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SYNTHESIS OF OPTIMAL MULTIDIMENSIONAL STABILIZATION SYSTEM OF HELICOPTER IN HOVERING MODE UNDER STOCHASTIC INFLUENCES

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Abstract*—*Proposed algorithm of synthesis of optimal stochastic system for helicopter stabilization in hovering mode. New problem and algorithm of synthesis for multidimensional unstable system under stochastic disturbances were considered. Method of synthesis provide receiving the system with optimal quality of stabilization.

Index Terms—Optimal controller; unstable system; stochastic disturbances.

I. INTRODUCTION

Significant increasing of requirements to level of accuracy of navigation, stabilization and control by many plant motions requires development of new modern technologies and searching of automatic control systems structure and parameters. In many cases research plants represent significantly unstable multidimensional dynamic systems, which are subjected to action of stochastic influences. So, a helicopter with suspended cargo in hovering mode may be believed an important example of such plant. Lately helicopters took an important place in aviation, where increasing of competition among basic developers of aviation technic causes to find continually new methods and technologies to achieve optimal stabilization quality indexes. Due to this it is possible to expand significantly application range of helicopters during implementation of aviation works in the air.

Hovering mode is one of important modes of helicopter application. This mode requires high precision stabilization at the given point in a space, as a pilot had to carry out many tasks in the same time under stochastic external disturbances. In this mode the high precision system of automatic stabilization is needed to improve a pilot work and make him free from some operations. Therefore expediency of solving the optimal stabilization problem of such system is of great importance.

A helicopter is a complex dynamic system with significantly unstable properties. To achieve a minimum of the functional, it is necessary to consider this system like multidimensional and multiply connected, taking into account the cross connections between channels. Due to the aerodynamics features, a helicopter can not be divided [2] into longitudinal and lateral channels in contrast to airplane configuration, because of close connections between the helicopter longitudinal and lateral motion in comparison with the airplane motion. This is caused by the principle of operation of the main rotor. However, earlier the approximate analysis of stabilization modes in some cases was implemented separately for longitudinal and lateral helicopter motion. For the new problem statement we will believe that a helicopter does not consist of separate channels, but represents a multidimensional dynamic system with cross connections between channels. In addition, during stabilization system synthesis it is necessary to provide not exceeding of real power limitations for control signals and take into account real dynamic characteristics of the stabilization plant, noise of motion parameters measurements and disturbances, which are acting in a flight.

II. PROBLEM STATEMENT

A problem of synthesis of an optimal multidimensional stabilization system of a helicopter in hovering mode under stochastic influences action may be stated similar to [1].

The typical structural scheme of the stabilization system looks like on Fig. 1.

Suppose, that motion of the multidimensional unstable stabilization plant may be described by a set of differential equations with constant coefficients:

$$P(s)x(s) = M(s)u(s) + \psi(s), \qquad (1)$$

where $x(s)$ is the Fourier transform of the plant output responses; $u(s)$ is the Fourier transform of the control signals; $\psi(s)$ is the Fourier transform of the disturbances (turbulent influence of the wind), which represents a centered random process, with the known spectral density matrix $S_{\psi\psi}(s)$; $P(s)$ and $M(s)$ are polynomial matrices with an argument $s = j\omega$ and dimensions $n \times n$ and $n \times m$ respectively, determinant det $P(s)$ has both the stable and unstable roots. Parameters of motion are measured with the measuring system described by the matrix transfer functions $K(s)$, and measurements are accompanied with disturbances; $W(s)$ is the sought matrix of the stabilization system transfer functions (Fig. 1).

The problem lies in determination of the optimal structure of controller W, which will provide stability of the closed loop system and ensure the minimum of the stabilization quality functional, which may be given by the expression [1]

$$e=\langle x^2\rangle+\lambda\langle u^2\rangle=\frac{1}{j}\int_{-j\infty}^{j\infty}\left[S_{xx}(s)+\lambda S_{uu}(s)\right]ds, \quad (2)$$

where $S_{xx}(s)$ and $S_{uu}(s)$ are matrices of spectral densities for output and control signal vectors, λ is a varying weighting coefficient which characterizes the operational limits on control.

