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# DESIGN OF ROBUST GAIN SCHEDULED HOLD CONTROL SYSTEM

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***Absract—****This paper represents an strategy to tune robust control system on the example of pitch hold mode for small unmanned aerial vehicles by using gain scheduling proportional-integral-derivative control algorithms. The simulation of controlled flight dynamics modeling with changing flight conditions and taking into account the action of stochastic disturbances prove the efficiency of the proposed procedure. This technique is useful to obtain flexibility and robustness in the controller's automatic design.*

**Index terms—**gain scheduling; PID controller; robustness; penalty function; unmanned aerial vehicle.

## I. INTRODUCTION

The control system developers give a particular attention to the widespread usage of unmanned aerial vehicles (UAVs). Moreover, the last generation of UAV provide to multifunction application and can be produced as for civil missions as for military tasks. Such UAVs are subjected to various disturbances within the flight envelope.

The UAV control law should be simple enough to be implemented on a board computer with restricted abilities. It is obvious that the structure of such controllers is simple and easy to implement. A proportional-integral-derivative (PID) controller has simple structure and can effectively provide pitch hold mode.

Over the past decades, the gain-scheduled (GS) control problem has been extensively discovered both from theoretical and practical viewpoint, see, for example, [7]. The GS approach has been found to be one of the most effective ones for the controller design problems for parameter-varying systems. The main idea is to design controller which can be tuned for any number of operating ranges. In this case an optimal set of tuning parameters can be automatically plugged into the controller depending on the current value of the process variable.

To satisfy the aforementioned usage range, it is proposed to use gain-scheduled PID-controller instead of singular one.

## II. PROBLEM STATEMENT

As stated before, an effective approach to solve the nonlinear control problem is using gain scheduling with linear controllers. For example in an aircraft flight control system, the altitude and Mach number might be the scheduling variables, with different linear controller parameters available for various combinations of these two variables [1], [6]. In the present paper proposed alternative scheduling parameter, which is the dynamic pressure (DP) $\overline{q}$ . The DP is related with both flight altitude and speed. To prove the efficiency of the proposed technique, the pitch hold mode is considered as a case study. The aircraft studied in this paper is the UAV Aerosonde for which the Aerosim Matlab toolbox was developed. In the article is shown the results of modelling the aircraft dynamic within changing flight conditions by using Simulink software.

## III. PROPORTIONAL-INTEGRAL-DERIVATIVE GS CONTROLLER

The designing a GS control system takes steps, which are shown on Fig. 1

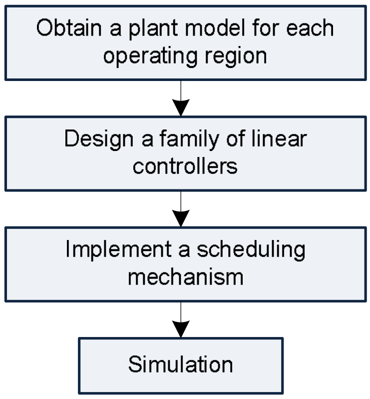


Fig. 1. Block-diagram of GS control system design

A PID-controller which is one of the most powerful but complex controller mode operations combines the proportional, integral, and derivative modes [3]. The analytical expression of GS PID-controller is given as,

$$W_{CSq}(z) = P_{pid}(\overline{q}) + I_{pid}(\overline{q}) \cdot T \frac{z}{z-1} + D_{pid}(\overline{q}) \frac{z-1}{z}$$

The scheme of a pitch hold GS-feedback loop is shown on Fig. 2.

Accordingly, the controller gain matrix has the following form

$$K = [P_{pid}, I_{pid}, D_{pid}, K_q, K_p]$$


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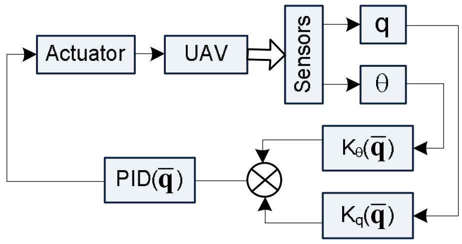


Fig. 2. The scheme of a pitch hold feedback loop

## IV. NELDER-MEAD METHOD FOR THE PARAMETRIC ROBUST OPTIMIZATION

In the controller design, Nelder-Mead's simplex method is adopted to ensure the control system has the best disturbance rejection performance and robustness for any perturbed model of the closed loop system within any particular DP range. The Nelder-Mead method is direct research method based on the original paper of J. A. Nelder and R. Mead [4]. A method minimizes a function of n variables, which depends on the comparison of function values at the $(n+1)$ vertices of a general simplex, followed by the replacement of the vertex with the highest value by another point. The simplex adapts itself to the local landscape, and contracts on to the final minimum. In order to get better result during a small time, it is necessary to start the optimization procedure from the initial values of controller parameters closed to the optimal ones.

