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MODELING OF CONVERGENT NETWORK OPERATION PROCESS

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Abstract—*We are studying the convergent network operation process with various types of traffic being transferred. As a mathematical model of the convergent network we are using the retrial queuing system with the flow of demands of different types. During the modelling we are taking into account such a feature of the information transfer process as the diversity of the input flow and presence of the retries for data transfer. The formulas determining the transition probabilities of system states have been developed.*

Index terms—Convergent network; orbit; transition probabilities; flow of different types; retrial queuing system.

I. INTRODUCTION

The current trend of convergence of different types of networks, such as computer and telecommunication, has led to the need to transfer all types of traffic via network. Convergent network – is a computer network that combines the transfer of voice and data, including multimedia, video, etc., through a common channel. The effective operation of the convergent network requires sufficient bandwidth with the possibility of videoconferencing and accessing corporate resources, services and databases.

The transferred traffic in a convergent network is divided into the traffic which is sensitive to packet transfer delays, and the traffic sensitive to packet loss and distortion. Traffic which is sensitive to packet transfer delays includes: E-mail applications; text editor, working with remote files; voice transfer applications – when exceeding the delay variation threshold the voice quality degrades dramatically; applications hypersensitive for delays – managing technical objects in real time. Traffic sensitive to data loss includes: text documents, program codes, numeric arrays, as well as all the traditional network applications (file services, database services, etc.). Resistant to data loss applications are multiple applications transferring traffic with information about the inertia physical processes, such as applications that work with multimedia traffic [1].

Research problems regarding efficiency of the data transfer in a convergent network, taking into account the diversity of traffic, require creation of mathematical models, taking into account the stochastic nature of the processes. For modelling of real processes taking place during transfer of different types of information in convergent networks it is recommended to use the single-channel retrial queuing system with the flow of demands of different types.

II. ANALYSIS OF STUDIES AND PUBLICATIONS

In order to analyze and study the functioning of various networks there are queuing system models with retransmission [2]. The input demand flow enters the service channel. The demand received by the system is being served immediately in case there is a free service channel. Otherwise, when the demand reaches the system and all service channels are busy, it goes to the orbit. Orbit is a virtual environment which accumulates demands that did not get access to the service channels at the time of entering the system. Demands that are in orbit are trying to get back to the system to get served after deterministic or random time period. This forms the secondary flow of demands. Demand which enters the system after repeated calls is served as any other demand that enters the system for the first time. In case there's a free channel such demands are immediately served and form the output flow.

The principal difference between retrial queuing system and classical queuing system is that demands that enter the system and find the service channel busy - do not leave the system but go for retransmission in order to try to occupy the channel later.

Note that the presence of repeated attempts is an essential feature of information transfer systems and networks. Ignoring this effect can lead to significant errors while receiving their functioning indexes. Consideration of repetition in constructing mathematical models of such systems makes them more adequate and allows to get new operation indicators.

A large number of papers are devoted to retrial queuing systems research. The most common for such systems is the model presented in the review article by T. Yang and Y. G. C. Templeton [3]. For example, the paper [4] studies the M/M/1 system with recurrence of demands serving two types of demands. And the paper [5] studies the priority queuing system with recurrence. In [6] I. M. Kovalenko and O. V. Koba are classifying typical classes of retrial queuing systems that originate from the practical problems. They give a brief comparative description of different classes of retrial queuing systems. In [7] an algorithm for statistical mode-ling of $GI/G/m/0//1/G$ multi-channel queuing system with returns has been developed, with the purpose of evaluation of the indicators of the system operation effectiveness, in particular, for evaluation of stationary probability of demand losses. The values of the stationary probabilities of denial of service have been obtained. A significant research of retrial queuing systems is presented in papers by D. Y. Kuznetsov, A. A. Nazarov, V. V. Anisimov, L. Lakatosh and others [8] – [10].

