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MATHEMATICAL MODEL OF DISTRIBUTED TRANSFER INFORMATION HIGHWAYS

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Abstract**—***A mathematical model of distributed information transmission highways based on the irregular transmission lines with a casual wave resistance is developed.*

Index Terms—Wave resistance; distributed transfer; information highways.

I. INTRODUCTION

In today's high-speed information systems the wave nature of the processes in the transmission of information highways should be taken into account. Currently, segments of transmission lines with constant characteristic impedance (scheduled) are used as models of such highways. This model is very approximate and takes into account the generally regular random errors in the implementation of continuous nominal wave impedance irregular and limited class of perturbations [1]. This model is not applicable to the analysis of processes in broadband information highways with variable along the length of the wave resistance, since the processes in this case are described by equations of irregular lines [1].

During the technical realization of irregular lines due to various technical inaccuracies inevitably there is an error in the reproduction of the desired value of the wave resistance, which leads to the rejection of the transmission characteristics of the line set. In general, this error is a random variable value Fig. 1. Therefore, the wave resistance is not a deterministic function, but a random process. In general, the expectation of this process is different from the nominal values. Therefore, in the production line necessary correction process (optimization) in the implementation of the wave resistance, which compensates for this difference.

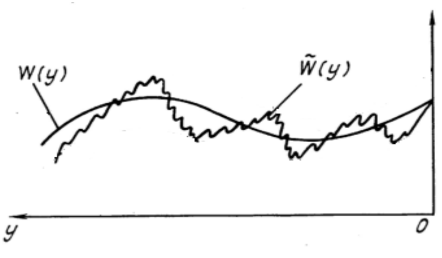

Fig. 1. Line with a casual wave resistance $\tilde{W}(y)$, $\tilde{W}(y)$ is the nominal characteristic impedance the geometric coordinate

In this paper, the stochastic description of irregular lines with random distributed inhomogeneities based on the theory of Markov processes [3] is given.

II. CONCLUSION OF THE STOCHASTIC EQUATION FOR THE WAVE RESISTANCE OF THE INHOMOGENEOUS LINE

It is known [4] that the processes of differential impedances are determined entirely in non-uniform lines $N(\tau)=\dfrac{W'(\tau)}{2W(\tau)}$, where W is the characteristic impedance line; τ is the time of the delay line. From the expression $N(\tau)$ follows that all the properties of the line is defined as the value of the wave resistance $W(\tau)$ at this point τ, and the rate of change $W'(\tau)=\dfrac{\partial W(\tau)}{\partial \tau}$.

By its nature, mistakes in the implementation of the line conductors are very diverse. For example, the implementation of coaxial lines available as unforeseen abrupt changes in diameter of the conductor (jumps), and quite a slow change in the coordinate error. When lines are implemented in a stripline structures, width of strips and dielectric permittivity vary randomly. Therefore, if the starting take random function $N=\dfrac{W'}{2W}$, the range of change is within

$$-\infty < N(\tau) < \infty.$$

Error in the playback function $N(\tau)$ is a result of many different unrelated factors. Therefore, according to limit theorem [3], we can assume that the error is a normal distribution. The interval error is determined correlation feature manufacturing process line and its value is generally much smaller than the length of a geometric line. For example, when grinding conductors correlation interval defined grain size; striplines when implementing correlation interval depends on the size of the microparticles forming the conductive layer, etc. It follows from the foregoing that the error in the playback function $N=\frac{W'}{2W}$ can be approximately regarded as normal white stationary noise with zero mean. The last assertion follows from the fact that the probability of positive and negative error is the same.

In the future, for the convenience of the following symbols are used: the random variables are indicated above "~" sign in - actual geometrical length, *y*-axis is directed to the left. For deterministic mined-functions (for nominal values) denote left unchanged.

Thus, on the basis of the above, we can write the stochastic equation for the wave resistance

$$\tilde{N}(y)=N(y)+\Delta_1(y), \quad \tilde{N}(y)=\frac{\tilde{W}'(y)}{2\tilde{W}(y)}, \tag{1}$$

where $N(y)=\frac{W(y)}{2W(y)}$ is the deterministic function; $\tilde{W}(y)$ is the random function of the wave resistance,

$$\Delta_1(y)=g(y)\Delta(y), \tag{2}$$

where $\Delta(y)$ is the normal stationary white noise with the correlation function

$$K_\Delta(y_1,y_2)=\frac{N_0}{2}\delta(y_2-y_1),$$

and zero expectation $m\{\Delta\}=0$, $g(y)$ is the some function characterizing the statistical properties of the process of realization of the line, $g(y)\geq 0$.