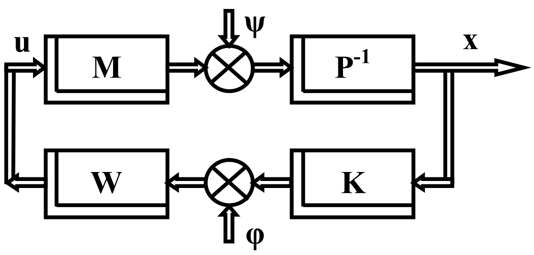


Fig.1. Structural scheme of the multidimensional stabilization system of unstable plant

III. SOLUTION OF THE PROBLEM

The problem of synthesis of the optimal multidimensional stabilization system we will reduce to the functional minimization problem (2). For this purpose we will use the modernized algorithm similar to mentioned in [2].

Taking into account the above stated notations the control signal may be represented in the following form

$$u=W(Kx+\varphi).$$

The algorithm of determination of an optimal structure W may be written in the form

$$W=F_u^{\psi}\left(F_x^{\psi}\right)^{-1},$$

where matrices of transfer functions F_u^{ψ} and F_x^{ψ} of the closed loop system must be physically realizable (this means that all poles must be located in the left half-plane (LHP) of the argument $s=j\omega$). The equation of constraints taking into consideration all unstable poles may be rewritten in the following way

$$F_x^{\psi}=\left\lfloor F_x^{\psi}\right\rfloor_+=\left\lfloor P^{-1}\left(MF_u^{\psi}+E\right)\right\rfloor_+$$

or $\left\lfloor P^{-1}\left(MF_u^{\psi}+E\right)\right\rfloor_-=0,$

where « + » and « – » is are separation signs.

Since stabilization plant dynamics is arbitrary and the characteristic equation has unstable roots, the following equation is true

$$P^{-1}=P_+^{-1}+P_-^{-1},$$

where P_+^{-1} and P_-^{-1} are fractionally rational functions, with poles only in LHP and RHP accordingly.

Using the operation of one-sided poles elimination, the function $P^{-1}M$ may be transformed into the form

$$P^{-1}M=\breve{M}B^{-1}, \quad (3)$$

where B^{-1} has poles only in RHP.

In accordance to [1] the function F_u^{ψ} may be represented in the form:

$$F_u^{\psi}=BF+A, \quad (4)$$

where F is chosen as a varying function, A is a polynomial function, that it is necessary to determine on the basis of incoming information by the condition

$$\left(P^{-1}MA\right)_-=-P_-^{-1}. \quad (5)$$

Thus, using the equations (3) and (5) we will determine the matrices A and B. By substituting these matrices in the equation (4), the function F_u^{ψ} expressed through the varying function F can be found. Notice, F_u^{ψ} will always have its poles only in LHP. Taking into account the function (4) and the equation (5), F_x^{ψ} can be rewritten using varying function in the following form

$$F_x^{\psi}=P^{-1}MBF+P^{-1}(MA+E). \quad (6)$$

Analysis of the expression (6) shows, that varying function F_x^{ψ} has no poles in RHP due to choice of the polynomial matrices *A* and *B* under conditions (3) and (5). Such approach ensures stability of the closed loop system.

The problem of optimal stabilization system synthesis may be lead to the problem of minimization of the quality index (2) for the class of varying function *F* based on the algorithm [1]

$$F = -G^{-1}\left(L_0 + L_+\right)D^{-1}, \quad (7)$$

where G is the result of factorization

$$G_* G = B_*\left(M_* P_*^{-1} R P^{-1} M + C\right)B;$$

D is the result of factorization

$$DD_* = S_{\psi\psi} + P \cdot S_{rr} \cdot P_*,$$

$\left(L_0 + L_+\right)$ is the result of separation.

$$L = L_0 + L_+ + L_- = G_*^{-1} B\left[M_* P_*^{-1} R P^{-1}\left(MA + E\right) - MP_*^{-1} R S_{rr} P^{-1} + CA\right]D.$$

After substitution of determined functions F_u^{ψ} and F_x^{ψ} into the quality index (2) we can define its minimum, which ensures achievement of the maximum quality of the stabilization system.

