MATHEMATICAL MODELING OF PROCESSES AND SYSTEMS

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ENTROPICAL ANALYSIS OF MACROSYSTEMS OUTSIDE THE MAIN POSTULET OF STATISTICAL MECHANICS

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Abstract—*It is shown that for most nonphysical macrosystems the basic postulate of statistical mechanics (the postulate of equal a priori probability of microstates) loses its power and the role of entropy should be fulfilled by a more general characteristic. Examples of distribution laws are considered.*

Index Terms—Entropy analysis; entropy divergence; macrosystems; complex systems.

I. INTRODUCTION

The variational principle of entropy maximum allows us to find the most probable system configuration, and this is precisely its macrostate, which can be reconstituted by the maximum number of distinct microstates. Having appeared in the depths of statistical physics, this principle soon became applicable in other fields of economic, humanitarian, and natural sciences.

However, not everybody considers such borrowing valid. Many physicists are very distrustful to the idea of using the entropy maximum principle for such objects kind analysis. Although to ignore the examples of successful application of this principle for the analysis of non-physical systems is quite difficult. "Non-physicists" treat it as an especially popular since Janes formalism appearance [1], which allows calculating a conditional entropy maximum as an integral criterion of the most probable configuration of the macrosystem.

II. REVIEW

The objections of physicists, who have been growing on Boltzmann and Gibbs ideas, are sufficiently substantiated. Statistical mechanics considers in general a priori uniform probability systems, where each single element has the same initial probability of all phase space cells settlement that in turn determines the equal probability of all possible microstates of the system realization. In addition, such a limitation as a condition of a priori equal probability of microstates is the main postulate of statistical physics [2] – [5].

However, this "convenient" characteristic is not always typical for the above-mentioned non-physical macrosystems. For them the use of a priori equal probability hypothesis is often simply invalid. A priori asymmetry of phase cells settling conditions is explained by the fundamental inability of these systems to be isolated. Typically, each of them with certain necessity is included into the scheme of a causal interaction with many other macrosystems, mutually distorting the conditions of their formation. For example, an economic system cannot exist outside social or political system, and demographic system cannot be isolated from economic or environmental system.

An asymmetry phenomenon of a priori conditions and hence of the inequality of a priori microstates probabilities are just not considered by many authors as usual. When studying the complex objects (for example, in economics, in case of deriving the law of social incomes distribution [6], [7] there is used, mainly traditional entropy schemes, without changes, borrowed from statistical mechanics. Equally, such comment can be attributed to other existing entropy systems modeling attempts: transport [8], ecological [9], social [10].

The aim to use the entropy principle reasonably for macrosystems of different nature requires a generalization of traditional statistical entropy form. Such "corrected" function should take into account not only a uniform (as Boltzmann entropy), but also an arbitrary distribution character of a priori probabilities, stemming from the mutual influence of macrosystems.

The purpose of the paper is to find and substantiate the more general function for an extended range of macro systems analysis compared to the statistical entropy (not limited by the requirement of a priori equal probability hypothesis); "Complex Systems" and "Macrosystems" expressions are not classified as established. In the paper, they are considered as equivalent and related to "Statistical Systems" name, which is defined in a Physics Encyclopedia [11] as the sum of a large number of particles that are studied by the methods of statistical physics.

Aiming to find something common, which is inherent in most of considered macrosystems, we can note, that they all usually are the objects, where on the final set of "agents" there is distributed a limited variety of "resources" [12], [13]. For example, among the gas molecules an energy is distributed and among the galaxies – their weight and among the cities – their inhabitants are distributed and among the people – their material goods and among the election candidates – their voters, etc. Therefore, the expressions like "agents" and "resources" are useful to be used for the analysis of some abstract macrosystem, which could be applied to a wide range of tasks.

Both of these expressions are "interchangeable." People are looking for the capital, but also the capital is "looking" people. Status of such system agents depends on the ratio of minimum parts of their reasonable fragmentation. For example, in social geography "the agents" are the cities, and the population is "the resources". At the same time, the inhabitants of these cities can be considered as "agents" of such "resources" as square meters of housing.

