UDC 629.3.025.2(045)

O. A. Sushchenko

ROBUST TWO-AXIS STABILIZATION OF UNMANNED AERIAL VEHICLES PAYLOAD

Educational-Scientific Institute of Air Navigation, National Aviation University, Kyiv, Ukraine E-mail: sushoa@ukr.net

Abstract—The paper deals with basic principles of robust two-axis stabilization of payload (cameras, laser locators) operated at unmanned aerial vehicles. The mathematical description of the system taking into consideration interconnection between channels is suggested. Approach to the system structural robust optimization is given. The optimization functional for two-degree-of-freedom system is represented.

Index terms—Payload; robust optimization; structural synthesis; two-axis stabilization.

I. INTRODUCTION

One of actual problems of unmanned aerial vehicles (UAV) development is achieving high accuracy and efficiency of such payload as television camera of the high resolution, digital camera and laser scanner operated at UAVs in the difficult conditions of external disturbances (action of the turbulent wind). The high image quality in this situation may be achieved by means of stabilization and control by attitude of the above stated payload lines-of-sight. The modern approach to such systems design lies in using concepts of robust performance and robust stability ensuring. Approaches to design of the robust systems for control by the aircraft motion are widely represented in many books and papers [1]-[3]. But usage of the same approaches to systems for stabilization of UAV payload has own peculiarities.

Now in many countries development of small UAVs is widespread [4]. The sufficiently high accuracy of tracking and stabilization processes taking into consideration difficult operation conditions may be achieved by means of robust stabilization. Such approach differs from described in [5] by using two-axis stabilization and perspective system components such as the Coriolis vibratory gyros and gearless drives.

This problem has two aspects. In the first place, the basic principles of creating the two-axis stabilization system mathematical description taking into consideration connection between channels must be considered. In the second place, approach to the structural robust optimization of a system of the studied type must be researched.

Traditionally design of robust systems is implemented in for two stages. At the first stage the synthesis of a system is implemented based on linear models and principles of the robust optimization (for example, structural optimization using the H_{∞} synthesis). At the second stage, simulation of the synthesized system is carried out. Results of this simulation allow estimating efficiency of synthesis procedures. In the case of insufficient results, it is necessary to return to the first stage and repeat synthesis changing conditions of stabilization law parameters determination or even choosing new devices for the system instrumental implementation. Of course, the latter approach must be applied in the extreme case only.

II. PROBLEM STATEMENT

Mathematical description of the stabilization systems assigned for control by UAV payload linesof-sight attitude must take into consideration presence of two interconnected channels. Notice, that connection between two channels for the studied system disappears after the model linearization. The linearized model of the two-axis stabilization system is divided into two separate channels. Such situation is caused by those factors, that the connection between channels is mathematically expressed through the trigonometric functions of the angles, which define the platform attitude. And these angles are usually accepted small for mathematical description linearization.

It should be noted, that use of the two-axis gimbals is the most acceptable for many applications, as two mutually-orthogonal axes represent the minimum quantity necessary for determination of the direction in the three-dimensional space [6].

In the paper the concepts, which define interconnection between the stabilization system channels are as follows. At the first place, interconnection between channels caused motion of UAV, at which the platform with information-measuring devices is mounted, is taken into account. At the second place, this interconnection becomes apparent due to a moment counteractive to the motor rotation. And at the third place, influence of cross input angular rate must be taken into consideration too.

As regards to control features basing, in this case it is convenient to use feedback by signals of the rate gyros [6]. The corresponding structural scheme is represented in Fig. 1.

III. PURPOSE OF RESEARCH

The purpose of this study is determinations of ways to stabilize payload functioned at UAVs in difficult conditions of real operation.

To achieve this purpose it is necessary to create the mathematical description and define an approach to the structural robust optimization.

IV. MATHEMATICAL DESCRIPTION OF THE SYSTEM

Stabilization system provides control by the attitude of payload line-of-sight mounted at the gimbaled platform.

