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ESTIMATION OF THE RESULTS OF STATISTICAL MODELING OF AUTOMATIC CONTROL SYSTEMS

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***Abstract*—*Probabilistic methods of processing the results of statistical modeling of the determining parameters of the automatic control system characterizing the position of the aircraft in the automatic approach and landing mode for the purpose of determining the accuracy characteristics of automatic control are considered. Relations are obtained for determining a nonparametric two-dimensional tolerant region in which the probability measure of the controlled parameter with an unknown probability distribution is concentrated no less than the given one. Relations are proposed for the fraction of the probability of a parameter with a normal distribution law in the tolerant interval with bounderaries determined by normative documents. The obtained relations can be used to estimate the accuracy of the automatic landing system during statistical modeling of its mathematical model.*

Index Terms—Statistical modeling; accuracy estimation; tolerant interval; probability measure; tolerant area; nonparametric estimation; order statistics; modeling volume.

I. INTRODUCTION

To create new control systems for prospective trunk airplanes, it is necessary to carry out various types of flight tests, including the statistical modeling. All of them are aimed at achieving a single goal, which is to improve the characteristics of the control system and to establish its concordance with specified requirements for accuracy and reliability to ensure flight safety. One of the main tasks is the accuracy estimation of functioning the control system at all stages of the flight.

All this suggests that a well-developed mathematical apparatus is required to determine the probabilistic characteristics of the measured parameters with the necessary reliability. At the same time, the task of metrological provision of statistical measurements and the development of effective procedures for statistical processing of the received information becomes no less important.

II. PROBLEM STATEMENT

To estimate the accuracy of the results of any type of tests (flight and operational tests, statistical modeling) for the purpose of certification of an automatic control system for take-off and landing aircraft, the following statement of the problem can be formulated.

The problem of estimating the accuracy of a system will be understood as the estimate of the probability P of a random variable X falling into an acceptable area D with verification of the inequality $P \geq P_{\text{req}}$, where X is the measurement results of a certain determining parameter characterizing the position and state of the aircraft at the touchdown point of the runway, P_{req} is the required fraction of the probability distribution of the random variable X in the acceptable area D. The inequality is evaluated with some given confidence probability γ (reliability of estimation).

It should be noted that rather strict requirements are made on the automatic approach and landing process, the fulfillment of which it is necessary to confirm at the control system certification. In particular, the lateral deviation of the trunk aircraft at the touchdown point of the runway should be in the given area D with a very high probability 0.9_6. This means that in 10^6 automatic landings only one outcome beyond area D is allowed, since this outcome can be catastrophic.

Obviously, it is impossible to confirm such a probability by the flight tests due to the need for a huge number of tests (several million). Only statistical modeling allows to obtain the required volume of tests (the simplest Monte Carlo method or modeling methods that take into account a priori information about landing parameters [1]).

III. ALGORITHMS FOR ESTIMATING THE ACCURACY OF A CONTROL SYSTEMS

With the unknown distribution law of the general population, only nonparametric methods will be correct. Let's consider two basic methods.

A. Method of the probability estimation using the observed frequency of falling the measurement results within the acceptable limits

The probability estimation is determined by a simple algorithm:

$$P^* = \frac{r}{n},$$

where r is the number of measurements falling within the limits.

The boundaries of the confidence interval for the required probability P for sufficiently large n (for n > 1000 the binomial distribution of the random variable

$$\frac{(P^* - P)\sqrt{n}}{\sqrt{P(1-P)}}$$

arbitrarily little differs from the normal law) are determined in accordance with the expression:

$$[P_1, P_2] = \frac{P^* + \frac{u^2_{\frac{1+\gamma}{2}}}{2n}}{1 + \frac{u^2_{\frac{1+\gamma}{2}}}{n}} \pm \frac{u^2_{\frac{1+\gamma}{2}}}{1 + \frac{u^2_{\frac{1+\gamma}{2}}}{n}} \sqrt{\frac{P^*(1-P^*)}{n} + \frac{u^2_{\frac{1+\gamma}{2}}}{4n^2}}$$