From equations (1) and (2) we can find the characteristic impedance

$$W(y)=A(y)X\,,\; A(y)=\exp\left\{2\int_0^y N(y)dy\right\}, \tag{3}$$

$$X=\tilde{W}(0)\exp\left\{2\int_0^y \Delta_1(y)dy\right\}.$$

Represent a process X

$$X=\exp\{2V\}, \tag{4}$$

where

$$V=\int_0^y \Delta_1(y)dy+\frac{1}{2}\ln\tilde{W}(0). \tag{5}$$

From equation (5) it follows that V is a Markov process with diffusion coefficient

$$b(y)=\frac{N_0 g^2(y)}{2}, \tag{6}$$

and zero drift coefficient [3]. Instead of equation (5) is often convenient to use a different form of writing

$$\frac{dV}{dy}=\Delta_1(y), \qquad V(0)=\lambda_0=\frac{1}{2}\ln\tilde{W}(0),$$

where $V(0)=\lambda_0$ is the initial random value.

From equations (3), (4) follows that the statistical characteristics of the wave resistance

$$\tilde{W}(y)=A(y)e^{2V} \tag{7}$$

completely determined by a Markov process V. Let us turn to the study of the basic properties of the process.

III. Determination of the Density Probability of the Markov Process V, Concluded Between Two Reflective Boundaries

The density probability $P(v,y)$ of the Markov process $V(y)$ satisfies the Fokker–Planck–Kolmogorov [3]. In our case, this equation takes the form

$$\frac{\partial}{\partial y}P(v,y)=\frac{1}{2}b(y)\frac{\partial^2}{\partial V^2}P(v,y). \tag{8}$$

Equation (8) admits separation of variables. Therefore, assuming

$$P(v,y)=V(v)Y(y), \tag{9}$$

of equation (8) we get

$$\frac{1}{b(y)Y(y)}\frac{\partial Y}{\partial y}=\frac{1}{2}\frac{1}{V(v)}\frac{\partial^2 V(v)}{\partial V^2}=-\lambda^2, \tag{10}$$

where λ^2 is a positive number. Since equation (10) is valid for all y and v, instead of (10) can be considered a pair of equations

$$V''+\lambda^2 V=0, \tag{11}$$

$$Y'+\frac{\lambda^2}{2}b(y)Y=0. \tag{12}$$

Decision (12) is a function of

$$Y(y)=Y(0)\exp\left\{-\frac{1}{2}\lambda^2\int_0^y b(y)dy\right\} \tag{13}$$

We assume that the reflective boundaries are located at the points $V=0$ and $V=2h$. Reflection condition is the vanishing of the stream function $G(v,y)$ [3]. For the process $V(y)$ in question.

$$G(v,y) = -\frac{1}{2}\frac{d}{dV}\left[b(y)P(v,y)\right].$$

Hence we find that the density probability $P(v,y)$ must satisfy the boundary conditions

$$\frac{\partial}{\partial V}P(v,y)\big|_{v=0} = \frac{\partial}{\partial V}P(v,y)\big|_{v=2h} = 0$$

Taking into account (9), we find

$$V'(0) = V'(2h) = 0. \qquad (14)$$

Decision (11) under the conditions (14) is known to the system orthonormal functions ϕ_k:

$$\phi_0(V) = \frac{1}{\sqrt{2h}}, \quad \phi_k(V) = \frac{1}{\sqrt{h}}\cos\lambda_k V, \quad \lambda_k = \frac{k\pi}{2h}.$$

Therefore, according to the separation of variables [3], the general solution that satisfies (6), equal to

$$P(v,y) = \sum_{k=0}^{\infty} C_k e^{-\frac{1}{2}\lambda_k^2\int_0^y b(y)dy}\cos\lambda_k V$$

Constant C_k determined by the initial conditions. For example, if the process V at the point $y = 0$ determined, i.e. $V(0) = \lambda_0$ then

$$P(V,0) = \delta(V - \lambda_0),$$

where $\delta(v)$ is the Dirac delta function. In this case, the decomposition [3]

$$\delta(V - \lambda_0) = \sum_{k=0}\phi_k(V)\phi_k(0),$$

follows that

$$C_0 = \frac{1}{2h}, \quad C_k = \frac{1}{h}\cos k\pi\frac{\lambda_0}{2h}.$$

Consequently,

$$P(v,y,\lambda_0) = \frac{1}{2h} + \frac{1}{h}\sum_{k=1}^{\infty}\cos\left[\frac{k\pi}{2h}\lambda_0\right]\cos\left[\frac{k\pi}{2h}v\right]\exp\left\{-\frac{k^2\pi^2}{8h^2}\int_0^y b(y)dy\right\}, \quad 0 < \lambda_0 < 2h, \quad 0 < v < 2h.$$

If the process is viewed in –*h*, *h* space,

$$P_{-h,h}(v,y,\lambda_0) = \frac{1}{2h} + \cos\left[\frac{k\pi}{2h}(\lambda_0 + h)\right]\cos\left[\frac{k\pi}{2h}(v + h)\right] + \frac{1}{h}\sum_{k=1}^{\infty}\exp\left\{-\frac{k^2\pi^2}{8h^2}\int_0^y b(y)dy\right\}, \qquad (15)$$
$$-h < \lambda_0 < h, \quad -h < v < h.$$

When considering the process of arbitrary boundaries between *c*, *d*, *c* < *d*, in (15) should produce a known change of variables [3]:

$$P_{c,d}(v,y,\lambda_0) = P_{-\frac{d-c}{2},\frac{d-c}{2}}\left(v,y,\lambda_0 - \frac{c+d}{2}\right),$$
$$c < \lambda_0 < d, \quad c < v < d.$$

If the initial condition $\lambda_0 = v(0)$ is a random variable, according to the method of separation of variables [3] the general solution will be equal to:

$$P_{c,d}(v,y) = \int_c^d P_{c,d}(v,y,\lambda_0)P_0(\lambda_0)d\lambda_0,$$
$$c < v < d,$$

where $P_0(\lambda)$ is the probability density value λ_0.