III. Problem Statement

We can also consider the general case when N elements are distributed among M cells. Let one of possible macrostates of such system be implemented with distribution in cells, respectively $n_1, n_2, ...n_M$ (provided $\sum_{i=1}^{M} n_i = N$). The number of combinations of such distribution option implementation, i.e. the statistical weight of relevant macrostate, is equal to the value of the polynomial (more precisely, "M-nomial") index of Nth degree, which can be calculated using D. Bernoulli formula

$$W(n_1, n_2, ...n_M) = \frac{N!}{n_1!n_2!...n_M!}. \quad (3)$$

This macrostate probability is equal to

$$Q(n_1, n_2, ...n_M; p_1, p_2, ...p_M) = \frac{N!}{n_1!n_2!...n_M!} \cdot p_1^{n_1} p_2^{n_2} ... p_M^{n_M}, \quad (4)$$

here $P:\{p_1, p_2, ...p_M\}$ is a variety of a priori probabilities.

In the particular case, when a priori probabilities are equal, that is $p_i = \text{const} = \frac{1}{M}$, the expression (4) takes the form

$$Q(n_1, n_2, ...n_M) = \frac{N!}{n_1!n_2!...n_M!} \cdot \frac{1}{M^N}. \quad (5)$$

Speaking about the instantaneous state of the system with a large number of interacting elements, it is possible to operate only probabilistic concepts. Here, along with individual a priori probabilities p_i and probabilities of microstates $\prod_{i=1}^{M} p_i^{n_i}$, an integral characteristic of the system is allocated - the probability of its macrostate $Q(n_1, n_2, ...n_M; p_1, p_2, ...p_M)$. Entropy is the characteristic reflecting that value. However, not reflecting in full, as it would be shown hereafter.

Although Boltzmann entropy is considered a probability function of the system macrostate, it is written (6) through the statistical weight W, which is often named as thermodynamic probability. However, W is not normalized in the classic sense and is not a probability. Normalized value is the probability of the macrostate Q. To be sure, it is enough to sum (4) and (5) expressions by the total number of macrostates:

$$\sum_{n_1, n_2, ...n_M} Q(n_1, n_2, ...n_M; p_1, p_2, ...p_M) = \sum_{n_1, n_2, ...n_M} \frac{N!}{n_1!n_2!...n_M!} \cdot p_1^{n_1} p_2^{n_2} ... p_M^{n_M} = (p_1 + p_2 + ... + p_M)^N = 1, \quad (7)$$

$$\sum_{n_1, n_2, ...n_M} Q(n_1, n_2, ...n_M) = \sum_{n_1, n_2, ...n_M} \frac{N!}{n_1!n_2!...n_M!} \cdot \frac{1}{M^N} = \Bigg(\underbrace{\frac{1}{M} + \frac{1}{M} + ... + \frac{1}{M}}_{M}\Bigg)^N = 1. \quad (8)$$

Essentially, the desired system macrostate probability function should depend on the probability of macrostate Q, but not of the statistical weight W.

The truth is that this comment is not fundamental for Boltzmann entropy obtained subject to *equal* a priori probability of microstates (constant multiplier $1/M^N$ in equation (8) may be put before the summation sign). However, as it is seen from (7) expression, in more general case of unequal a priori probability, the replacement of the macrostate Q probability into the statistical weight W is not fundamentally allowed.

IV. Development of Entropy Conception

Renunciation of the postulate of equal a priori probability leads to that instead of the expression (5) for macrostate Q probability, it is needed to write "complicated" by a priori probabilities $p_1, p_2, ...p_M$

expression (4). In this case, the statistical weight $W = \dfrac{N!}{n_1!n_2!...n_M!}$ already "has no right" to determine independently the probability of the macrostate, as it was earlier, when $p_1, p_2, ...p_M$ were equal. Therefore, it is not enough even classical Boltzmann entropy $S = lnW$, to characterize independently the probability of such a system macrostate, where the postulate of equal a priori probability "does not work" already. Nevertheless, what should be instead?