The feedback is implemented by means of the gyro sensors measuring absolute angular rates of the platform with mounted on it payload. Control by the drive is implemented by means of the moment contactless engines.

In the general case, the system dynamics may be described by the Euler equations [7]

$$\begin{split} \dot{\omega}_x J_x + \omega_y \omega_z (J_z - J_y) - (\omega_y^2 - \omega_z^2) J_{yz} \\ - (\omega_x \omega_y + \dot{\omega}_z) J_{xz} + (\omega_x \omega_z - \dot{\omega}_y) J_{xy} = M_x; \\ \dot{\omega}_y J_y + \omega_x \omega_z (J_x - J_z) - (\omega_z^2 - \omega_x^2) J_{xz} \\ - (\omega_z \omega_y + \dot{\omega}_x) J_{xy} + (\omega_x \omega_y - \dot{\omega}_z) J_{yz} = M_y; \\ \dot{\omega}_z J_z + \omega_x \omega_y (J_y - J_x) - (\omega_x^2 - \omega_y^2) J_{xy} \\ - (\omega_x \omega_z + \dot{\omega}_y) J_{yz} + (\omega_x \omega_z - \dot{\omega}_x) J_{xz} = M_z, \end{split}$$

$$(1)$$

where $\omega_x, \omega_y, \omega_z$ are projections of the platform angular rates onto its own axes; J_x, J_y, J_z are the inertia moments of the platform with mounted payload relative to the gimbals axes; J_{yz}, J_{xz}, J_{xy} are the centrifugal inertia moments relative to the gimbals axes; $\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z$ are projections of the platform angular accelerations onto its own axes; M_x, M_y, M_z are moments acting by gimbals axes.

The mutual position of the coordinate axes for the two-axis stabilization system is represented in Fig. 2.

In accordance with Fig. 2 the angular rate projections of the platform onto its own axes may be represented in the following form

$$\omega_x = \beta,$$

$$\omega_y = \dot{\alpha} \sin \beta,$$
 (2)

$$\omega_z = \dot{\alpha} \cos \beta.$$

The platform motion relative to the axis z is not stabilized and the platform is deviated from this axis together with UAV. Based on the relationships (2) the Euler kinematics equations, which correspond to the sequence of turns represented in Fig. 1, become

$$\dot{\alpha} = \omega_z \cos\beta + \omega_y \sin\beta,$$

$$\dot{\beta} = \omega_x.$$
(3)



Fig. 1. Structural scheme of the two-axis stabilization system: C_i are controllers; A_i are actuators; G_i are gyros; ω_{id} are external disturbances (i=x, z); J_i are platform inertia moments; ω_i are platform angular rates (i = x, y, z); α , β , γ are platform attitude angles; $\dot{9}_{ref}$, $\dot{\psi}_{ref}$ are reference signals; k is the cross-connection coefficient; M_{zd} are

disturbance moments





Modern methods of the stabilization system design, as a rule, are based on the robust structural synthesis [2]. In this case, the mathematical description of the stabilization plant is of great importance. To achieve this goal is possible using the expressions (1) - (3) and the mathematical

model of the one-axis system for stabilization and tracking of the information-measuring devices.

For creating this model it is expedient to accept some simplifications for the set of equations (1) such as neglect by the centrifugal inertia moments. Taking into account this simplification, the mathematical model of the plant of the two-axis system for stabilization and control by attitude of the lines-of-sight of the UAV payload becomes