The set of linearized equations [3] is chosen as a model of the helicopter dynamics.

$$\begin{aligned}
&\Delta\dot{\omega}_{z1} + a_{mz}^{\omega z}\Delta\omega_{z1} + a_{mz}^{Vx}\Delta V_{x1} + a_{mz}^{Vy}\Delta V_{y1} = a_{mz}^{\delta n}\Delta\delta_n + a_{mz}^{\delta r}\Delta\delta_r,\\
&\Delta\dot{V}_{x1} + a_x^{vx}\mathrm{H}V_{x1} + a_x^{Vy}\Delta V_{y1} + a_x^{\omega z}\Delta\omega_{z1} + a_x^{\vartheta}\Delta\vartheta = a_x^{\delta n}\Delta\delta_n + a_x^{\delta r}\Delta\delta_r,\\
&\Delta\dot{V}_{y1} + a_y^{\omega z}\Delta\omega_{z1} + a_y^{Vy}\Delta V_{y1} + a_y^{Vx}\Delta V_{x2} = a_y^{\delta n}\Delta\delta_n + a_y^{\delta r}\Delta\delta_r.
\end{aligned} \quad (8)$$

where V_{x1}, V_{y1} are disturbed speeds along the OX_1 and OY_1 axes correspondingly; ω_{z1} is the pitch angular rate; ϑ is an angle of the pitch; δ_n is deviation of a washer of the main rotor swash plate; δ_r is a step of the main rotor ($\Delta\delta_r$ is a change of the rotor step).

After substituting numerical values of coefficients for the set of equations (8) and the Laplace transformation, we can write matrices of transfer functions of the multidimensional unstable plant in the following form

$$\mathbf{P} = \begin{bmatrix} 7.7\times10^{-4} & 4.75\times10^{-3} & s^2 + 0.32s \\ s + 0.031 & -0.0034 & 3.4s + 9.8 \\ -0.049 & s + 0.62 & 0.3s \end{bmatrix}; \qquad \mathbf{M} = \begin{bmatrix} 3.3 & 1 \\ 15 & 1.04 \\ 47 & 74 \end{bmatrix}.$$

The matrix of normalized spectral densities of noise measurements looks like

$$S_{\varphi\varphi} = \begin{bmatrix} \dfrac{-3.1831}{(s-1)(s+1)} & 0 & 0 \\ 0 & \dfrac{-3.9789}{(s-1.118)(s+1.118)} & 0 \\ 0 & 0 & \dfrac{-2.6526}{(s-0.9129)(s+0.9129)} \end{bmatrix}.$$

The matrix of normalized spectral densities of external disturbances may be represented in the form

$$S_{\psi\psi} = \begin{bmatrix} \dfrac{0.4}{\left|0.29s + 0.5\right|^2} & 0 & 0 \\ 0 & \dfrac{0.2}{\left|0.36s + 0.5\right|^2} & 0 \\ 0 & 0 & \dfrac{0.15}{\left|0.48s + 0.5\right|^2} \end{bmatrix}.$$

So, all the initial data for synthesis are obtained. Repeating the procedure of synthesis for various operational conditions, that is changing the coefficients γ and λ, the components $e1$ and $e2$ of the functional e and optimal structures of the controller W were obtained and represented in the tables 1, 2.

TABLE 1

VALUES OF FUNCTIONAL DEPENDING ON THE OPERATIONAL PARAMETERS

λ / γ^2	0.1		1		10		100	
	$e1$	$e2$	$e1$	$e2$	$e1$	$e2$	$e1$	$e2$
0.3378	291.1296	8.2619	239.4406	25.1886	25.3151	79.6713	220.4205	256.8622
	e = 299.3915		e = 264.6292		e = 304.9864		e = 477.2827	
$3.378\cdot10^2$	196.9724	6.0002	163.1068	7.7605	151.5954	55.0094	147.9696	173.1048
	e = 202.9726		e = 180.8673		e = 206.6048		e = 321.0744	
$3.378\cdot10^2$	136.7869	4.3036	109.1500	12.2410	101.4189	37.6353	98.9413	118.2156
	e = 141.0905		e = 121.391		e = 139.0542		e = 217.1569	