The used method is based on $H_\infty / H_2$-robust optimization [8], [9]. Flight various uncertainties could be external and/or internal, structural and/or unstructured, which produce certain deviation from the nominal behavior to perturbed one. The design algorithm is to estimate the performance and robustness of the closed loop system using $H_2$-norm of the sensitivity function and $H_\infty$-norm of the complementary sensitivity function, then try to find the compromise between this two properties. For this reason, we use a multi-objectives optimization problem, based on insert several objectives in one cost function and try to satisfy them at the same time [2].Therefore the penalty function (PF), restricting location's area of the closed loop system poles in the predefined region in the complex plan, the references [10] gives a mathematical model of this penalty function.

The expression of the cost function is given as,

$$J_\Sigma = \lambda_d J_d^2 + \lambda_d^p \left(J_d^p\right)^2 + \lambda_s J_s^2 + \lambda_s^p \left(J_s^p\right)^2 + \\ + \lambda_\infty \|T\|^2_\infty + \lambda_\infty^p \|T^p\|^2_\infty + Pf \qquad (1)$$

where $\lambda_d$, $\lambda_d^p$, $\lambda_s$, $\lambda_s^p$ $\lambda_\infty, \lambda_{\infty p}$ – the LaGrange factors, $J_d$ and $J_{dp}$ defines the $H_2$-norms of the models in deterministic cases for particular DP range. $J_s$ and $J_{sp}$-defines the performances of the stochastic models. $\|T\|_\infty$ and $\|T^p\|_\infty$ are the $H_\infty$-norms and give the estimation of the robustness of the particular two parametrically disturbed plants. Pf is the penalty function.

The cost functions (1) is the function of a controller gain vector $\overrightarrow{C_a}$, that's why the result of optimization procedure will be following $\vec{C}_a^* = \arg\min J_\Sigma(\vec{C}_a)$, $\vec{C}_a \in M_c$, where $M_c$ is the stability range of controller gains.

## V. CASE STUDY

The state space vector of the longnitudal channel is $x = [V_T, \alpha, q, \theta]^T$, where $V_T$ is the true speed, m/s, $\alpha$ is the angle of attack, deg, $q$ is the pitch rate, deg/s, $\theta$ is the pitch angle, deg.

The control input vector is $u = [\delta_e]^T$ represented by elevator deflections. The nonlinear model of the Aerosonde is linearized for range of operating conditions respected to the range of DP from 200 to 650 $\frac{\text{kg}}{\text{m}\cdot\text{s}^2}$ with a granularity of 50 $\frac{\text{kg}}{\text{m}\cdot\text{s}^2}$.

The state space matrices A and B in general form filled with longnitudal coefficients and stability derivatives are given below:

$$A = \begin{bmatrix} X_V + X_{Tv}\cos\alpha_e & X_\alpha - g_0\cos\gamma_e & 0 \\ Z_V + Z_{Tv}\sin\alpha_e & Z_\alpha - g_0\sin\gamma_e & V_T + Z_q \\ 0 & 0 & 0 & 1 \\ M_V + M_{Tv} & M_\alpha & 0 & M_q \end{bmatrix};$$

$$B = \begin{bmatrix} X_{\delta e} \\ Z_{\delta e} \\ 0 \\ M_{\delta e} \end{bmatrix}$$

where $X_V = -\dfrac{\bar{q}S}{mV_T}(2C_L - C_{Dv})$;

$$X_\alpha = \frac{\bar{q}S}{m}(C_L - C_{D\alpha});\ X_{\delta e} = \frac{\bar{q}S}{m}C_{D\delta e};$$

$$Z_V = -\frac{\bar{q}S}{mV_T}(2C_L - C_{Lv});$$

$$Z_{\alpha} = -\frac{\overline{q}S}{m}(C_D - C_{L\alpha});$$

$$Z_{\delta e} = -\frac{\overline{q}S}{m}C_{L\delta e};\ Z_q = -\frac{\overline{q}Sc}{2mV_T}C_{Lq};$$

$$M_V = \frac{\overline{q}Sc}{J_y m V_T}(2C_M + C_{mv});\qquad M_{\alpha} = \frac{\overline{q}Sc}{J_y}C_{m\alpha};$$

$$M_{\delta e} = -\frac{\overline{q}Sc}{J_y}C_{m\delta e};\qquad M_q = -\frac{\overline{q}Sc}{J_Y}\frac{\overline{c}}{2V_T}C_{mq};$$

$\gamma = \gamma_e$, $\alpha = \alpha_e$ are equilibrium (steady-state) conditions.

As seen from the description of space matrices coefficients, the aircraft flight dynamic depends on DP changes.