As the required function instead of the entropy lnW, we use a positive value $-lnQ$, because it does directly contain the probability of the macrostate (expression (4)). Given that $0 \le Q \le 1$, as well, because of the monotony of the logarithm, probability Q corresponds to the minimum of the function:

$$-\ln Q(n_1, n_2, ...n_M; p_1, p_2, ...p_M) = -\ln\left(\frac{N!}{n_1!n_2!...n_M!} \cdot p_1^{n_1} p_2^{n_2} ... p_M^{n_M}\right) \quad (9)$$

Moreover, given (3), we can see its relation to Boltzmann entropy $S = lnW$:

$$-\ln Q(n_1, n_2, ...n_M; p_1, p_2, ...p_M) = -\sum_{i=1}^{M} n_i \ln p_i - S. \quad (10)$$

Including the designation:

$$D_d(n_1, n_2, ...n_M; p_1, p_2, ...p_M) = \frac{-\ln Q(n_1, n_2, ...n_M; p_1, p_2, ...p_M)}{N},$$

and writing it in a short form, taking into account (10), we can obtain

$$D_d = \frac{-\ln Q}{N} = -\sum_{i=1}^{M} \frac{n_i}{N} \ln p_i - \frac{S}{N}. \quad (11)$$

The designation D_d is of the word "divergence" (because the expression (11) may be deduced to the form, matching Kullback–Leibler divergence [14]). Subscript d is of the word "different", indicating *different* values of a priori probability. D_d value may be conveniently named an entropy divergence, having in mind its continuity with the entropy.

The name of entropy divergence is quite valid, because (11) represents the difference between two entropies – the current value of entropy is subtracted from its maximum possible value, that depends on the distribution of a priori probabilities. The value D_d always stays positive. The proof of these assertions is shown in the following section.

Let us show that the expression (11) can really be presented in the form of Kullback divergence. To this end, for (9) we apply the formula of Stirling $\ln m! \approx m \cdot (\ln m - 1)$. Then

$$D_d = -\sum_{i=1}^{M} \frac{n_i}{N} \ln p_i - \ln N + \sum_{i=1}^{M} \frac{n_i}{N} \ln n_i.$$

Bracing the positive value, we can obtain

$$D_d = \frac{-\ln Q}{N} = -\sum_{i=1}^{M} \frac{n_i}{N} \ln p_i - \left(-\sum_{i=1}^{M} \frac{n_i}{N} \ln \frac{n_i}{N}\right), \quad (12)$$

or

$$D_d = \sum_{i=1}^{M} \frac{n_i}{N} \ln \frac{n_i/N}{p_i}. \quad (13)$$

This expression constitutes the entropy divergence as the statistical distance between two distributions - current and its maximum achievable - equilibrium distribution. The visibility to the above specified is of an individual (classic) case, where the postulate of equal a priori probability remains valid. In this case: $p_i = \text{const} = 1/M$, then from (11) there should be:

$$D_{nd} = \ln M - \frac{S}{N}. \quad (14)$$

Alternatively, given (12), we can write:

$$D_{nd} = \ln M - \left(-\sum_{i=1}^{M} \frac{n_i}{N} \ln \frac{n_i}{N}\right). \quad (15)$$

Here the index of entropy divergence for equal a priori probabilities is changed: *nd* means "no different".

Out of this case, it is clear, that the entropy divergence (14) represents the difference between the maximum possible value of entropy $\ln M$ and its current value S/N. Hence, the pursuit of the entropy to a maximum corresponds to the pursuit of the entropy divergence to a minimum.

Thus, when the postulate of a priori probabilities equality becomes invalid, the statistical analysis of the systems cannot be focused only in the entropy criterion. More general criterion, which takes into account asymmetrical a priori conditions of elements distribution within the system, is the entropy divergence. Its inclusion conforms to the principle of continuity – the entropy is an additive component of the criterion and in its usual role is used in the case where the main postulate of statistical mechanics remains valid.

V. Proof of Basic Theorems

Theorem 1. A necessary and sufficient condition of equal probability of microstates of the system is the equality of a priori probabilities of its elements distribution in the space of individual states.

Substantiation. A necessity of the condition is substantiated by contradiction. Let an equal probability of *all* the microstates $\left(p_1^{n_1} p_2^{n_2} ... p_M^{n_M}\right)$ to be satisfied under unequal values of a priori probabilities $p_1, p_2, ..., p_M$ of elements distribution among the cells of conditions' space. Then the probabilities of all *M* different microstates should be equal among themselves, whose elements are collected in a single cell, that is $p_1^N = p_2^N = ... = p_M^N$. Nevertheless, it is possible only when all a priori probabilities $p_1, p_2, ..., p_M$ are equal among themselves that contradicts the original conclusion. The necessity of the theorem condition is substantiated.