$$= \frac{n}{u^2_{\frac{1+\gamma}{2}} + n}\left[P^* + \frac{u^2_{\frac{1+\gamma}{2}}}{2n} \pm u_{\frac{1+\gamma}{2}} \sqrt{\frac{P^*(1-P^*)}{n} + \frac{u^2_{\frac{1+\gamma}{2}}}{4n^2}}\right], \quad (1)$$

where $u_{\frac{1+\gamma}{2}}$ is the quintile of the standard normal distribution $N(0,1)$ of the $\frac{1+\gamma}{2}$ level, γ is the confidence probability with which the interval $[P_1, P_2]$ contains the true value of P.

For example, for $n = 100, r = 100, \gamma = 0{,}95$ we have $P^* = 1$, $u_{0{,}975} = 1.96$ and the boundaries of the interval are equal to $[0.963;\ 0.9_5 4 \approx 1]$, and for $n = 500,\ r = 500$ the interval is $[0.992;\ 1]$. The lower bound of the interval is compared with the probability P_{req} and if it is less than the required one, then the volume of the modeling needs to be increased.

B. Method of the probability estimation with the use of the nonparametric tolerant interval

Consider the construction of a nonparametric tolerant interval for which the probability measure of an unknown distribution concentrated in it would be no less than a given value P with a confidence probability γ. The boundaries of the interval L and U are random, and the following relation holds:

$$\Pr\left\{\int_L^U f(x)dx \geq P\right\} = \gamma. \quad (2)$$

The left-hand side of the equation has a value that does not depend on $f(x)$ if L and U are the ordinal statistics. Denoting the boundaries through order statistics $L = x_{(r)}$ and $U = x_{(s)}$, where $s > r$, we can write that

$$\Pr\{[F(x_{(s)}) - F(x_{(r)})] \geq P\} = \gamma. \quad (3)$$

In [1] the general expression for the probability is obtained:

$$\Pr\{[F(x_{(s)}) - F(x_{(r)})] \geq P\} = 1 - I_{P_\gamma}(s - r, n - s + r + 1) = \gamma, \quad (4)$$

or

$$I_{P_\gamma}(s - r, n - s + r + 1) = 1 - \gamma, \quad (5)$$

where P is the probability measure, concentrated in the tolerant interval $[x_{(r)}, x_{(s)}]$, γ is the probability that this interval contains the fraction of the distribution P, r and s are the positions of ordinal statistics in the sample of measurements. If any four from (P, n, r, s, γ) are given, then the equation (5) can be solved with respect to the fifth variable.

In practice, as a rule, the extreme values of the sample of measurements $x_{(1)}$ and $x_{(n)}$ are used as the order statistics. In this case the length of the nonparametric interval corresponds to the range of the sample $w = x_{(n)} - x_{(1)}$. Then the expression (5) takes the form:

$$I_{P_\gamma}(n-1, 2) = 1 - \gamma. \quad (6)$$

Introducing the incomplete B-function as [4]:

$$I_z(p,q) = \frac{B_z(p,q)}{B(p,q)} = \frac{1}{B(p,q)}\int_0^z t^{p-1}(1-t)^{q-1}dt, \quad (7)$$

we may write:

$$\frac{1}{B(n-1, 2)}\int_0^P t^{n-2}(1-t)dt = 1 - \gamma. \quad (8)$$

Since

$$\frac{1}{B(n-1, 2)} = 2\frac{n!}{(n-2)!2!} = n(n-1),$$

we finally get

$$1-\gamma = nP^{n-1} - (n-1)P^{n}. \qquad (9)$$

In particular, for $n = 500$, $\gamma = 0.95$ the solution of this equation gives a value P = 0.9906, i.e. the probability measure, concentrated in the interval $[x_{(n)} - x_{(1)}]$, will not be less than the obtained value with a confidence probability $\gamma = 0.95$.

If the required probability measure P_{req} exceeds the value 0.9906 (for example P_{req} = 0.9999 or P_{req} = 0.9_4), then it is necessary to significantly increase the test volume. The solution of the inverse problem in the formulation of (9) gives the value $n = 5\cdot 10^4$ that, with probability $\gamma = 0.95$, the share of the unknown distribution of the parameter in a given tolerance interval was equal to or exceeded the value 0.9_4.

