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A. A. Ziganshin

SIMILARITY PARAMETRICAL METHOD TO CALCULATE AERODYNAMIC CHARACTERISTICS FOR SAVONIUS ROTORS

Aviation Computer-Integrated Complexes Department, National Aviation University, Kyiv, Ukraine
E-mail: anwarzihan@gmail.com

Abstract**—***Computer aided method of obtaining the analytical empiric formula in the polynomial form to determine the torque coefficient with the rotor similarity parameters such as the Reynolds number, the aspect ratio, the buckets overlap ratio and the tip speed ratio was created. Polynomial coefficients were derived from the solution of linear algebraic equations system built on the base of the least-square method. Initial amount of data was taken by digitizing the charts with the wind tunnel tests results obtained for Savonius rotors.*

Index Terms—aerodynamics; digitizing; least-square method; multidimensional approximation polynomial; Savonius rotor.

I. INTRODUCTION

It is known that the empirical methods give averaged characteristics, are simple and may be successfully used in the prototype design and at the preliminary stage of various designing processes. In spite of having restricted area of application they also may be used in designing the rotors of vertical axis wind turbines in particular of Savonius rotors.

II. REVIEW OF EXISTING APPROACHES

There are several approaches among the empirical methods of calculating the aerodynamic characteristics of wings and blades (buckets) of wind turbines.

The empirical expression for the power coefficient (efficiency) of the Savonius rotors was obtained by the authors in [1] conducting their experiments and others' ones [2] resulting in

$$\left.\begin{aligned} C_P &= C_{Pm}\left[2.5\left(\frac{\lambda}{\lambda_m}\right)^{1.5} - 1.5\left(\frac{\lambda}{\lambda_m}\right)^{2.5}\right],\ \lambda \le \lambda_m, \\ C_P &= C_{Pm}\left[1-\left(\frac{\lambda-\lambda_m}{\lambda_k-\lambda_m}\right)^2\right],\ \lambda_m < \lambda \le \lambda_k, \end{aligned}\right\}$$

where C_P is the power coefficient; C_{Pm} is the maximal value of power coefficient; λ is the tip speed ratio; λ_m is the tip speed ratio providing the value of C_{Pm}; λ_k is the autorotation (windmilling) tip speed ratio. As it was shown in [1] the values of λ_m and λ_k depend on specific geometric parameters of rotors and airflow conditions.

Tip speed ratio is the complex parameter presenting the ratio between the circumferential tip velocity ωR and the incoming wind flow speed V:

$$\lambda = \frac{\omega R}{V},$$

where ω is the rotational (angular) speed of rotor; R is the rotor bucket radius.

III. TASK STATEMENT

The power of rotor depends on the several factors as the incoming wind speed V, the angular speed ω, the air density ρ, cinematic coefficient ν of the air viscosity, the number N of blades, the height H of the rotor, the form and geometry of the blades:

$$P = f\left(V,\ \omega,\ \rho,\ \nu,\ N,\ H,\ \text{form, geometry}\right).$$

The air viscosity depends on temperature t:

$$\nu = f_1(t).$$

As it is well-known the air density depends on the atmospheric pressure p, the temperature t and the relative humidity φ:

$$\rho = f_2(p, t, \varphi).$$

As usually the effects from the pressure and the humidity changes to rotor torque and power are negligible at the definite place above the sea level.

For the conventional Savonius rotor (Fig. 1) we have the multidimensional dependence:

$$P = f\left(V,\ \omega,\ \rho,\ \nu,\ N,\ H,\ d,\ s\right),$$

where d is the bucket chord; s is the overlap between the buckets.

Our goal is to obtain the definite expression with definite dimensionless parameters. On the other hand the aerodynamic power P and the torque Q on the rotor axis may respectively be written as follows:

$$P = C_P \rho HRV^3, \quad Q = C_Q \rho HR^2 V^2,$$

where C_Q is the torque coefficient.

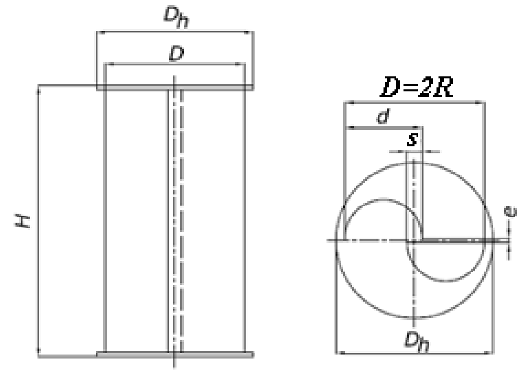

Fig. 1. Savonius rotor with the two buckets ($N = 2$).
D is the diameter ($D = 2R$); D_h is the end plate diameter;
e is the gap (accepted as $e = 0$)

As it is known relations between the power and the torque and also between their coefficients are

$$P = Q\omega; \quad C_P = C_Q \lambda.$$

From the analyses of the several experimental results [3], [4] the following dimensionless parameters of Savonius rotors were chosen: the Reynolds number *Re* (traditionally used for the low speeds of the airflow); the aspect ratio of the bucket $AR = H/D$ regarding it as a complex wing; the overlap ratio between the buckets $\beta = s/d$ (traditionally used for the Savonius rotor) and the tip speed ratio λ as the complex parameter.

