of reliability, durability and storability of technical systems.

REFERENCES

1. Vorobiev, V.G. *New methods of technical operation (overview).* Moscow. MIIGA, 1987. 28 p. (in Russian).

2. Gribov, V.M.; Hryshchenko, Yu.V.; Skripets, A.V.; Strelnikov, V.P. Dependability theory of avionics. Study Guide, Part. I. Kyiv: NAU, 2006. 324 p. (in Ukrainian).

3. Gribov, V.M. *Estimation and prediction of dependability of onboard aerospace equipment* / Gribov, V. M.; Kofanov, Y.N.; Strelnikov, V.P. Edited by prof. NRU Kofanov Y.N. M.: Higher School of Economics, 2013. 496 p. (in Ukrainian).

4. Dependability engineering. Models of failures. The main provisions. State Standards of Ukraine 3433-96. – Kyiv: State Standards of Ukraine, Kyiv, 1996. 42 p. (in Ukrainian).

5. Pogrebensky, S.B.; Strelnikov, V.P. *Design and dependability of multiprocessor computers.* M.: Radio and Communications, 1988. 168 p. (in Russian).

6. *Problems of dependability in aeronautical engineering* / Results of science and technology, a series of "Air transport", Volume 20. M.: Transport, 1990. 48 p. (in Russian).

7. Strelnikov, V.P. *Methodological failures of calculating the systems dependability* / magazine "Dependability". M.: LLC "Journal of Dependability ", 2005, № 3. (in Russian).

8. Strelnikov, V.P. *Status and prospects theory and practice of the dependability* / International scientific and technical journal "Dependability and durability of machines and structures". – Kyiv: National Academy of Sciences, Kyiv, 2005, №№ 2 and 3. (in Russian).

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Ю.В. Грищенко, О. В. Кожохіна, В. М. Грібов. До питання про похибки розрахунку надійності на основі експоненціальної моделі розподілу відмов

Представлені кількісні оцінки похибок лямбда-методу при прогнозуванні середнього наробітку до відмови і інтенсивності відмов технічних систем в залежності від безвідмовності елементної бази, складності систем та тривалості їх експлуатації.

Ключові слова: похибки лямбда-методу, ймовірносно-фізичний метод, середня наробітка до відмови, інтенсивність відмов

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