A multidimensional case is of interest when the acceptable area D^m is given in the form of a m-dimensional parallelepiped. Obviously, such area for two independent parameters (m = 2) is a rectangle. For example, for lateral and longitudinal deviations of the aircraft at the height of decision making and at the touchdown point of the runway, such areas are specified by rectangles, Fig. 1.

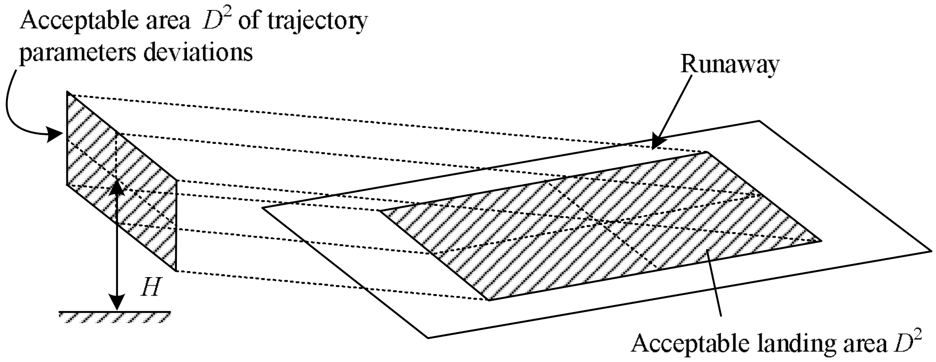

Fig. 1. Landing area on the runaway

If the acceptable tolerance area is constructed according to the ranges w of the measurement samples ($w_1 = x_{1(n)} - x_{1(1)}$ and $w_2 = x_{2(n)} - x_{2(1)}$), then the previously obtained relation (6) for the one-dimensional tolerant interval is completely transferred to the multidimensional case:

$$I_{P_\gamma}(n+1-2k, 2k) = 1-\gamma.$$

When using two ordinal statistics for each parameter k = 2 and, consequently, we get:

$$I_{P_\gamma}(n-3, 4) = 1-\gamma.$$

Carrying out analogous calculations on relations (6) – (9), we can determine the probability measure of the distribution concentrated in the acceptable area bounded by extreme values. In particular, for $n = 500$, $\gamma = 0.95$ the solution of this equation gives a value $P = 0.9845$, for $n = 1000$ and for the same confidence probability we have $P = 0.9923$.

The considered estimation method does not require the storage of the entire sample of measurements, but only the extreme values $x_{(1)}$ and $x_{(n)}$ accumulated over the entire volume.

It is obvious that an accurate estimation of the distribution of a random variable by the results of an experiment is fundamentally impossible, and therefore in practice different hypotheses about the distribution of the measured parameter are tested. As the results of many tests show, some parameters that characterize the accuracy of the control system functioning have a normal distribution with unknown probabilistic characteristics (m_x, σ_x).

C. Method of the probability estimation with the use of the parametric tolerant interval

At first we will consider a method for constructing a one-sided tolerant limit, which can be represented as the critical value of the corresponding random variable with the distribution function $F(x)$. Constructing an upper (lower) tolerant limit means that in about $100\gamma\,\%$ of cases the corresponding half-interval will be a critical multiplicity for the investigated parameter (for example, the vertical descent speed of the aircraft in touchdown point $V_y > 0$) with the required level of significance.

If the normality of the distribution law is assumed then as the upper tolerant interval, one can choose a function $U = m^* + k\sigma^*$ such that

$$\Pr\left\{\Phi\left[\frac{m^* + k\sigma^* - m}{\sigma}\right] > P\right\} = \gamma, \qquad (10)$$

where $\Phi(\bullet)$ is a function of the standard normal distribution $N(0,1)$.