$$\begin{split} \dot{\alpha} &= \omega_{y} \sin \beta + \omega_{z} \cos \beta; \\ \dot{\beta} &= \omega_{x}; \\ \dot{\alpha}_{e} &= \omega_{e\alpha}; \\ \dot{\beta}_{e} &= \omega_{e\beta}; \\ \dot{U}_{\omega\alpha} &= U_{\omega d\alpha}; \\ \dot{U}_{\omega\beta} &= U_{\omega d\beta}; \\ \dot{\omega}_{\sigma\beta} &= U_{\omega d\beta}; \\ \dot{\omega}_{z} &= \left[-(J_{y} - J_{x})\omega_{x}\omega_{y} - M_{frz} \mathrm{sign}\omega_{z} + c_{r}(\alpha_{e} - \alpha) \right] / J_{z}; \\ \dot{\omega}_{y} &= \left[-(J_{x} - J_{z})\omega_{x}\omega_{z} - M_{fry} \mathrm{sign}\omega_{y} \right] / J_{y}; \\ \dot{\omega}_{x} &= \left[-(J_{z} - J_{y})\omega_{y}\omega_{z} - M_{frx} \mathrm{sign}\omega_{x} + c_{r}(\beta_{e} - \beta) \right] / J_{z}; \\ \dot{\omega}_{e\alpha} &= \left[-M_{fr\alpha} \mathrm{sign}\omega_{e\alpha} + c_{m}U_{\alpha} / R_{w} + c_{r}(\alpha_{e} - \alpha) \right] / J_{e\alpha}; \\ \dot{\omega}_{e\beta} &= \left[-M_{fr\beta} \mathrm{sign}\omega_{e\beta} + c_{m}U_{\beta} / R_{w} + c_{r}(\beta_{e} - \beta) \right] / J_{e\beta}; \\ \dot{U}_{\alpha} &= \left[-U_{\alpha} + k_{U\alpha}U_{con\alpha} - c_{ed}\omega_{e\alpha} \right] / T_{arm}; \\ \dot{U}_{\beta} &= \left[-U_{\beta} + k_{U\beta}U_{con\beta} - c_{ed}\omega_{e\beta} \right] / T_{arm}; \\ \dot{U}_{\omega d\alpha} &= -2v\Delta_{\omega}U_{\omega e\alpha} - U_{\alpha}\Delta_{\omega}^{2} + \Delta_{\omega}k_{ars}\omega_{z}; \\ \dot{U}_{\omega d\beta} &= -2v\Delta_{\omega}U_{\omega e\beta} - U_{\beta}\Delta_{\omega}^{2} + \Delta_{\omega}k_{ars}\omega_{x}, \end{split}$$
(4)

where α , β are angles of the platform turns; ω_z , ω_x are the platform angular rates in the horizontal and vertical planes respectively; α_e , β_e are the angles of turns of the motors mounted at the axes z, x; $\omega_{e\alpha}$, $\omega_{e\beta}$ are the motor rates; $U_{\omega\alpha}$, $U_{\omega\beta}$ are the output signals of gyro sensors measuring the platform angular rates by the axes z, x; $U_{\omega d\alpha}$, $U_{\omega d\beta}$ are the derivatives of the sensor output signals; J_x , J_y , J_z are the inertia moments of the platform with mounted on it information-measuring devices relative its own axes; M_{frx} , M_{fry} , M_{frz} are the nominal dry friction moments acting at the gimbals axes; c_r is the rigidity of elastic connection between the motor and a base, on which the plant is mounted; $M_{\rm fra}$, $M_{\rm fr\beta}$ are the nominal dry friction moments of motors installed at the gimbals axes z, x; $c_{\rm m}$ is the constant of the load moment at the motor shaft; $R_{\rm w}$ is the resistance of the motor armature winding; J_{ea} , $J_{e\beta}$ are the moments of the motor inertia; U_{α} , U_{β} are the armature voltages of motors mounted at the gimbals axes; $k_{U\alpha}$, $k_{U\beta}$ are the amplifier transfer constants; $U_{\rm con\alpha}$, $U_{\rm con\beta}$ are the voltages at the controller outputs; c_{ed} is the coefficient of proportionality between the motor angular rate and the electromotive force; $T_{\rm arm}$ is the time constant of the motor armature circuit; ν is the relative damping coefficient; Δ_{ω} is the bandwidth of the rate sensor, $k_{\rm ars}$ is the transfer constant of the rate sensor.