The Reynolds number of the airflow is

$$Re = \frac{Vl}{\nu},$$

where l is the reference length (usually it is the diameter D of the rotor).

The rotor radius is calculating by the equation:

$$R = d(1 - \beta/2).$$

Thus we formulate the task statement: it is necessary to find an analytical formula expressing non-linear and multidimensional dependence of the torque coefficient C_Q with the four dimensionless parameters of the Savonius rotor:

$$C_Q = f(Re, AR, \beta, \lambda)$$

IV. Digitizing the Charts

Suppose we have the definite experimental set of function values with theirs mentioned above four parameters.

Usually the experimental data are presented by charts. Create a bitmap type file of the charts. Open the bitmap file. Do the maximum scale of the picture. Find the graphic coordinates in the left-bottom (X_1, Y_1) right-top (X_2, Y_2), left-top (X_3, Y_3) and right-bottom (X_4, Y_4) corners.

The graphic field may not be a rectangle one:

$$X_1 \neq X_3 \vee X_2 \neq X_4 \vee Y_1 \neq Y_4 \vee Y_2 \neq Y_3.$$

Then we calculate the extreme points coordinates:

$$X_{\min} = \frac{X_1 + X_3}{2}, \quad X_{\max} = \frac{X_2 + X_4}{2},$$

$$Y_{\min} = \frac{Y_2 + Y_3}{2}, \quad Y_{\max} = \frac{Y_1 + Y_4}{2}.$$

Write from the chart the tests extreme points coordinates as:

$$x_{\min}, x_{\max}, y_{\min}, y_{\max}.$$

Middle point coordinates:

$$X_C = \frac{X_1 + X_2 + X_3 + X_4}{4}; \quad Y_C = \frac{Y_1 + Y_2 + Y_3 + Y_4}{4};$$

$$x_C = \frac{x_{\min} + x_{\max}}{2}; \quad y_C = \frac{y_{\min} + y_{\max}}{2}.$$

Find the scales of coordinates:

$$k_X = \frac{(x_{\max} - x_{\min})}{(X_{\max} - X_{\min})}; \quad k_Y = \frac{(y_{\max} - y_{\min})}{(Y_{\max} - Y_{\min})}.$$

Get the graphic coordinates (X, Y) of the experiment.

Then the physical (real) coordinates of the chart function and its parameter:

$$y = y_C + (Y_C - Y)k_Y; \quad x = x_C + (X - X_C)k_X.$$

V. Description of Method

From the analysis of experimental data [3], [4] it is supposed that the functional dependences of C_Q with the parameters *Re* and AR are near to straight lines and the rest ones with β and λ are near to the quadratic and the fourth order parabolas respectively.

Introduce the notations:

$$y = C_Q, \quad x_1 = Re, \quad x_2 = AR, \quad x_3 = \beta, \quad x_4 = \lambda,$$

$$\mathbf{X} = \{x_1, x_2, x_3, x_4\}.$$

Polynomials still give lots of flexibility. Thus such a multidimensional function in the form of the fourth-order tensor product polynomial in $\mathbf{X}$ is proposed:

$$y = \mathrm{F}(\mathbf{X}) = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{2}\sum_{m=0}^{4} a_{i,j,k,m} x_1^i x_2^j x_3^k x_4^m \,. \quad (1)$$

Use the least-square method to define the unknown coefficients $a_{i,j,k,m}$:

$$\sum_{i=1}^{n} \left[F(\mathbf{X}_i) - y_i \right]^2 \to \min \,, \quad (2)$$

where n is the number of tests.

We have the four-dimensional space of parameters. The proportional transforming of the tests parameters into this multidimensional unit hypercube is done. The partial derivatives of this expression with respect to the unknown coefficients $a_{i,j,k,m}$ give the system of linear algebraic equations to be solved.

VI. Discussion of Results

The suitable computer aided algorithm was produced for digitizing the tests data, building and solving the equations system by the Gauss method. The highest order of the polynomial (1) is 8. Traditionally the low order powers are more important. Actually the high order powers bring larger derivations in the extrapolating situations. So, the fourth order polynomial was chosen, simultaneously taking into account (2). It means that we must take in the formula (1) only the definite degrees satisfying the condition:

$$i + j + k + m \le 4 \,.$$

Therefore we obtained polynomial with 36 coefficients. In Figure 2 the comparison with the wind tunnel results [5] was presented.

