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FAST CONVOLUTION ALGORITHMS IN THE REDUCTION OF THE VISUAL SIGNAL AND IMAGE PROCESSING SYSTEMS

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Abstract*—The mathematical model of building recognition systems and signal restoration of visual systems. The basis for the construction of invariant systems are ortonormal discrete functions of Kravchuk and algorithms, based on number-theoretical transformations of Fermat.*

Index Terms—Invariant processing; Kruwtchuk functions; ortonormal system; functional of energy; rotation; integral transforms.

I. INTRODUCTION

The problem of automatic recognition, classification and diagnosis-related problem deals with complete system invariant features of signals and images, which allows to reliably carry out the procedure of automatic recognition. Systems of invariants as a function of generalized spectral coefficients may be based on the decomposition of the signal system of classical orthogonal polynomials [1] (Chebyshev, Legendre, Hermite and others).

II. THE ANALYSIS OF THE LATEST RESEARCHES AND PUBLICATIONS

In case where the basis functions are functions of multiple harmonic frequencies, and a periodic function, or a superposition of multiple harmonics, a basis is quite effective for the analysis signal, but does not guarantee invariance characteristics of the signal with respect to all linear transformations (shear, scaling, rotation). Calculation of the coefficients of the expansion (of the signal) by the classical orthogonal polynomials (the functions of a continuous argument) requires considerable numerical integration operations.

In view of the above as a basis on which the spectral coefficients are calculated generalized signal of the visual system, the basic functions offered Kravchuk, which are inherently discrete orthonormal system functions.

Functions of Kravchuk are discrete orthonormal functions with weight

$$\rho(k)=\frac{N!\,p^k q^{N-k}}{k!(N-k)!},\quad q=1-p,$$

are determined by the recurrence formula,

$$F_0^{(p)}(k,N)=\sqrt{\frac{N!\,p^k q^{N-k}}{k!(N-k)!}};$$

$$F_1^{(p)}(k,N)=(k-pN)\sqrt{\frac{(N-1)!\,p^{k-1}q^{N-k-1}}{k!(N-k)!}}.$$

$$F_{n+1}^{(p)}(k,N)=\frac{k-n-p(N-2n)}{\sqrt{(N-n)(n+1)pq}}F_n^{(p)}(k,N)-\sqrt{\frac{(N-n+1)n}{(N-n)(n+1)}}F_{n-1}^{(p)}(k,N),$$

where. $k=\overline{0,N-1},\quad n=\overline{2,N-1};\quad 0<p<1$.

$$F_0^{(p)}(k,N)=\sqrt{\frac{N!\,p^k q^{N-k}}{k!(N-k)!}};$$

$$F_1^{(p)}(k,N)=(k-pN)\sqrt{\frac{(N-1)!\,p^{k-1}q^{N-k-1}}{k!(N-k)!}}$$

Two-dimensional function Kravchuk determined by the formula

$$F_{k,l}^{(p_1,p_2)}(i,j)=F_k^{(p_1)}(i,N)F_l^{(p_2)}(j,N),$$

$$i,k=\overline{0,N_1-1};\ j,l=\overline{0,N_2-1};\ 0<p_1<1,0<p_2<1.$$

Two-dimensional function Kravchuk will be used for image compression (obtain the spectrum image). In general, the spectral coefficients of the signal relative to the selected set of linear transformations of the basis is calculated as the convolution of

$$c_k(s,s_0)=\left(R^{s_0}y(i)R^s\varphi_k(i)\right)=\sum_Q R^{s_0}y(i)\varphi_k^p(i)\mu(i).$$

After determining the optimal values of the spectral coefficients of the criterion of maximum energy functional

$$W(s,s_0)=\sum_{k\in M}|c_k(s,s_0)|^2,$$

signal $\tilde{y}(i)$ is recovered using the formula

$$\tilde{y}(i)=\sum_{k\in M}c_k(s_0,s_0)R^{s_0}\varphi_k(i).$$

A similar scheme is restored the two-dimensional signal-image.

In [2], [3] an algorithm for allocation of a complete system of signs of the signal invariant under the shift, scaling and rotation.

The distributions of spectrum of signal are showned on Fig. 1 (p = 0,2) and 2 (p = 0,4).