The sufficiency of the theorem condition is obvious: if the values of a priori probabilities of the elements distribution in the cells of the states' space are equal among themselves $p_1 = p_2 = ... = p_M$, so the probabilities of all the microstates, i.e. the probabilities of any combinations of $p_1^{n_1} p_2^{n_2} ... p_M^{n_M}$ (under $\sum_{i=1}^{M} n_i = N$), remain equal. The theorem is proved.

Theorem 2. If the distribution of a priori probabilities is defined by a variety of $P:\{p_1, p_2, ... p_M\}$, then the maximum of system macrostate is implemented under such equilibrium distribution of the number of the elements of the system $\left(n_1, n_2, ... n_M\right)_{max}$ under $\sum_{i=1}^{M} n_i = N$ that satisfies the condition:

$$\left(\frac{n_i}{N}\right)_{\max} = p_i . \quad (16)$$

Substantiation. If the distribution of a priori probabilities is defined by a variety of $P:\{p_1, p_2, ... p_M\}$, then the probability of macrostate Q is expressed by the expression (4). Because of $0 \le Q \le 1$, and while the logarithm is a monotonically increasing function, so the maximum value Q corresponds to the minimum of the function – ln Q. Let us find a conditional extremum of this function under $\sum_{i=1}^{M} n_i = N$, using the method of Lagrange multipliers.

To do this, we will request $\frac{\partial}{\partial n_i}\left(-\ln Q + \alpha\left(\sum_{i=1}^{M} n_i - N\right)\right) = 0$, that, considering the expression (12), is equivalent to the request:

$$\frac{\partial}{\partial n_i}\left(-\sum_{i=1}^{M} n_i \ln p_i + \sum_{i=1}^{M} n_i \ln \frac{n_i}{N} + \alpha\left(\sum_{i=1}^{M} n_i - N\right)\right) = 0 ,$$

From this we obtain the extremum conditions

$$\ln \frac{n_i}{N} = \ln p_i - 1 - \alpha ,$$

or

$$\frac{n_i}{N} = p_i e^{-1-\alpha} = p_i C . \quad (17)$$

Multiplier $C = 1$ that comes out of normalization:

$$\sum_{i=1}^{M} \frac{n_i}{N} = C \sum_{i=1}^{M} p_i = 1 .$$

Obtained extremum condition, taking into account the positive second derivative, provides a conditional minimum of the function $-\ln Q$, and, consequently, a conditional maximum of the macrostate probability Q. Then, as can be seen from (17), the maximum of macrostate probability $Q = Q_{\max}$ is achieved under the extremal distribution $\left(n_1, n_2, ... n_M\right)_{\max}$ that satisfies the condition (16). The theorem is proved.

In the particular case of equal a priori probabilities $p_i = \text{const} = 1/M$ from the equality (16), it comes out, that macrostate probability maximum (so the entropy maximum) is achieved with an uniform distribution

$$\left(\frac{n_i}{N}\right)_{\max} = \frac{1}{M} = \text{const} , \quad (18)$$

that is a well-known fact.

Theorem 3 (*consequence*). If the distribution of a priori probabilities is set by a variety $P:\{p_1, p_2, ... p_M\}$, then the maximum value that can be taken by Boltzmann statistical entropy is

$$\frac{S_{\max}}{N} = -\sum_{i=1}^{M} p_i \ln p_i . \quad (19)$$

Indeed, if we substitute (3) into the entropy formula (6) and then further convert it using Stirling formula (see (12)) it would lead to such a form

$$\frac{S}{N} = -\sum_{i=1}^{M} \frac{n_i}{N} \ln \frac{n_i}{N} ,$$

then considering theorem 2 we should obtain (19). The theorem is proved.

In the particular case of equal a priori probabilities $p_i = \text{const} = 1/M$ out of (19) we get, that Boltzmann entropy maximum is a known value:

$$\frac{S_{\max}}{N} = \ln M . \qquad (20)$$

Theorem 4. Entropy divergence is always non-negative $D_d \ge 0$.