An example to be calculated had the parameters:

$$V = 4 \text{ m/s}; \quad \omega = 8 \text{ rad/s}; \quad t = 20^{\circ}\mathrm{C};$$
$$\rho = 1.205 \text{ kg/m}^3; \quad \nu = 1.506 \cdot 10^{-5} \text{ m}^2/\text{s};$$
$$N = 2; \quad H = 1.5 \text{ m}; \quad d = 0.5 \text{ m}; \quad s = 0.075 \text{ m}.$$

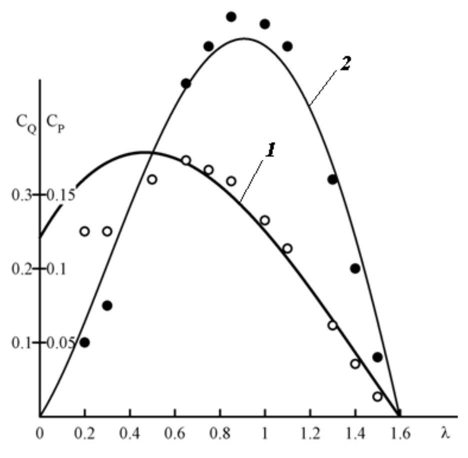

Fig. 2. Torque coefficient (*1*) and efficiency (*2*) for: $\mathrm{Re} = 4.12 \cdot 10^5$; $AR = 1.575$; $\beta = 0.1$

Define the overlap ratio: $\beta = 0.075/0.5 = 0.15$. The rotor radius: $R = 0.5(1 - 0.15/2) = 0.4625$ m. The diameter and the reference length: $D = l = 2 \cdot 0.4625 = 0.925$ m. The aspect ratio: $AR = 1.5/0.925 = 1.622$. The Reynolds number: $\mathrm{Re} = 4 \cdot 0.925/(1.506 \cdot 10^{-5}) = 2.46 \cdot 10^5$. The tip speed ratio: $\lambda = 8 \cdot 0.4625/4 = 0.925$. The torque coefficient by the polynomial: $C_Q = 0.289$. The torque: $Q = 0.289 \cdot 1.205 \cdot 1.5 \cdot 0.4625^2 \cdot 4^2 = 1.78 N \cdot m.$ The power coefficient (efficiency) of the rotor: $C_P = 0.289 \cdot 0.925 = 0.267$. The rotor power: $P = 1.78 \cdot 8 = 14.3 W$.

VII. Description Conclusions

The dimensionless parameters describing the airflow with the Savonius rotors has been chosen.

The computer aided simple algorithm was done to find the polynomial coefficients using the least-square method on the base of experimental tests data. The analytical empiric formula was obtained to determine the torque coefficient with the rotor parameters as the Reynolds number, the rotor aspect ratio, the buckets overlap ratio and the rotor tip speed ratio.

The operation of digitizing the charts with the data derived from the wind tunnel tests of the Savonius rotors was added into the algorithm.

The method for obtaining the analytical expression to determine the torque and power coefficients may be used for the parametrical optimization in the prototype design, at the preliminary stage of the computer aided design or for verification of some method calculating the aerodynamic characteristics of the Savonius rotors.

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Ziganshin Anwar. Assistant Professor.
Aviation Computer-Integrated Complexes Department, National Aviation University, Kyiv, Ukraine.
Education: Kazan Aviation Institute. Kazan, Russia, (1978).
Research interests: computer aided design systems, numerical methods.
Publications: 17.
E-mail: anwarzihan@gmail.com

А. А. Зіганшин. Метод параметрів подібності для обчислення аеродинамічних характеристик роторів Савоніуса

Створено комп'ютерний метод для отримання аналітичної емпіричної формули в формі полінома для визначення коефіцієнту обертального моменту в залежності від параметрів подібності ротору, таких як число Рейнольдса, відносна довжина, відносне перекриття лопатей та швидкохідність. Коефіцієнти полінома отримано розв'язанням системи лінійних алгебраїчних рівнянь, побудованих на основі методу найменших квадратів. Обсяг вихідних даних взятий шляхом оцифрування графіків по результатам випробувань роторів Савоніуса в аеродинамічних трубах.

Ключові слова: аеродинаміка; оцифрування; метод найменших квадратів; поліном багатовимірної апроксимації; ротор Савоніуса.

Зіганшин Анвар Абдуллович. Старший викладач.
Кафедра авіаційних комп'ютерно-інтегрованих комплексів, Національний авіаційний університет, Київ, Україна.
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Кількість публікацій: 17.
E-mail: anwarzihan@gmail.com

А. А. Зиганшин. Метод параметров подобия для вычисления аеродинамических характеристик роторов Савониуса

Создан компьютерный метод для получения аналитической эмпирической формулы в форме полинома для определения коефициента крутящего момента в зависимости от параметров подобия ротора, таких как число Рейнольдса, удлинение, относительное перекрытие лопастей и быстроходность. Коефициенты полинома получены решением системы линейных алгебраических уравнений, построенных на основе метода наименьших квадратов. Объем исходных данных взят путем оцифровки графиков с результатами испытаний роторов Савониуса в аєродинамических трубах.

Ключови слова: аэродинамика; оцифровка; метод наименьших квадратов; полином многомерной апроксимации; ротор Савониуса.

Зиганшин Анвар Абдуллович. Старший преподаватель.
Кафедра авиационных компьютерно-интегрированных комплексов, Национальный авиационный университет, Киев, Украина.
Образование: Казанский авиационный институт. Казань, Россия, (1978).
Направление научной деятельности: системы автоматизации проектных работ, численные методы.
Количество публикаций: 17.
E-mail: anwarzihan@gmail.com