TABLE 2

MATRIX OF TRANSFER FUNCTIONS FOR SYNTHESIZED OPTIMAL CONTROLLER

λ	W		
R = 0.1	$\frac{7.2526 (s^2 + 0.9731s + 0.3127) (s^2 + 3.543s + 5.975)}{(s+19.21) (s+0.8879) (s^2 + 3.562s + 6.244)}$	$\frac{0.67236 (s+3.104) (s+0.7886) (s^2 + 3.52s + 7.908)}{(s+19.21) (s+0.8879) (s^2 + 3.562s + 6.244)}$	$\frac{3.1011 (s-6.146) (s+0.6078) (s-0.5379) (s^2 + 3.717s + 6.345)}{(s+19.21) (s+0.8879) (s^2 + 3.562s + 6.244)}$
	$\frac{-1.2379 (s-1.189) (s+0.5581) (s^2 + 3.731s + 4.219)}{(s+19.21) (s+0.8879) (s^2 + 3.562s + 6.244)}$	$\frac{0.30547 (s+12.35) (s+0.8789) (s^2 + 4.466s + 9.252)}{(s+19.21) (s+0.8879) (s^2 + 3.562s + 6.244)}$	$\frac{-0.73128 (s+4.161) (s+1.749) (s-2.854) (s-0.6552) (s+0.2397)}{(s+19.21) (s+0.8879) (s^2 + 3.562s + 6.244)}$
R = 1	$\frac{16.4653 (s+4.431) (s+2.19) (s\hat{}2 + 1.084s + 0.4323)}{(s+38.09) (s+3.79) (s+2.403) (s+0.8344)}$	$\frac{1.5252 (s+9.189) (s+0.7944) (s^2 + 3.656s + 8.299)}{(s+38.09) (s+3.79) (s+2.403) (s+0.8344)}$	$\frac{6.3291 (s-6.575) (s+4.701) (s+2.102) (s+0.5701) (s-0.4053)}{(s+38.09) (s+3.79) (s+2.403) (s+0.8344)}$
	$\frac{-5.4506 (s+6.962) (s+1.843) (s+0.3731) (s+0.2499)}{(s+38.09) (s+3.79) (s+2.403) (s+0.8344)}$	$\frac{-0.9762 (s+23.57) (s+0.8264) (s^2 + 4.645s + 11.8)}{(s+38.09) (s+3.79) (s+2.403) (s+0.8344)}$	$\frac{-2.21 (s+8.378) (s-6.014) (s+1.639) (s+0.5557) (s-0.6513)}{(s+38.09) (s+3.79) (s+2.403) (s+0.8344)}$
R = 10	$\frac{42.7771 (s^2 + 1.232s + 0.586) (s^2 + 8.19s + 25.12)}{(s+87.09) (s+0.8132) (s^2 + 7.884s + 23.44)}$	$\frac{2.3289 (s+48.9) (s+0.9408) (s^2 + 3.237s + 7.422)}{(s+87.09) (s+0.8132) (s^2 + 7.884s + 23.44)}$	$\frac{15.9606 (s-6.677) (s+0.5261) (s-0.2731) (s^2 + 8.11s + 24.89)}{(s+87.09) (s+0.8132) (s^2 + 7.884s + 23.44)}$
	$\frac{-17.0638 (s^2 + 0.9892s + 0.4106) (s^2 + 9.873s + 34)}{(s+87.09) (s+0.8132) (s^2 + 7.884s + 23.44)}$	$\frac{-3.501 (s+77.16) (s+0.8419) (s_2 + 3.046s + 7.979)}{(s+87.09) (s+0.8132) (s^2 + 7.884s + 23.44)}$	$\frac{6.297 (s-7.238) (s+0.5602) (s-0.4989) (s^2 + 9.337s + 32.39)}{(s+87.09) (s+0.8132) (s^2 + 7.884s + 23.44)}$

Analysis of surfaces of functional changes e as a function of λ and γ^2 (Fig. 2) shows that quality of helicopter stabilization depends on the relationship between variances of external disturbances and noise measurements γ and on limitations of control actions λ.

Surface of change of quality by γ is monotonous and has the global extremum by λ for $\lambda = 1$. Therefore the optimal matrix of transfer functions of the controller W is defined for this constraint. Determined optimal structures of the controller W ensure the stability of closed loop system and the minimum of the stabilization quality functional (2) at the same time.

Logarithmic amplitude-frequency and phase-frequency characteristics of closed loop system F_u, F_x and appropriate transient processes are represented on Fig. 3.