For further researches it is necessary to implement linearization of the equations (4) relative to the nominal values of the phase coordinates. Such linearization must include the following steps:

1) linearization of the expressions for the dry friction moments of the motor and stabilization plant determination;

2) neglect by the zero drift of the angular rate sensors;

3) assumption of smallness of the platform turn angles for linearization of the trigonometric functions.

After these transformations the set of equations (4) may be represented in the linearized form.

The linearized model of the stabilization plant will correspond to the state space model with the vector of state, control and observation

$$\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} \alpha & \beta & \alpha_{e} & \beta_{e} & U_{\omega\alpha} & U_{\omega\beta} & \omega_{z} \\ \omega_{y} & \omega_{x} & \omega_{e\alpha} & \omega_{e\beta} & U_{\alpha} & U_{\beta} & U_{\omega d\alpha} & U_{\omega d\beta} \end{bmatrix}, \\ \mathbf{u}^{\mathrm{T}} = \begin{bmatrix} U_{\mathrm{con}\,\alpha} & U_{\mathrm{con}\,\beta} \end{bmatrix}; \ \mathbf{y}^{\mathrm{T}} = \begin{bmatrix} \omega_{z} & \omega_{x} \end{bmatrix},$$
(5)

and quadruple of matrices **A**, **B**, **C**, **D**, which may be determined based on the set of equations (4).

The non-linear moment of the dry friction forces acting on the drive may be approximated by the linear dependence in accordance with the method of the harmonic linearization [8] $M_{\rm fri} {\rm sign}\omega_i = f_i \omega_i$,

 $i = x, y, z; M_{\text{fij}} \text{sign}\omega_j = f_j \omega_{ej}, j = \alpha, \beta.$

The approximating coefficients may be determined as a ratio of the first harmonic of the amplitude of the friction moment to the amplitude of the rate

$$f_i = 4M_{ai}/(\pi\omega_i), f_j = 4M_{aei}/(\pi\omega_{ej})$$

V. ROBUST STRUCTURAL SYNTHESIS

Modern procedures for design of systems for stabilization of lines-of-sight of payload operated at UAVs must be created taking into consideration action of external disturbance and measurement noise in real operation conditions. This problem may be solved as a problem of the robust structural H_{∞} -synthesis [2].

To ensure the high accuracy of both stabilization and tracking processes it is necessary to use the twodegree-of-freedom stabilization system [2].

There are different approaches to structural synthesis of the two-degree-of-freedom robust systems. The method suggested by K. Glover and D. McFarlane [3] and developed in [4] is based on the robust stabilization and determination of the parametrical disturbances by means of the normal

left co-prime factorization. It is based on forming desired frequency responses of the closed-loop robust control system by means of augmentation of the open-loop system. In the paper this approach is developed in the part of taking into consideration of the external disturbances and measurement noise.

The structural scheme of the designed system is represented in Fig. 3.

In accordance with Fig. 3, connection between input and output vectors of the studied stabilization system looks like

$$\begin{bmatrix} u_s \\ y_s \\ e \end{bmatrix} = \begin{bmatrix} \rho W_1 K_1 & K_2 W_2 G_{ds} & K_2 W & K_2 W_4 \\ \rho W G_s K_1 & W G_{ds} & W & W_4 \\ \rho^2 W_3 & \rho W G_{ds} & \rho W & \rho W_4 \end{bmatrix} \begin{bmatrix} r \\ d \\ n \\ \varphi \end{bmatrix}$$
or $\mathbf{z} = \mathbf{\Phi} \mathbf{w}$. (7)

