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FLOOD PROTECTION DESIGN DISCHARGE AT THE CONFLUENCE OF THE DANUBE AND THE DRAVA

Keywords: *coinciding flood discharges, lines of equal probabilities of occurrence, exceedance probabilities, design flood discharges*

Introduction. It is standard practice to assess the flood risk on the basis of the probability that the flood will exceed a pre-defined flood wave characteristic. In effect, this is equivalent to determining the flood return period. The approach includes statistical analysis of hydrologic data from the nearest hydrologic station that can provide flood discharge data. From an engineering perspective, the approach yields satisfactory results in a large number of tasks, especially in the case of flood defenses where there are no tributaries along the considered river reach. However, when the protected area includes the mouth of a tributary, the approach does not reliably estimate the considered flood wave characteristics because the rise and development of flood waves on the rivers differ as a rule. Maximum flood waves do not occur simultaneously on both rivers but a flood wave on one can have a significant effect on the flow of the other. It should be kept in mind that hydrologic data are generally collected by hydrologic stations located beyond the zone of mutual influence of the considered rivers. In such cases it is especially important to assess coinciding (concurrent) floods on the recipient and the tributary, and to size flood defenses for the discharge of a certain return period defined by two-dimensional probability analysis.

Methodology.

Coincidence of two random variables. In order to determine the design water levels in the zone of mutual influence of the recipient and a tributary, it is necessary to define the probability of instantaneous occurrence of floods on both of them, which represent random events (random variables X and Y), or, in other words, coincidence [5].

If two-dimensional random variables are normally distributed, the probability distribution function (lines of the same probability of occurrence of random variables *X* and *Y*) can be written as [2]:

$$f(x,y)=\frac{1}{2\pi\cdot\sigma_x\cdot\sigma_y\cdot\sqrt{1-\rho^2}}\cdot$$
$$\cdot\exp\left(-\frac{1}{2\cdot\left(1-\rho^2\right)}\cdot\left[\frac{\left(x-\mu_x\right)^2}{\sigma_x^2}-\frac{2\rho\cdot\left(x-\mu_x\right)\cdot\left(y-\mu_y\right)}{\sigma_x\cdot\sigma_y}+\frac{\left(y-\mu_y\right)^2}{\sigma_y^2}\right]\right) \quad (1)$$

The symbols in Eq. (1) stand for the following: x, y – simultaneous realization of random variables X and Y, respectively; μ_x, μ_y – expected values of X and Y; σ_x, σ_y – standard deviation of X and Y; ρ – coefficient of correlation between X and Y.

For a joint probability density function (jpdf), $f(x,y)$, the marginal densities $f(x,\bullet)$ and $f(\bullet,y)$ are defined by:

$$f(x,\bullet) = \int_{y=-\infty}^{y=\infty} f(x,y)dy, \tag{2}$$

$$f(\bullet,y) = \int_{x=-\infty}^{x=\infty} f(x,y)dx. \tag{3}$$

The marginal cumulative probability functions are determined from:

$$F(x,\bullet) = \int_{t=-\infty}^{t=x} f(t,\bullet)dt \tag{4}$$

and

$$F(x,\bullet) = \int_{t=-\infty}^{t=x} f(t,\bullet)dt. \tag{5}$$

The cumulative probability density function (cpdf), $F(x,y)$, is obtained from:

$$F(x,y) = P\left[X \le x \bigcap Y \le y\right] = \int_{t=-\infty}^{t=x} \int_{z=-\infty}^{z=y} f(t,z)dtdz. \tag{6}$$

The cumulative exceedance probability $\Phi(x,y)$ can be obtained from the following relation [2]:

$$\begin{aligned}\Phi(x,y) &= \int_{t=x}^{t=+\infty} \int_{z=y}^{z=+\infty} f(t,z)dtdz = P\left[X > x \bigcap Y > y\right] = 1 - P\left[X < x \bigcup Y < y\right] = \\ &= 1 - F(x,\bullet) - F(\bullet,y) + F(x,y)\end{aligned} \tag{7}$$

In bivariate statistical analyses of flood characteristics, hydrologists encounter two basic obstacles which must be overcome in a practical implementation of the proposed model.