The validity of this conclusion follows directly from the definition of entropy divergence (11). If the probability of the macrostate Q satisfies the condition $0 \le Q \le 1$, (and hence $\ln Q \le 0$), for all the distributions of $n_1, n_2, \ldots n_M$ $\left(\sum_{i=1}^{M} n_i = N\right)$, $D_d = \dfrac{-\ln Q}{N} \ge 0$ is valid.

VI. Results

Results of research represent distributions, obtained on the basis of the proposed criterion such as conditional minimum of entropy divergence.

Popular formalism of Janes [1], used to build the equilibrium distributions, is actually the use of classical Lagrange multipliers method to find the conditional entropy maximum. As conditions there may be various restrictions, specified using the relation equations.

New formalism described here contains the calculation procedure for another criterion – conditional minimum of the entropy divergence. Here are the examples of its use to output two distribution laws, adapted considering random values of a priori probabilities.

Exponential distribution law. By the use of the adopted terminology in Section 4, we can find such a distribution of the number of "agents" $n_1, n_2, \ldots n_M$ that corresponds to the maximum probability of the system macrostate, or is equivalently to the minimum entropy divergence. New formalism suggests the search procedure of conditional entropy divergence minimum $D_d(n_1, n_2, \ldots n_M; p_1, p_2, \ldots p_M)$ through finding an *unconditional* minimum of some function $\Phi(n_1, n_2, \ldots n_M; p_1, p_2, \ldots p_M)$, that additively includes the divergence D_d itself and a number of other restrictions, weighted with Lagrange multipliers $\alpha, \beta, \gamma, \ldots$. We will take into account only natural limitations (1) and (2), considering total number of "agents" and "resources".

Further, by reducing the designations, we shall write the minimum Φ conditions through the requirement of equality to a zero of each partial derivative for all $i \in \{1, 2, \ldots M\}$:

$$\frac{\partial}{\partial n_i}\Phi = \frac{\partial}{\partial n_i}D_d + \alpha\left(\frac{\sum_{i=1}^{M} n_i}{N} - 1\right) + \beta\left(\frac{\sum_{i=1}^{M} n_i \varepsilon_i}{E} - 1\right) = 0, \qquad (21)$$

here D_d is defined by the expression (12). This requirement leads to the equality

$$-ln\, p_i + ln\frac{n_i}{N} + 1 + \alpha + \beta\frac{\varepsilon_i}{E/N} = 0 .$$

Designating a constant $e^{-1-\alpha} = C$, we can find its solution as known exponential distribution, but now it considers the distribution of a priori probabilities p_i

$$\frac{n_i}{N} = p_i \cdot C \cdot e^{-\beta\frac{\varepsilon_i}{E/N}} . \qquad (22)$$

We can show that this expression takes the form:

$$\frac{n_i}{n_{**}} = \frac{p_i}{p_{**}} \cdot e^{1-\frac{\varepsilon_i}{\varepsilon_{**}}}, \qquad (23)$$

here $p_{**} = p(\varepsilon_{**})$ designation refers to the cell with coordinate $\varepsilon_i = \varepsilon_{**}$. Here ε_{**} is the coordinate of the extremum (mode), and $n_{**}\varepsilon_{**}$ is the value of the extremum of the function $E_i(\varepsilon_i) = n_i\varepsilon_i$, describing the distribution of "resources".

Limitary hyperbolic distribution law. The macrosystem can be regarded as limited variety of "resources" distribution on the final variety of "agents". Moreover, the names of these varieties are conditional, and their status is changing frequently. In addition, we can also speak about their various "activity" [13]. More active is such a variety, in which the relaxation process is implemented more quickly. It has been shown that for such a case the corresponding equilibrium distribution $n_1, n_2, \ldots n_M$ has the exponential form (23).

However, there also are the opposite cases, where a variety conditionally named "resources" has the higher activity. This seemingly paradoxical conclusion is easier perceived after providing some examples. In particular, if the capital is considered just as a set of "resources", it is significantly more active than its "agents" – the people. Even brighter example of social geography, where the population as a variety of "resources" is more active than its "agents" – the cities. For these examples, as it was found empirically, there is not exponential distribution observed already, but hyperbolic (exponential). In the first example, it is named as Pareto law, and in the second example – Auerbach law.