To calculate the tolerant factor *k* the following formula may be recommended:

$$k = \sqrt{\frac{n-1}{x_\gamma(n-1)}}u_{\frac{1+P}{2}}\left\{1+\frac{1}{2n}-\frac{2\left(u_{\frac{1+P}{2}}\right)^2-3}{24n^2}\right\}, \quad (11)$$

where *P* is the probability of not exceeding by random value *X* of a given critical value x_{accept}; $x_\gamma(n-1)$ is $100\gamma\,\%$ percentage point of the χ^2-distribution with (*n* – 1) degrees of freedom. For fixed values of γ and *n*, the value of $x(\gamma, n)$ is defined as the root *x* of the equation $1-F_n(x)=\gamma$, where $F_n(x)=\Pr\{\chi_n^2<x\}$ there is the χ^2-distribution function.

It should also be noted that from the χ^2-distribution properties for $n\to\infty$ and $\gamma\to 1$ it follows that the following approximation of the quintile may be used:

$$x_\gamma(n) = n\left(1-\frac{2}{9n}+u_{1-\gamma}\sqrt{\frac{2}{9n}}\right)^3. \quad (12)$$

As an example, we determine which limiting value can reach a certain parameter *x* at the level of reliability $\gamma=0.9$ and given values $P=0.9_6$, x_{accept} = 4 if as a result of modeling of *n* = 1000 realizations of the parameter the following statistical characteristics are obtained:

$$m_x^* = 1{,}228,\ \sigma_x^* = 0{,}5742\ .$$

According to (11) we get:

$$k=\sqrt{\frac{999}{942.1}}4.89\left[1+\frac{1}{2000}-\frac{2\cdot 4.89^2-3}{24\cdot 10^6}\right]=5.04.$$

And upper tolerant limit is:

$$U = m^* + k\sigma^* = 1.228 + 5.04\cdot\ 0.574 = 4.12.$$

This value exceeds x_{accept} Obviously, for a given probability $P=0.9_6$, it is necessary to increase the volume of modeling to obtain new characteristics $\left(m^*, \sigma^*\right)$.

For sufficiently large values n, which are characteristic of statistical modeling, we can use another asymptotic expression for the tolerant factor, which is determined through the quintile of the normal distribution:

$$k = u_{\frac{1+P}{2}}\sqrt{\frac{n-\frac{1}{n}}{x_\gamma(n-1)}}. \quad (13)$$

For example, for the considered above example *k* = 5.043, which almost coincides with the value obtained earlier.

In those cases where it is necessary to estimate the probability *P* of a random variable hitting into the given tolerance limits $[a_1,\ a_2]$, it is necessary to find the values of two tolerant factors:

$$k_1 = \frac{a_1 - m_x^*}{\sigma_x^*},\quad k_2 = \frac{a_2 - m_x^*}{\sigma_x^*}. \quad (14)$$

The values of the quintiles of the normal distribution are found in accordance with (12):

$$u_{\frac{1+P_2}{2}} = \frac{k_2}{\sqrt{\frac{n-\frac{1}{n}}{x_\gamma(n-1)}}},\quad u_{\frac{1+P_1}{2}} = \frac{k_1}{\sqrt{\frac{n-\frac{1}{n}}{x_\gamma(n-1)}}}. \quad (15)$$

In accordance with the distribution $N(0,1)$ the obtained values of quintiles allow to find the probability values P_2, P_1, and the probability measure $P=P_2-P_1$, concentrated in the given interval $[a_1,\ a_2]$.

Let according to the regulatory requirements 95% of the distribution of the measured values of a certain parameter (for example, the lateral deviation of the aircraft at the touchdown point) should be within the tolerance limits [–8, 8] with a confidence probability $\gamma = 0{,}95$. When *n* = 1000 realizations of a random variable were obtained, as a result of the modeling, statistical estimates of the parameter $m^*=3.24$, $\sigma^*=2.36$ were determined assuming the normality of the distribution.