The first stems from the fact that most flood characteristics are not normally distributed. It is, however, customarily assumed that the considered variables follow the Log-normal distribution. Therefore, their logarithmic transformations in expression (8) are said to be normally distributed:

$$U = \log X; \; W = \log Y. \tag{8}$$

Evaluation of cumulative distribution functions involves extensive calculations in a three-dimensional space, *X*, *Y* and *ρ*, and implementation of a graphoanalytical scheme. This scheme has been described in [1] and is briefly discussed in the ensuing text.

The scheme deals with standard normal variables. Non-standard variables can be transformed into standardized variables by the well-known procedure, namely:

$$\psi = (u - \mu_u)/\sigma_u; \; \xi = (w - \mu_w)/\sigma_w. \tag{9}$$

Based on the above assumption, the variables ψ and ξ are normally distributed, with the expected values $\mu_\psi = \mu_\xi = 0$ and standard deviations $\sigma_\psi = \sigma_\xi = 1$.

With the above transformations, the joint probability density function can be defined as:

$$f(\psi,\xi) = \frac{1}{2\pi \cdot \sqrt{(1-\rho^2)}} \exp\left(-\frac{1}{2\cdot(1-\rho^2)}\left[\psi^2 - 2\rho\psi\xi + \xi^2\right]\right). \tag{10}$$

The values of the correlation coefficient ρ should be replaced by R, which can be calculated from observed data using the standardized series ψ and ξ. With this parameter, and after simplifying the notation in Eq. (10), the following relation can be written:

$$\iint_A f(\psi,\xi)\mathrm{d}\psi\mathrm{d}\xi = 1 - \exp\left(-\frac{\lambda^2}{2-(1-\rho^2)}\right). \tag{11}$$

The integral given by Eq. (11) over an area A, i.e. the integral over the space ψ, ξ from A, represents the probability that the realization of events $\psi = h$ and $\xi = k$ will fall within the area A, which is contoured by an ellipse described by the following equation [10]:

$$\psi^2 - 2\rho\cdot\psi\cdot\xi + \xi^2 = \lambda^2. \tag{12}$$

The newly-introduced symbol λ is obviously related to the constant value of the integral in Eq. (11). Consequently, it is related to the variables ψ and ξ, as well as to the correlation coefficient.

Hence, the probability contained within the ellipse of Eq. (12) can be calculated for each value of $\lambda = \text{const}$.

Equating the variable part of the exponent in Eq. (10) to the exponent of Eq. (11) yields the relation

$$\psi^2 - 2\rho\cdot\psi\cdot\xi + \xi^2 = \lambda^2, \tag{13}$$

$$\xi^2 - 2\rho\cdot\psi\cdot\xi + (\psi^2 - \lambda^2) = 0. \tag{14}$$

As previously stated, any particular value of $\lambda = \text{const}$ corresponds to an ellipse. Furthermore, any given value, $\psi = h$, intersects the ellipse at two different values of ξ, let us say $\xi_1 = k_1$ and $\xi_2 = k_2$.

Hence, solving the quadratic Eq. (14) for any particular value of $\lambda = \text{const}$ corresponding to the required level of probability given by Eq. (11) results in two particular coordinates ($\xi_1 = k_1, \xi_2 = k_2$), which represent the intersection of the ellipse and the straight line $\psi = h_0$. A series of ellipses can be constructed by repeating the calculations for several selected values of λ while varying the values of $\psi = h_0$. After each calculation step, a transformation corresponding to Eq. (9) should be performed to obtain unstandardized values of the flood characteristics, instead of standardized logarithmic values.

The described computational scheme is rather direct. However, the results have no great use, except to give the analyst general insight into the relation of the considered flood characteristics.