Thus, the retrial queuing system models can be used in solving the problems of technical design of modern communication systems that require calculating the probability-time and economic indicators of their functioning.

In this paper for the purpose of modelling of convergent network operation process it is proposed to use the retrial queuing system with regard to such a peculiarity of data transfer as the diverse nature of the input flow.

III. Statement of the Problem

A single-channel retrial queuing system is considered. The initial input flow consists of demands of s types. Let us define system states as:

$$\bar{k} = (k_0, k_1, ..., k_s),$$

where $k_0 = \begin{cases} 0, & \text{the chanel is free,} \\ j, & \text{the chanel serves a demand} \\ & \text{of the } j\text{th type } (j = 1, 2, ..., s). \end{cases}$

For $1 \le j \le s$, k_j is the number of demands of jth type in the channel and in orbit, $\lambda_j(\bar{k})\Delta + o(\Delta)$ is the probability that a demand of jth type will arrive (from the outside of the system) in a small interval $(t, t+\Delta)$ provided that at the time t its state is $\bar{k}$.

Denote by $\bar{k}(t) = (k_0(t), k_1(t)..., k_s(t))$ the state of the system at time t; by t_n the terminal time of the nth service; and by $\bar{k}_n = (0, k_{n1}, ..., k_{ns})$ the state of the system at the time $t_n + 0$.

Let us define $B_j(x)$ as a distribution function of the holding time of a demand of j th type.

Let $|\bar{k}_n| = k_{n1} + ... + k_{ns}$ and always $|\bar{k}_n| \le N$, where N is a given natural number.

If a demand of j th type does not appears in the channel immediately, it arrives in orbit, whence it returns in an exponential time with parameter $\theta_j > 0$.

The problem is as follows: deduce formulas to evaluate the steady-state distribution $p(\bar{k})$ on the set Y_0 of states of embedded Markov chain $(\bar{k}_n)$.

III. Exact Formulas

Let us write the steady-state equation:

$$p(\bar{k}) = \sum_{r \in Y_0} p(\bar{r}) p(\bar{r}, \bar{k}), \ \bar{k} \in Y_0 \qquad (1)$$

$$\sum_{k \in Y_0} p(\bar{k}) = 1, \qquad (2)$$

where $p(\bar{k})$) is the steady-state probability of state $\bar{k}$, $p(\bar{r}, \bar{k})$ is the transition probability from $\bar{r}$ to $\bar{k}$ in one step.

Let us consider system states at the moments $t_{n-1} + o$, and $t_n^{'} + o$, $t_n + o$, where t_{n-1} is the $(n-1)$th moment of the termination of service of the demand, $t_n^{'}$ is the nth moment of the beginning of service of the demand, and, t_n is the nth moment of the termination of its service. These states are equal, respectively, to $\bar{r} = (0, r_1, ..., r_s)$ and $\bar{l} = (l_0, l_1, ..., l_s)$, $1 \le l_0 \le s$, $\bar{k} = (0, k_1, ..., k_s)$.

Denote by $u(\bar{r}, \bar{l})$ the transition probability from the state $\bar{r}$ to the state $\bar{l}$; and by $v(\bar{l}, \bar{k})$ the transition probability from the state $\bar{l}$ to the state $\bar{k}$.

From the description of the system operation we have

$$u(\bar{r}, \bar{l}) = \begin{cases} \dfrac{r_j \theta_j}{\sum_{i=1}^{s} [r_i \theta_i + \lambda_i(\bar{r})]}, & \bar{r} = (0, r_1, ..., r_s), \quad \bar{l} = (j, r_1, ..., r_s), \quad 1 \le j \le s, \\ \dfrac{\lambda_j(\bar{r})}{\sum_{i=1}^{s} [r_i \theta_i + \lambda_i(\bar{r})]}, & \bar{r} = (0, r_1, ..., r_s), \quad \bar{l} = (j, r_1, ..., r_j + 1, ..., r_s), \quad 1 \le j \le s, \\ 0 \quad \text{otherwise.} \end{cases} \qquad (3)$$