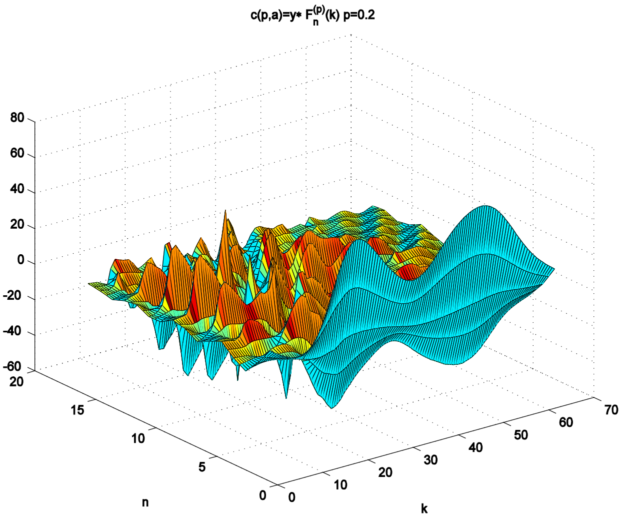


Fig. 1. The distributions of spectrum of signal (p = 0,2)

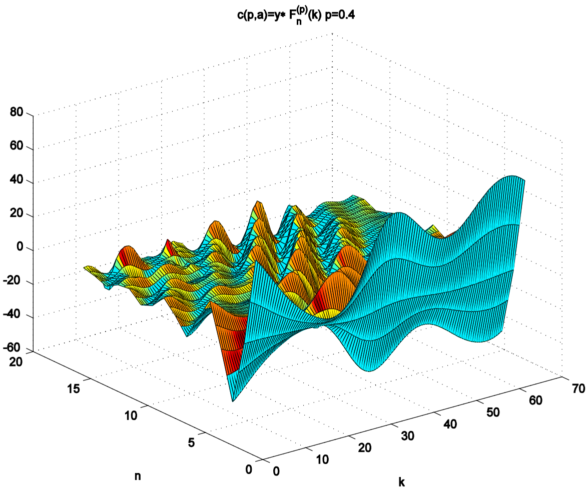


Fig. 2. The distributions of spectrum of signal (p = 0,4)

Therefore, as described, the main problems related to the restoration of signals and images of visual systems, is to calculate the convolution of discrete signals and calculating the inverse transform.

III. Task Solution

In the case of relatively small amounts of discrete signal is known to be quite effective it is a fast Fourier transform. But for sufficiently large sample to ensure the implementation of reforms in the real time necessary to look for more efficient algorithms for computing the convolution.

There are efficient algorithms based on number-theoretic transform [3]. Such algorithms are converting Mersenne and Fermat. Conversion Mersenne have several properties desired in the calculation of convolution.

These transformations using arithmetic, which is easy to implement, and can be computed without multiplications. The main drawback is that it lacks an algorithm of fast calculation, as well as that between word length and the length of the conversion there is a rigid connection. From this lack of free Fermat conversions, where the module is used the number of Fermat $F_t = 2^{2^t} + 1$ and number-theoretic transform, defined by a module F_t, called number-theoretic transforms Fermat (TCHPF).

$$\bar{X}_k \equiv \sum_{m=0}^{2^{t+1}-1} x_m 2^{mk} \bmod \left(2^{2^t}+1\right).$$

Accordingly, the reverse TCHPF:

$$x_m \equiv -2^{2^t-1} \sum_{k=0}^{2^{t+1}+1} \bar{X}_k 2^{-mk} \bmod \left(2^{2^t}+1\right),\ 2^{-mk} \equiv -2^{(2^t-1)mk} \bmod \left(2^{2^t}+1\right).$$

Length TCHPF power of 2, and can be calculated using only the operations of addition and multiplication by a power of 2 (shift by one digit) for lengths up to $N = 2^{t+2}$.. Consequently, the structure similar to that of computing TCHPF conventional FFT, but multiplication by complex exponential replaced by simple shift operations. This means that to calculate N -point TCHPF necessary additions $N \log_2 N$ and $(N/2)\log_2 N$ shifts.

In the case of very long sequences sampled to compute discrete convolution it is advisable to use algorithms that are based on iterative schemes.

All the above discussed algorithms are of the form

$$y = Dx, \tag{1}$$

where x is the unknown input signal; y is the known output; D is the known operator distortion known or conversion. These equations are valid for continuous signals – one-dimensional or multidimensional, and for the digital signals. Operator D is a main operator, who performs the mapping function in the function or sequence in the sequence. The task of restoring the signal is reduced to finding the known and D. Formally,

$$x = D^{-1} y.$$

But in practice, the determination of the operator D^{-1} may be situations where the operator is defined approximately, then even if the operator D a good approximation of the result of its action on y may differ significantly from the true solution, if a job is performed with an error signal caused by, for example, additive noise. Furthermore, the operator distortion may be such that the multiple input signals generate the same output signal; in this case the inverse does not exist. In such situations, the ambiguity can be avoided with some preliminary information about the properties of the input signal x.

Therefore practical interest other ways to restore signals. One of them is the method of successive approximations, easy to implement on a PC.

$$x^{k+1} = Fx^{k}, \quad (2)$$

where F is the operator, which is derived from the expression (1). Practical interest in the iterations of this type due to their flexibility to all sorts of combinations and distortions.