In the case of evaluation of the cumulative distribution function, the direct method, as previously outlined, is not convenient. To overcome computational difficulties, the Abramowitz and Stegun [1] procedure was implemented in this study. The computational scheme uses a grapho-analytical procedure that defines the cumulative probability, $\Phi(h,k,\rho)$, in terms of the probabilities $\Phi(h,0,r)$ and $\Phi(k,0,r)$, where instead of the correlation coefficient, ρ, the value $r = r(h,k,\rho)$ is used. The value r is related to h and k, as well as to ρ itself. More specifically, the probability $\Phi(h,k,\rho)$ can be assessed from:

$$\Phi(h,k,\rho) = \Phi\left(h,0,\frac{(\rho h - k)\cdot \operatorname{sgn} h}{\sqrt{h^2 - 2\rho hk + k^2}}\right) + \Phi\left(k,0,\frac{(\rho k - h)\cdot \operatorname{sgn} k}{\sqrt{h^2 - 2\rho hk + k^2}}\right) - \begin{cases} 0 & \text{if } hk \geq 0 \text{ and } h + k \geq 0 \\ \frac{1}{2} & \text{for all other cases} \end{cases}, \qquad (15)$$

where ($\operatorname{sgn} h$) and ($\operatorname{sgn} k$) are equal to 1 if h or k, respectively, are greater than or equal to zero, and they become -1 whenever h and k are less than zero.

It should be reiterated that the described procedure requires that the variables X and Y be logarithmed and properly transformed into standard normal variables. Therefore, the particular values of h_0 and k_0, for which the exceedance probability is calculated, must consequently be converted into natural values, namely:

$$x = 10^u = 10^{\left(\sigma_u \psi + \bar{U}\right)} = 10^{\left(\sigma_u h_0 + \bar{U}\right)}, \qquad (16)$$

$$y = 10^w = 10^{\left(\sigma_w \xi + \bar{W}\right)} = 10^{\left(\sigma_w k_0 + \bar{W}\right)}. \qquad (17)$$

Any value obtained according to the above-described model represents the probability that a flood event, which corresponds to particular magnitudes x_0 and y_0, will exceed a chosen combination of X and Y.

A model, based on the described procedure and utilizing the charts presented in [1], has been developed to perform the above calculations related to the two-dimensional distribution function. It contains the correlation coefficient, as a measure of dependence of the flood events in question. In order to assess the strength of that correlation, the error of the computed correlation coefficients needs to be estimated. To that end, relation (18) was used [11]:

$$\sigma_R = \left(1 - R^2\right) / \sqrt{N}, \qquad (18)$$

where: σ_R - error of the correlation coefficient R; N - total number of data.

In this paper the following criterion was adopted: the correlation coefficient, R, is significantly different from zero if its absolute value is greater than the triple value of the error, σ_R that is $|R| \geq 3 \cdot \sigma_R$.

Based on the absolute value of the correlation coefficient, three degrees of statistical significance of the coincidence of two random variables are distinguished:

(1) Nearly statistically-significant coincidence:

$$|R| \geq 0.95 - 1.0. \qquad (19)$$

(2) Statistically significant coincidence:

$$|R| \geq 3 \cdot \sigma_R - 0.95 . \quad \textbf{(20)}$$

(3) Statistically insignificant coincidence:

$$|R| \leq 3 \cdot \sigma_R \quad \textbf{(21)}$$

Defining the variables. The analysis of coinciding flood discharges of the recipient and a tributary is founded upon the definition of a two-parametric law of distribution of the combinations of variables shown in Table 1 [4].