In paper [12] there was performed the search process for the entropy maximum of the distribution, but of not the "agents" $n_1, n_2, ...n_M$ already, but of the "resources" $E_1, E_2, ...E_M$, where $E_i = n_i \varepsilon_i$. The solution is so-called limitary hyperbolic distribution law

$$\frac{n_i}{n_*} = \frac{\varepsilon_*}{\varepsilon_i} e^{1-\frac{\varepsilon_*}{\varepsilon_i}}, \qquad (24)$$

that under the extremum coordinates ε_* striving to a zero, turns into a clear hyperbolic distribution law. Here $n_* = n(\varepsilon_*)$.

As a next step, we define how the limitary hyperbolic distribution (24) changes its form under the condition, the a priori probabilities are not equal and are set by $P:\{p_1, p_2, ...p_M\}$ variety. For this purpose instead of the conditional maximum of the entropy, we shall search for the conditional minimum of the entropy divergence $D_d(E_1, E_2, ...E_M; p_1, p_2, ...p_M)$ under the same limiting conditions (1) and (2).

Conditional minimum requirement can be written in the same mathematical form as (21). However, taking into account the above, the entropy divergence instead of the formula (12) will have different form:

$$D_d = -\sum_{i=1}^{M} \frac{E_i}{E} \ln p_i - \left(-\sum_{i=1}^{M} \frac{E_i}{E} \ln \frac{E_i}{E} \right), \qquad (25)$$

here $E_i = n_i \varepsilon_i$. This requirement leads to the equality

$$-\ln p_i + \ln \frac{n_i \varepsilon_i}{E} + 1 + \frac{\alpha}{\varepsilon_i} + \beta = 0 .$$

Its solution (including the designation $\frac{E}{N} e^{-1-\beta} = C$) is:

$$\frac{n_i}{N} = p_i \frac{C}{\varepsilon_i} e^{-\frac{\alpha}{\varepsilon_i}} . \qquad (26)$$

Furthermore, by defining the constants C and α in the manner it was made in [12], we obtain the final form of the *limitary hyperbolic distribution law,* if a priori probabilities p_i are set by a variety $P:\{p_1, p_2, ...p_M\}$

$$\frac{n_i}{n_*} = \frac{p_i}{p_*} \frac{\varepsilon_*}{\varepsilon_i} e^{1-\frac{\varepsilon_*}{\varepsilon_i}}, \qquad (27)$$

here ε_* is the coordinate of the extremum, n_* is the value of the extremum of distribution (27), $p_* = p(\varepsilon_*)$ is an a priori probability of the cell with this coordinate settlement. It would be fair to refer the work [15], after reading which the author was prompted to start the researches that have lead to the distribution (27).

VI. Conclusions

Formal borrowing of the entropy approaches from the thermodynamics and statistical physics to analyze non-physical macrosystems (economic, social) is not always valid and may cause errors. It is important to remember that in the physical sciences, such an assumption is valid, as a condition of a priori equal probability of the microstates, and it has the status of the basic postulate of statistical physics.

However, this ideal characteristic, which is due to the phase cells settlement conditions symmetry, is not always the case in the above-mentioned non-physical macrosystems due to their fundamental inability to be isolated. Typically, each of them with certain necessity is included into the scheme of a causal interaction with many other macrosystems, mutually distorting the conditions of their formation. For example, an economic system cannot exist outside social or political system, and demographic system cannot be isolated from economic or environmental system.

When the postulate of equal a priori probabilities becomes invalid, Boltzmann entropy $S = \ln W$ (and other derivatives of its form) cannot longer play the role of a function, characterizing the probability of the macrostate of the system. Such a task can be handled by only more general characteristic, which is called here the entropy divergence D_d. In accordance with the principle of continuity it contains the entropy as a component. In relation to the properties of the entropy divergence a number of theorems are proved.

It is shown that the criterion of the equilibrium state of the system in the general case is a conditional minimum of the entropy divergence $(D_d)_{\min}$. In addition, such a widely used criteria as the conditional maximum of the entropy $S_{\max}$, corresponds to it in the particular case of equal a priori probabilities.