(6)

where $\mathbf{z}^{\mathrm{T}} = \begin{bmatrix} u_s & y_s & e \end{bmatrix}$ represents the vector of output signals, which allow to estimate the functional of the system quality, and includes signals of the control, output and error respectively. The input vector $\mathbf{w}^{\mathrm{T}} = \begin{bmatrix} r & d & n & \phi \end{bmatrix}$ includes the reference signal, coordinate disturbance, noise and parametrical disturbance; Φ is the matrix transfer function of the closed-loop system ($G_{ds} = G_d G_s$, $W_3 = [W_2 G_s K_1 - T_{ref}])$. H_{∞} -norm of the matrix transfer function Φ , described by the expression (6), represents the new functional of the designed system, as elements of this matrix determine accuracy, robustness and costs for control [2]. The transfer functions $W_1 = (I + G_s K_2)^{-1}$, $W_2 = (I + G_s K_2)^{-1}$ $+K_{2}G_{s}$)⁻¹ are the input and output sensitivities functions [4].



Fig. 3. Structural scheme of the two-degree-of-freedom stabilization system: K_1 , K_2 , $N_s M_s^{-1}$ are the transfer functions of the controllers and the plant respectively

In the state space the matrix of the generalized system becomes

$$\mathbf{P} = \begin{bmatrix} A_s & 0 & 0 & B_d \\ 0 & A_r & B_r & 0 \\ 0 & 0 & 0 & 0 \\ C_s & 0 & 0 & D_d \\ \rho C_s & -\rho^2 C_r & -\rho^2 D_r & \rho D_d \\ 0 & 0 & \rho I & 0 \\ C_s & 0 & 0 & D_d \end{bmatrix}$$
$$\begin{pmatrix} 0 & (B_s D_s^{\mathrm{T}} + Z_s C_s^{\mathrm{T}}) D_{M_s} & B_s \\ I & 0 & 0 \\ 0 & 0 & I \\ 0 & D_{M_s^{-1}} & D_s \\ 0 & 0 & 0 \\ I & D_{M_s^{-1}} & D_s \end{bmatrix}$$
$$\begin{pmatrix} (8) \end{pmatrix}$$

The procedure of the H_{∞} -synthesis includes such stages as creation of the system mathematical description (both linearized and taking into consideration non-linearities inherent to the real systems), choice of the weighting transfer functions, augmentation of the plant and properly structural synthesis. All these stages may be implemented by means of the Robust System Toolbox representing components of the calculating system MatLab.

Now usage of the discrete controllers is the most actual for the practical applications. There are two known approaches to creation of the discrete controllers.

Design of the discrete controller of the researched system was implemented based on the second approach. The flow chart of this process is represented in Fig. 4.



Fig. 4. Flow chart f the discrete controller design

The weighting coefficients were determine based on the heuristic methods. The structure of designed robust controller will be described by the quadruple of matrices in the space state. Check of synthesis results was carried out by means of the non-linear model, which was created by means of Simulink, is

given in Fig. 5. The simulation results are given in Fig. 6.



Fig. 5. The stabilization system model created by means of Simulink



Fig. 6. Transient processes: (a) under action of the constant disturbance moments; (b) under action of random disturbance moments

Represented simulation results of the designed system simulation (Fig. 6) prove the possibility of

the robust system to keep the accuracy requirements in conditions of disturbances and measurement noise action [9].

VI. CONCLUSIONS

The mathematical description of the two-axis stabilization system taking into consideration interconnection between channels is represented. Approach to design of the two-axis robust system for stabilization of information-measuring devices operated at UAV is represented. The new functional of the robust structural synthesis of the two-degreeof-freedom stabilization system, taking into consideration influence of the coordinate disturbance and measurement noise is suggested.

REFERENCES

[1] D. W. Gu, P. H. Petkov, and M. M. Konstantinov, *Robust Control Design with MATLAB*, London: Springer-Verlag, 2005.

[2] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control*, New York: Jonh Wiley, 1997.