Let us define the transition probability $v(\overline{l},\overline{k})$ from the state $\overline{l}$ to the state $\overline{k}$:

$$v(\overline{l},\overline{k})=\int_0^\infty dB_j(x)\psi(x,\overline{l},\overline{k}), \qquad (4)$$

where $\overline{l}=(j,l_1,...,l_s)$, $1\le j\le s$; $\psi(x,\overline{l},\overline{k})$ is the transition probability from $\overline{l}$ to $\overline{k}$ provided that $t_n-t'_n=x$.

Let us determine the probabilities $\psi(x,\overline{l},\overline{k})$ assuming that $\overline{l}=(j,l_1,...,l_s)$, where $1\le j\le s$, and $l_i\ge 0$, $1\le i\le s$, $l_j\ge 1$. For $i\in\{1,...,s\}$ denote by γ_i the number of new demands of ith type arrived in the system in time x. Then we obtain the equations

$$k_i=l_i+\gamma_i, 1\le i\le s, i\ne j,\ k_i=l_i+\gamma_i-1. \qquad (5)$$

From here we deduce the necessary conditions for possible values of k_i:

$$k_i\ge l_i,\ 1\le i\le s,\ i\ne j;\ k_j\ge l_j-1.$$

For given $\overline{l}$ and $\overline{k}$ satisfying these conditions $\gamma_i=l_i-k_i$ new demands should arrive in the interval (t'_n,t_n) of duration x for all $i\ne j$, and $\gamma_i=l_i-k_i+1$ new demands for $i=j$. The probability of this event is $\psi(x,\overline{l},\overline{k})$. The total number of new demands is $d=|\overline{k}|-|\overline{l}|+1$.

For $d=0$ we have

$$\psi(x,\overline{l},\overline{k})=\exp\left\{-\sum_{i=1}^{s}\lambda_i(\overline{l})x\right\}.$$

For $d>0$ we have

$$\psi(x,\overline{l},\overline{k})=\sum_{\Gamma}\psi_\Gamma(x,\overline{l},\overline{k}), \qquad (6)$$

where Γ are all possible chains $(i_1,...,i_d)$, composed from the numbers 1, 2,..., s and satisfying the following conditions.

Consider a state $\overline{l}=(j,l_1,...,l_s)$. Recurrently determine $\overline{l}_m$, $1\le m\le d$, where $\overline{l}_1=\overline{l}+\overline{e}(i_1)$ (in what follows, $\overline{e}(n)$ is the vector with unity at the n-place and remaining elements being zero):

$$\overline{l}_m=\overline{l}_{m-1}+\overline{e}(i_m),\ 2\le m\le d;\ \overline{k}+\overline{e}(j)=\overline{l}_d.$$

Then

$$\psi_\Gamma(x,\overline{l},\overline{k})=\lambda_{i_1}(\overline{l})\lambda_{i_2}(\overline{l}_1)...\lambda_{i_d}(\overline{l}_{d-1})$$
$$\cdot\int\limits_{0<x_1<...<x_d<x}\!\!...\!\int \exp\left\{-\left[\lambda(\overline{l})x_1+\lambda(\overline{l}_1)(x_2-x_1)+...+\lambda(\overline{l}_d)(x-x_2)\right]\right\}dx_1...dx_d, \qquad (7)$$

where $\lambda(\overline{l})=\sum_{i=1}^{s}\lambda_i(\overline{l})$.