IV. Determination of the Probability of Not Exceeding of Markov Process Beyond the Defined Boundaries

To determine the probability of not exceeding of the process v beyond the defined boundaries $q_{c,d}$ use the method [3], based on a decision of direct Fokker–Planck–Kolmogorov. For the process of *V*, this equation takes the form

$$\frac{\partial}{\partial y}\tilde{P}(v,y,\lambda_0) = \frac{1}{2}b(y)\frac{\partial^2}{\partial V^2}\tilde{P}(v,y,\lambda_0), \qquad (16)$$

where $\tilde{P}(v,y,\lambda_0)$ is the density, the probability of transition v from the initial point $\lambda_0 \in (c,d)$ to any point inside the interval (c,d) for the process trajectories $V(y)$ that never reached the boundaries of *c*, *d* smaller at coordinates.

To determine the probability $q_{c,d}$ of the points *c* and *d* must be observed absorption condition [3]

$$\tilde{P}(c,y,\lambda_0) = \tilde{P}(d,y,\lambda_0) = 0.$$

Equation (16) with the same form as equation (8). Therefore, relations (11) – (13) remain valid. absorption condition will be recorded in the form of

$$V(c) = V(d) = 0.$$

Following the method of separation of variables [3] with *a* = –*h*, *d* = *h* and deterministic initial condition, we find that the probability of absenteeism *V* process abroad –*h, h*:

$$q_{-h,h}(y,\lambda_0) = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos\left[\frac{(2n+1)\pi\lambda_0}{2h}\right]\exp\left\{-\frac{(2n+1)^2\pi^2}{8h^2}\int_0^y b(y)dy\right\}, \quad (17)$$

$$-h < \lambda_0 < h.$$

When considering the arbitrary region with in *c, d*

$$q_{c,d}(y,\lambda_0) = q_{-\frac{d-c}{2},\frac{d-c}{2}}\left(y,\lambda_0 - \frac{c+d}{2}\right). \quad (18)$$

If λ_0 a random variable with a density probability $P_0(\lambda_0)$, then

$$q_{c,d}(y) = \int_c^d q_{c,d}(y,\lambda_0)P_0(\lambda_0)d\lambda_0. \quad (19)$$

In the derivation of (17) – (19) the whole line of reasoning is the same as in sections II.

Note, if in the expression for the diffusion coefficient $b(y)$ (6) it would be taken $g(y) = 1$, all the formulas derived in sections II and III will be transferred to the corresponding formula of Wiener process [3].

V. Example

Random variable with range, uniformly distributed in

$$P_0(\lambda_0) = \frac{1}{d-c}. \quad (20)$$

From equations (19) – (20) we find that the probability of absenteeism *V* process beyond the boundaries *c*, *d*:

$$q_{c,d}(y) = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{2n+1}\exp\left\{-\frac{(2n+1)^2\pi^2}{2(d-c)^2}\int_0^y b(y)dy\right\}. \quad (21)$$

Thus, the equations (17) – (21) determine the probability that the random realization of distributed circuit characteristic impedance (7) will not exceed the set limits, that is, these formulas describe the percentage of product yield. Furthermore, using expression (17) – (19) at a predetermined reject rate to define the required accuracy can implement the wave resistance of irregular lines.

VI. Conclusions

Based on the theory of Markov processes derived stochastic description of irregular lines with random inhomogeneities distributed. For example, we can see that the obtained formulas determine the likelihood that will not exceed the set limits, that is, these formulas describe the percentage of product yield characteristic impedance at random realization of distributed circuits. Also, for a given percentage of the marriage can determine the required accuracy of the implementation of the wave resistance of irregular lines.

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В. В. Козловський, Д. П. Чирва, І. О. Басюк, Д. О. Волокітін. Математична модель розподілених магістралей передачі інформації

Розроблено математичну модель розподілених магістралей передачі інформації на основі нерегулярних ліній передачі з випадковим хвильовим опором.

Ключові слова: хвильовий опір; розподілені магістралі; передача інформації.

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В. В. Козловский, Д. П. Чирва, И. А. Басюк, Д. А. Волокитин. Математическая модель распределённых магистралей передачи информации

Разработана математическая модель распределённых магистралей передачи информации на основе нерегулярных линий передачи со случайным волновым сопротивлением.

Ключевые слова: волновое сопротивление; распределённые магистрали; передача информации.

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