[3] K. Zhou and J. Doyle, *Essential of robust control*, New Jersey: Prentice Hall, 1999.

[4] www.aviales.ru/popup.aspx. Market report of unmanned means which may beused for preservation of forests from fire.

[5] O. A. Sushchenko and A. A. Tunik, "Robust Stabilization of UAV Observation Equipment" in *Proc. IEEE 2nd Int. Conf. Actual Problems of Unmanned Air Vehicles Developments* October, 15–17, 2013, pp. 176–180.

[6] J. M. Hilkert, "Inertially stabilized platform technology". *IEEE Control Systems Magazine*, vol. 26, no. 1, 2008, pp. 26–46.

[7] A. I. Lurye, *Analytic mechanics*. Moscow: Gostechizdat, 1961. 822 p.

[8] V. A. Besekersky and E. P. Popov, *Theory of Automatic Control Systems*. Moscow: Nauka, 2004.

[9] O. A. Sushchenko and A. A. Tunik, "Robust Optimization of Inertially Stabilized Platforms", in *Proc. IEEE 2nd Int. Conf. Methods and Systems of Navigation and Motion Control*, MSNMC-2012, Kyiv, October, 9–12, 2012, pp. 101–105.

Received August 11, 2015.

Sushchenko Olha. Doctor of Science (Engineering), Professor.

Aircraft Control Systems Department, National Aviation University, Education-Scientific Institute of Air Navigation, Kyiv, Ukraine.

Education: Kyiv Politechnic Institute, Kyiv, Ukraine, (1980).

Research interests: systems for stabilization of information-measuring devices operated at vehicles of the wide class. Publications: 120.

E-mail: sushoa@ukr.net

О. А. Сущенко. Робастна двоосна стабілізация корисного навантаження безпілотних літальних апаратів

Статтю присвячено дослідженню основних принципів робастної двоосної стабілізації корисного навантаження (камери, лазерні локатори), що експлуатуються на безпілотних літальних апаратах. Наведено підхід до робастної структурної оптимізації. Запропоновано математичний опис системи, беручи до уваги взаємозв'язок між каналами. Представлено функціонал оптимізації системи із двома ступенями вільності (комбінованої системи).

Ключові слова: корисне навантаження; робастна оптимізація; структурний синтез; двоосна стабілізація.

Сущенко Ольга Андріївна. Доктор технічних наук. Професор.

Кафедра систем управління літальних апаратів, Національний аваційний університет, Навчально-науковий Інститут аеронавігації, Київ, Україна.

Освіта: Київський політехнічний інститут, Київ, Україна, (1980).

Напрям наукової діяльності: системи стабілізації інформаційно-вимірювальних пристроїв, експлуатованих на рухомих об'єктах широкого класу.

Кількість публікацій: 120.

E-mail: sushoa@ukr.net

О. А. Сущенко. Робастная двухосная стабилизация полезной нагрузки беспилотных летальных аппратов

Статья посвящена исследованию основних принципов робастной двухосной стабилизации полезной нагрузки (камеры, лазерные локаторы), эксплуатируемой на беспилотных летательных аппаратах. Приведен подход к робастной структурной оптимизации. Предложено математическое описание системы с учетом взаимосвязи между каналами. Представлен функционал оптимизации системы с двумя степенями свободы (комбинированной системы).

Ключевые слова: полезная нагрузка; робастная оптимизация; структурный синтез; двухосная стабилизация.

Сущенко Ольга Андреевна. Доктор технических наук. Профессор.

Кафедра систем управления летательных аппаратов, Национальний авиационный университет, Учебнонаучный Институт аэронавигации, Киев, Украина.

Образование: Киевский политехнический институт, Киев, Украина (1980).

Направление научной деятельности: системы стабилизации информационно-измерительных устройств, эксплуатируемых на подвижных объектах широкого класса.

Количество публикаций: 120.

E-mail: sushoa@ukr.net