V. The Final Formula for Transition Probabilities

Let us write the final formula:

$$p(\overline{r},\overline{k})=\sum_{l}u(\overline{r},\overline{l})v(\overline{l},\overline{k}), \qquad (8)$$

where $\overline{r}=(0,r_1,...,r_s)$, $\overline{k}=(0,k_1,...,k_s)$, $\overline{l}=(j,l_1,...,l_s)$, $j\in\{1,...,s\}$. The probabilities appearing in the right-hand side of formula (8) are defined by formulas (3), (4), (6), and (7). After $p(\overline{r},\overline{k})$, is calculated, it remains to find the solution of the system of equations (1), (2) to determine the steady-state probabilities $p(\overline{k})$ of the embedded Markov chain.

VI. Approximate Formulas

The calculations are considerably simplified if a two-sided estimate of the integrand based on the inequalities $1-\alpha<\exp\{-\alpha\}<1$ (where α is the expression in square brackets) is used in integrals (7). For example, for $d=1$ the integral factor on the right-hand side of formula (7) is estimated as follows:

$$\int\limits_{0<x_1<x}\left(1-\lambda(\overline{l})x_1-\lambda(\overline{l}_1)(x-x_1)\right)dx_1$$
$$<\int\limits_{0<x_1<x}\exp\left\{-\left[\lambda(\overline{l})x_1+\lambda(\overline{l}_1)(x-x_1)\right]\right\}dx_1<\int\limits_{0<x_1<x}dx_1.$$

Simple evaluations yield the two-sided estimate

$$x-\left(\lambda(\overline{l})+\lambda(\overline{l}_1)\right)\frac{x^2}{2}$$
$$<\int\limits_{0<x_1<x}\exp\left\{-\left[\lambda(\overline{l})x_1+\lambda(\overline{l}_1)(x-x_1)\right]\right\}dx_1<x.$$

Assume now that $\psi(x,\overline{l},\overline{k})$ admits a two-sided estimate

$$f\left(x-\lambda\frac{x^2}{2}\right)<\psi(x,\overline{l},\overline{k})<fx,$$

where λ is a small parameter and *f* is some function. By equation (4)

$$f\tau_j - \lambda\frac{\alpha_{j2}}{2} < v\left(\bar{l},\bar{k}\right) < f\tau_j,$$

where $\tau_j = \int_0^\infty x dB_j(x), \alpha_{j2} = \int_0^\infty x^2 dB_j(x)$.

If

$$\lambda\frac{\alpha_{j2}}{\tau_j} \to 0, \qquad (9)$$

the lower- and upper-bound estimates of $v(\bar{l},\bar{k})$ asymptotically approach each other. Condition (9) can be interpreted as a small probability that two or more demands arrive while one demand is served.

VII. Conclusions

For single-channel retrial queuing system and flow of demands of different types the formulas are defined to calculate transition probabilities of the embedded Markov chain. Thus, with the help of these formulas we can define the operational indicators of convergent network with different types of traffic (diverse traffic).

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О. М. Кучерява. Моделювання процесу функціонування конвергентної мережі

Досліджено процес функціонування конвергентної мережі з передачею різноманітного трафіку. В якості математичної моделі конвергентної мережі використано систему масового обслуговування з повторенням і потоком різнотипних заявок. При моделюванні враховано таку особливість процесу передачі інформації як різнотипність вхідного потоку та наявність повторних спроб під час передачі даних. Виведено формули визначення перехідних ймовірностей станів системи.

Ключові слова: конвергентна мережа; орбіта; перехідні ймовірності; потік різнотипних заявок; система масового обслуговування з повторенням заявок.

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О. Н. Кучерявая. Моделирование процесса функционирования конвергентной сети
Исследован процесс функционирования конвергентной сети с передачей разнообразного трафика. В качестве математической модели конвергентной сети использована система массового обслуживания с повторением и потоком разнотипных заявок. При моделировании учтена такая особенность процесса передачи информации как разнотипность входящего потока и наличие повторных попыток при передаче данных. Выведены формулы определения переходных вероятностей состояний системы.
Ключевые слова: конвергентная сеть; переходные вероятности; поток разнотипных заявок; система массового обслуживания с повторением.

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