As a result of the calculation using formulas (13) – (15) we obtain the values of tolerant factors $k_1=-4.763$, $k_2=2.017$ and the values of the quintiles of the distribution:

$$u_{\frac{1+P_2}{2}} = \frac{2.017}{0.965} = 2.09;\quad u_{\frac{1+P_1}{2}} = \frac{-4.763}{0.965} = -4.935\ .$$

From the found quintiles of the normal distribution, we find the probabilities P_2 = 0.9634 and $P_1\approx 0$, so that the share of the parameter distribution in the given interval is equal to $P = P_2 - P_1 = 0.9634$, that is $P > P_{\text{req}}$. This means that, according to the modeling results, it can be concluded that the control system for this parameter satisfies the requirements.

Let's consider an example when the acceptable interval for the above example is [–21, 21] and in accordance with regulatory requirements it is necessary that the probability measure of the distribution in this interval is not less than $P_{\text{req}} = 0.9_6$ with the confidence probability equaled $\gamma = 0{,}95$. As a result of modeling of the parameter realizations (n = 1000), the following characteristics were obtained: $m^* = 3$, $\sigma^* = 3.5$.

In this case we get:

$$k_1 = -6.86, \quad k_2 = 5.14,$$

$$u_{\frac{1+P_2}{2}} = 5.33, \quad u_{\frac{1+P_1}{2}} = -7.1, \quad P = 0.9_7 5.$$

As can be seen from the obtained results, even with a small volume of modeling, the control system for the considered parameter satisfies the hard accuracy requirements.

VI. Conclusions

The essentially limited possibilities of flight tests of automatic control systems exclude the achievement of the necessary volume of experiment in order to confirm the high requirements to the accuracy of control of these systems for ensuring the safety of the automatic approach and landing of the aircraft. This predetermines the significance of statistical modeling of the functioning of control systems in a wide range of perturbing influences and optimal algorithms for processing the obtained statistical information.

In the statistical processing of information obtained during modeling, it is necessary to use not only strict classical parametric algorithms, but also robust and nonparametric methods of processing, which allow to obtain sufficiently high reliability and stability of statistical conclusions.

Nonparametric methods (the restrictions on the form of distribution are not required) have a much greater stability in comparison with other methods and their effectiveness is rather high with a substantial increase of the experiment volume. It is possible with statistical modeling. However, when the results are interpreted, it should be taken into account that their reliability can not be higher than the reliability of the initial data and the made assumptions.

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О. А. Зеленков, О. О. Бунчук, А. П. Голік. Оцінка результатів статистичного моделювання систем автоматичного керування

Розглянуто ймовірні методи обробки результатів статистичного моделювання, визначальних параметрів системи автоматичного керування, які характеризують стан літака в автоматичному режимі заходу на посадку і посадки з метою визначення точностних характеристик автоматичного керування. Отримано співвідношення для визначення непараметричної двовимірної толерантної області, в якій зосереджена імовірнісна міра контролюючого параметра з невідомим розподілом імовірності не менше заданої. Запропоновано співвідношення для визначення частки ймовірності параметра з нормальним законом розподілу в толерантному інтервалі з межами, визначеними нормативними документами. Отримані співвідношення можуть бути застосовані для оцінки точності системи автоматичного приземлення при статистичному моделюванні її математичної моделі.

Ключові слова: статистичне моделювання; оцінка точності; толерантний інтервал; імовірнісна міра; толерантна область; непараметричне оцінювання; обсяг моделювання; порядкові статистики.

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А. А.Зеленков, А. А.Бунчук, А. П. Голик. Оценка результатов статистического моделирования систем автоматического управления

Рассмотрены вероятностные методы обработки результатов статистического моделирования, определяющих параметров системы автоматического управления, характеризующих положение самолета в автоматическом режиме захода на посадку и посадки с целью определения точностных характеристик автоматического управления. Получены соотношения для определения непараметрической двумерной толерантной области, в которой сосредоточена вероятностная мера контролирующего параметра с неизвестным распределением вероятности не меньше заданной. Предложены соотношения для определения доли вероятности параметра с нормальным законом распределения в толерантном интервале с границами, определенными нормативными документами. Полученные соотношения могут быть применены для оценки точности системы автоматического приземления при статистическом моделировании её математической модели.

Ключевые слова: статистическое моделирование; оценка точности; толерантный интервал; вероятностная мера; толерантная область; непараметрическое оценивание; порядковые статистики; объем моделирования.

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