Next step is to develop GS controller according to the granulated DP range. The DP range depends on type of an aircraft and may be different according to a flight operating range and aircraft aerodynamic characteristics. Stochastic disturbance are modeled by means of Dryden filter [5]. The controller gains after providing optimization procedure are shown in table 1.

TABLE 1

CONTROLLER GAINS

| IAP, $\frac{\text{kg}}{\text{m}\cdot\text{s}^2}$ | $P_{pid}$ | $I_{pid}$ | $D_{pid}$ | $K_q$ | $K_{\theta}$ |
|---|---|---|---|---|---|
| 200 | 0.9423 | 0.2712 | -0.0261 | 0.0273 | -3.926 |
| 250 | 0.5798 | 0.3332 | -0.0518 | 0.0273 | -6.408 |
| 300 | 0.4158 | 0.2847 | -0.0956 | 0.027 | -4.84 |
| 350 | 0.8952 | 0.3175 | -0.1502 | 0.027 | -6.879 |
| 400 | 0.9147 | 0.1763 | -0.0772 | 0.027 | -5.324 |
| 450 | 0.2262 | 0.132 | -0.0842 | 0.1507 | -5.902 |
| 500 | 0.434 | 0.3576 | -0.0882 | 0.1922 | -3.656 |
| 550 | 0.268 | 0.1565 | -0.0121 | 0.187 | -5.437 |
| 600 | 0.2563 | 0.1321 | -0.0188 | 0.1869 | -4.169 |
| 650 | 0.2332 | 0.1171 | -0.0195 | 0.1934 | -4.164 |

To demonstrate the pitch hold for changing flight conditions was chosen the following flight conditions: the flight altitude is 800 m and speed increasing from 32 m/s (115.2 km/h) to 34 m/s (122.4 km/h). The DP changes for the proposed case is shown on the Fig. 3

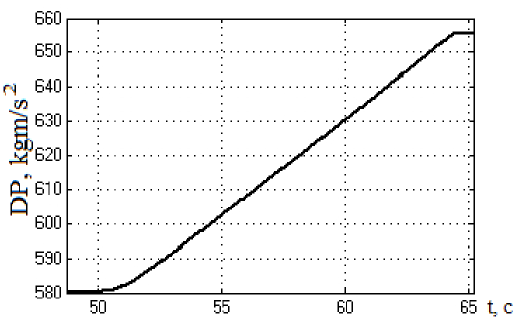


Fig. 3. The DP changes for speed increasing from 32 m/s to 34 m/s on the altitude 800 m

Performance indices, which describe the behavior of the system for the speed increasing from 32 m/s to 34 m/s on the altitude 800 m are following:

1. $H_2$ -norm for the deterministic cases is changing in the range [0.95-1.05].

2. $H_2$ -norm for the stochastic cases is changing in the range [0.89-0.83].

3. $H_{\infty}$ -norm is changing in the range [2.25-1.96].

The pitch angle step plots for the DP 550, 600, 650 $\frac{kg}{m\cdot s^2}$ are shown on the Fig. 4.

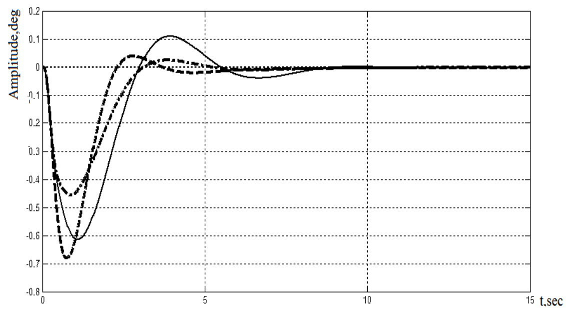


Fig. 4. System responses to step input pitch angle with scheduled DP 550,600, 650 $\frac{kg}{m\cdot s^2}$

Given step responses show that settling time values and the maximum overshoot values reflect the stability of the close-loop system with synthesized controller.

After developing and analyzing the behavior of the system at the first step, at the second step, the closed-loop system was created in Simulink.

The simulation results are shown in Figs 5–6.