Formalism, based on the definition of a conditional minimum of the entropy divergence makes it possible to calculate the equilibrium distributions, depending on the a priori probabilities, which in turn are caused by the influence of other systems. The multiplicative form of the combined distribution allows us to consider the process of the interaction of two or more macrosystems as the realization of a complex experience. Merging macrosystems related to this type by an adaptive statistical interaction may be analyzed from the perspective of a single meta-system.

REFERENCES

[1] E. T. Jaynes, "Information theory and statistical mechanics", *Physical Review*, 1957, vol. 106, pp. 620–630.

[2] M. Planck, *Introduction to the Theoretical Physics: Theory of Heat*. Moscow, Scientific and Technical Publishing House, 1935, part 5, 229 p. (in Russian)

[3] J. I. Frankel, *Statistical Physics*, Moscow, Publishing House of Academy of Sciences of USSR, 1948, 760 p. (in Russian)

[4[A. Sommerfeld, *Thermodynamics and Statistical Physics*, Moscow, Foreign Publishing House, 1955, 481 p.

[5] C. Kittel, *Statistical Thermodynamics*, Moscow, Prosveshenie, 1977, 336 p. (in Russian)

[6] V. Yakovenko, *Statistical Mechanics Approach to the Probability Distribution of Money*, Department of Physics, University of Maryland, 2010, 11 p.

[7] S. A. Galkin, O. I. Elagin, A. A. Kozlov, V. A. Potapenko, and M. J. Romanovsky, "Exponential distributions of individual incomes and expenses of the citizens: Observations and models", *Proceedings of the Institute of General Physics*, Moscow, Russian Academy of Sciences, 2009, vol. 65, pp. 29–49. (in Russian)

[8] A. J. Wilson, *Entropy Methods of Complex Systems Modeling*, Moscow, Nauka, 1978, 248 p. (in Russian)

[9] A. P. Levich, V. L. Alexeev, and V. A. Nikulin, "Mathematical aspects of the variational modeling in the ecology of communities", *Mathematical Modeling*, 1994, no. 5, pp. 55–76. (in Russian)

[10] I. V. Prangishvili, *Entropy and Other Systemic Regularities: Questions of Complex Systems Controlling*, Moscow, Nauka, 2003, 428 p. (in Russian)

[11] *Physical Encyclopedia* under edition of A. M. Prokhorov, Moscow, Sovietskaya Encyclopedia, 1995, vol. 4, 640 p. (in Russsian)

[12] N. I. Delas, and V. A. Kasyanov, "Limitary hyperbolic distribution law in self-organized systems", *East-European Journal of Advanced Technologies*, 2012, vol. 58, no. 4, pp. 13–18. (in Russsian)

[13] N. I. Delas, "Hyperbolic distribution complex systems evolution", East-European Journal of Advanced Technologies, 2013, vol. 63, no. 4, pp. 67–73. (in Russian)

[14] S. Kullback, and R. A. Leibler, "On information and sufficiency", The Annals of Mathematical Statistics, 1951, vol. 22, no.1, pp. pp. 79–86. (in Rusian)

[15] B. A. Trubnikov and O. B. Trubnikova, "Five great probabilities distributions", *Priroda*, 2004, no. 11, pp. 13–20. (in Russian)

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М. І. Делас. Ентропійний аналіз макросистем без основного постулату статистичної механіки

Показано, що для більшості нефізичних макросистем основний постулат статистичної механіки (постулат рівної апріорної імовірності мікростанів) втрачає силу, а роль ентропії має виконувати більш загальна характеристика. Розглянуто приклади законів розпділу.

Ключові слова: ентропійний аналіз; ентропійна дивергенція; макросистеми; складні системи.

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Н. И. Делас. Энтропийный анализ макросистем без основного постулата статистической механики

Показано, что для большинства нефизических макросистем основной постулат статистической механики (постулат равной априорной вероятности микросостояний) утрачивает силу и роль энтропии должна выполнять более общая характеристика. Рассмотрены примеры законов распределения.

Ключевые слова: энтропийный анализ; энтропийная дивергенция; макросистемы; сложные системы.

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