When restoring the signal x of known signals y using an iterative procedure are known in advance of the desired properties of the solutions. For example, it may be known that a signal with a finite range, or it is limited in space and time, or physical reasons can be assumed that there is only non-negative values. Such preliminary information can be expressed by the operator of the restrictions C that

$$x = Cx$$

when x satisfy this constraint. For example, if you know that a digital signal x is non-negative, can be defined by the operator P.

$$C[x(n)] = P[x(n)] = \begin{cases} x(n), & x(n) \geq 0, \\ 0, & \text{for the rest values.} \end{cases}$$

If you use such a representation a priori constraints to the signal, the equation (2) can be written as

$$x = DCx,$$

then

$$x = Cx + \lambda(y - DCx).$$

where λ there may be a constant parameter function of the independent variable or function of x. We have the equation

$$x = Fx, \quad (3)$$

where $Fx = \lambda y + Gx, \quad G = (E - \lambda D)C.$

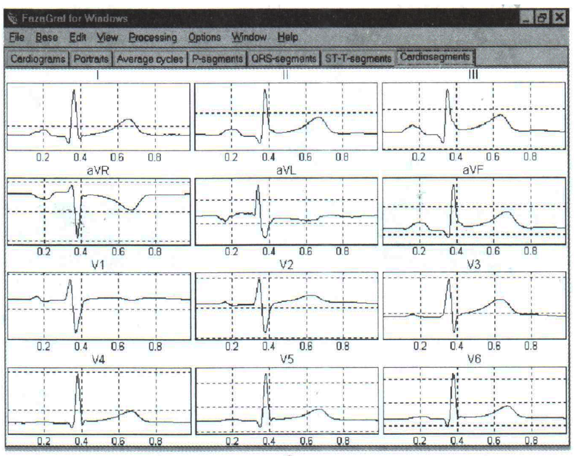
The signal x, satisfying equation (3) is called fixed-point of conversion F. The standard method of finding such solutions is the method of successive approximations, based on iterative equation

$$x^{k+1} = Fx^{k} = \lambda y + Gx^{k}.$$

This parameter λ is used to control the speed of convergence of the iteration process. In many cases it is convenient to choose an initial approximation $x^0 = \lambda y$.

Consequently, an iterative process may be more efficient from a computational point of view, compared with the calculation of convolution if determined a priori constraints on the desired input.

The fragments electrocardiogram in 12-lead, and an example of the recovery of the phase trajectories with the local selection shows Fig. 3.



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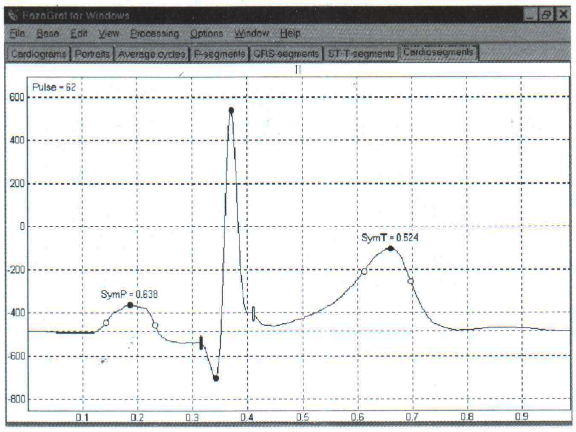

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Fig. 3. The fragments of fragments electrocardiogram in 12-leads, and an example of the recovery of the phase trajectories with the local selection

CONCLUSIONS

The polynomial transforms insure the effective methods of reflecting two-times DPF- convolution in one-times DPF-convolution. In fact the theoretic-number transforms are special case of polynomial transforms

The use of number-theoretic transforms for recovery and classification of complex signals such as electrocardiograms showed that they are optimal in terms of the minimum possible number of members in decomposition to the corresponding Fourier series

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Г. В. Кіт. Алгоритми швидкої згортки при відновленні сигналів та зображень зорових систем

Запропоновано математичні моделі побудови систем розпізнавання і відновлення сигналів зорових систем. Підґрунтям вирішення цієї задачі є ортонормовані базисні функції Кравчука, та алгоритми, що ґрунтуються на теоретико-числових перетвореннях Ферма.

Ключові слова: інтегральні перетворення; ітераційна схема; інваріантні перетворення; ортонормовані базисні функції; функції Кравчука.

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Г. В. Кит. Алгоритмы быстрой свертки при восстановлении сигналов и изображений зрительных систем

Предложены математические модели построения систем распознавания и восстановления сигналов зрительных систем. Основанием для построения инвариантных систем являются ортонормированные дискретные функции Кравчука и алгоритмы, основанные на теоретико-числовых преобразованиях Ферма.

Ключевые слова: интегральные преобразования, итерационная схема, инвариантные преобразования, ортонормированные функции, функции Кравчука.

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