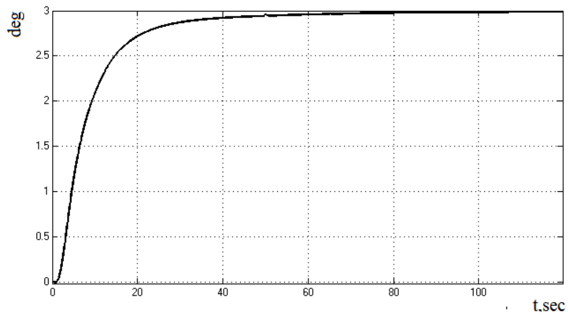


Fig. 5. Pitch hold without external disturbances

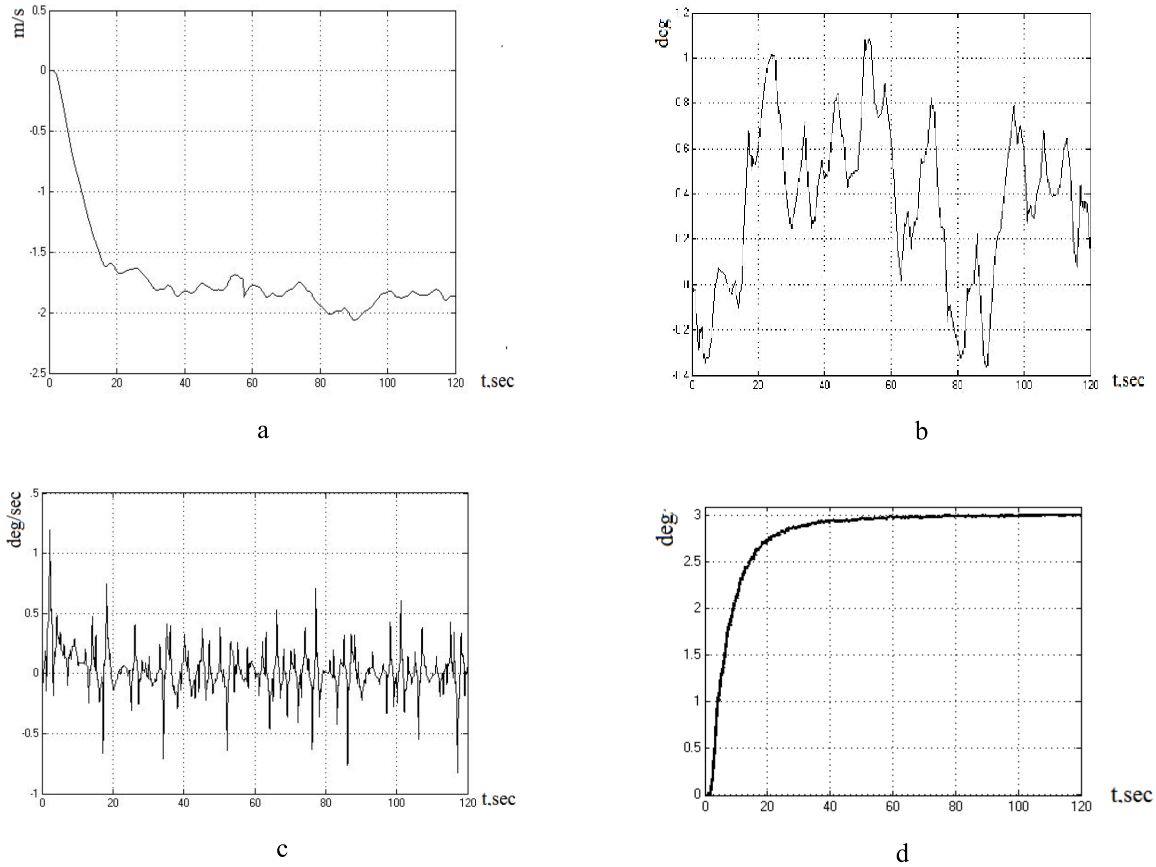


Fig. 6. Simulation results for longnitudal channel of the UAV in the presence of external disturbances: *a* is a velocity; *b* is an angle of attack; *c* is a pitch rate; *d* is a pitch angle

## CONCLUSION

The simulation of controlled flight dynamics modeling with changing flight conditions and taking into account the action of stochastic disturbances prove the efficiency of the proposed control method. The required flight performances are respected as well as the robustness of the control law. It can be seen that the handling quality of the nominal and the perturbed models are satisfied.

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**О. І. Надсадна. Проектування робастної системи стабілізації на базі табличного регулятора**

Запропоновано методику налаштування робастної системи керування на прикладі режиму стабілізації кута тангажу для невеликих безпілотних літальних апаратів за допомогою табличних ПІД-регуляторів. Моделювання замкнутої системи з урахуванням змін умов польоту та дії стохастичних збурень демонструє ефективність запропонованої процедури. Метод корисний для отримання системи з високим рівнем показників якості та робастності.

**Ключові слова:** табличний регулятор; ПІД-регулятор; робастність; штрафна функція; безпілотний літальний апарат.

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**О. И. Надсадная. Проектирование робастной системы стабилизации на основе табличного регулятора**

Предложена методика настройки робастной системы управления на примере режима стабилизации угла тангажа для небольших беспилотных летательных аппаратов с помощью табличных ПИД-регуляторов. Моделирование замкнутой системы с учетом изменений условий полета и действия стохастических возмущений демонстрирует эффективность предложенной процедуры. Метод полезен для получения системы с высоким уровнем показателей качества и робастности.

**Ключевые слова:** табличный регулятор; ПИД-регулятор; робастность; штрафная функция; беспилотный летательный аппарат.

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