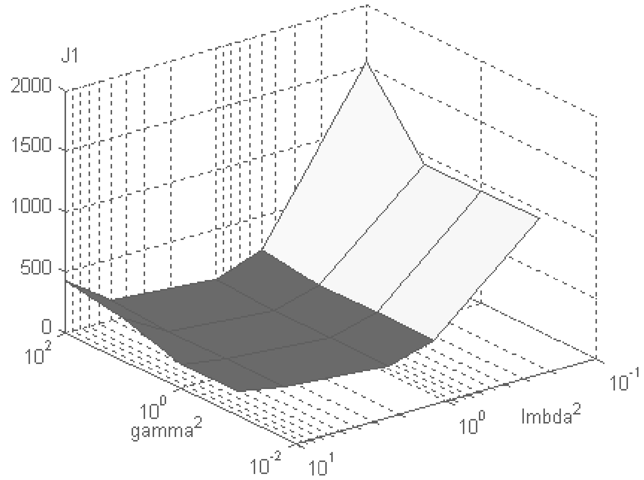
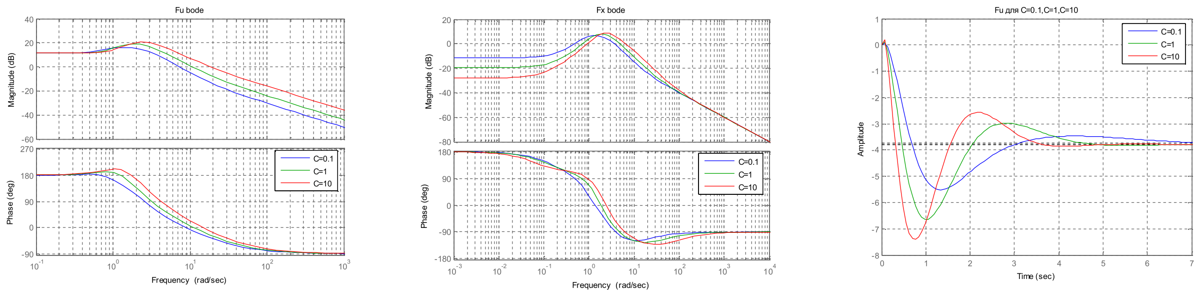


Fig. 2. The surface of changes of the stabilization quality index



a *b* *c*

Fig. 3. Transient processes and logarithmic amplitude-frequency and phase-frequency characteristics of closed loop system

CONCLUSIONS

The problem of optimal multidimensional and multiply connected stabilization system of unstable dynamic plant with cross connection between channels in stochastic conditions is stated and solved. For this modification of known methods of system synthesis of similar class was implemented for conditions when output coordinates vector is fully measured, intensity of noises of measuring instruments in comparison with useful signals is small and a plant has unstable poles. Analysis of the matrix of optimal transfer functions of the system by the output coordinates F_x^{Ψ} showed that most of channels of closed loop system have astatic properties. If we define structure and parameters of the controller for limitation on control power $\lambda = 1$, the system almost does not change stabilization quality when variation of relation between angle and linear disturbances is in range from 0,1 to 100.

Compared analysis of helicopter behavior in hovering mode with the standard and synthesized optimal controllers showed the high effectiveness of the synthesized system for accepted operational limitations. The standard system has regular divergent trends of output coordinates, which deviate from a program trajectory. In the proposed optimal system transient processes are damped and finished after 6-8 seconds that shows its significant effectiveness in comparison with not optimal one. Moreover variances of stabilization errors of the optimal system are decreased almost on order in comparison with the standard system.

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О. П. Кривоносенко, О. В. Савченко. Синтез оптимальної багатовимірної системи стабілізації гелікоптера у режимі висіння за наявності стохастичних впливів
Запропоновано алгоритм синтезу оптимальної системи стохастичної стабілізації вертольоту у режимі висіння. Детально розглянуто нову задачу і алгоритм синтезу багатовимірної нестійкої системи при стохастичних збуреннях. Метод синтезу забезпечує отримання системи з оптимальною якістю стабілізації. Результати синтезу підтверджуються за допомогою багатоетапного моделювання.
Ключові слова: оптимальний регулятор; нестійка система; стохастичні збурення.

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А. П. Кривоносенко, А. В. Савченко. Синтез оптимальной многомерной системы стабилизации вертолета в режиме висения при стохастических воздействиях
Предложен алгоритм синтеза оптимальной системы стохастической стабилизации вертолёта в режиме висения. Подробно рассмотрены новая задача и алгоритм синтеза многомерной неустойчивой системы при стохастических возмущениях. Метод синтеза обеспечивает синтез системы с оптимальным качеством стабилизации. Результаты синтеза подтверждаются многоэтапным моделированием.
Ключевые слова: оптимальный регулятор, неустойчивая система, стохастические возмущения.

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