Table 1. **Combinations of simultaneously occurring variables**

River Reach		Combination	Appellation
Gauging station 1	Gauging station 2		
The main river upstream from the tributary	The main river downstream from the tributary	max-cor	*QINmax - QOUTcor1*
		cor-max	*QINcor1 - QOUTmax*
The main river upstream from the tributary	Tributary	max-cor	*QINmax - QTRcor1*
		cor-max	*QINcor2 - QTRmax*
Tributary	The main river downstream from the tributary	max-cor	*QTRcor2 - QOUTmax*
		cor-max	*QTRmax - QOUTcor2*

The coincidence calculations result in a line of similar probabilities of the above combinations of the selected flood wave parameter (differential distribution laws), as well as lines that define the exceedance probabilities of the same combinations of variables:

$$P\left[X > x; Y_{cor} > y\right] = \int_{X_1}^{\infty}\int_{Y_1}^{\infty} g\left(X, Y_{cor}, R\right) dxdy , \quad \textbf{(22)}$$

$$P\left[X > x; Y_{cor1} > y\right] = \int_{X_1}^{\infty}\int_{Y_1}^{\infty} g\left(X, Y_{cor1}, R\right) dxdy_{cor1} , \quad \textbf{(23)}$$

$$P\left[X_{cor} > x; Y > y\right] = \int_{X_1}^{\infty}\int_{Y_1}^{\infty} g\left(X_{cor}, Y, R\right) dx_{cor} dy , \quad \textbf{(24)}$$

where: X , Y_{cor} - the highest annual flood wave peak of the recipient upstream from the mouth of the tributary and corresponding flood wave peak of the recipient downstream from the tributary, respectively; X , Y_{cor1} - the highest annual flood wave peak of the recipient and corresponding flood wave peak of the tributary; Y , X_{cor} - the highest annual flood wave peak of the tributary and corresponding flood wave peak of the recipient upstream from the tributary.

Determining analytical flood discharges of characteristic probabilities of occurrence. The results of calculations of analytical flood discharges of the recipient and the tributary in the zone of the mouth of the tributary can be used in practice to define: analytical water levels at a gauged confluence and analytical discharges at an insufficiently gauged mouth of the tributary – when there is no data on the downstream reach of the recipient [8].

The theoretical background for all the above aspects of the practical uses of the results of calculations of coinciding flood discharges of the recipient and the tributary in the zone of their confluence is provided below.

Estimation of coinciding flood discharges to define design water levels at gauged confluences

A confluence is said to be gauged if hydrologic data (hydrologic stations) are available on the input cross-sections (of the recipient and the tributary) and the output cross-section (of the recipient downstream from the mouth of the tributary) [8]. The following data are needed to calculate design water levels:

- time-series of annual maximum discharges at the entry and exit stations, and
- results of flood discharge coincidence calculations of the following combinations of variables:
  - the highest annual discharge of the recipient and the corresponding discharge of the tributary, and
  - the highest annual discharge of the tributary and the corresponding discharge of the recipient.

The design water levels of the recipient and the tributary in the extended area of the confluence are obtained from hydraulic calculations of the water level lines at selected design discharges. The design water levels in the case of gauged confluences are determined for [6, 9]:

- the reach of the recipient downstream from the confluence:
  - the design discharge is the theoretical value of the annual maximum discharges $QOUT_{max,p}$ for the selected probability of occurrence p at the hydrologic station on the recipient downstream from the mouth of the tributary;
- for the reach of the recipient upstream from the confluence, in the zone of mutual influence of the recipient and the tributary, the design water level is an envelope of the maximum water levels obtained by calculating the water level lines of the water surface for the following combinations of discharges:
  - theoretical value of the highest annual discharge of the recipient downstream from the confluence for the probability of occurrence p and corresponding discharge of the recipient upstream from the confluence for the same probability of coincidence $(QOUT_{max};QIN_{cor1})_p$,
  - corresponding discharge of the recipient downstream from the confluence and theoretical value of the highest annual discharge of the recipient upstream from the confluence, for the probability of occurrence p and the same coincidence probability $(QIN_{max};QOUT_{cor1})_p$,
- for the tributary upstream from the confluence, in the zone of mutual influence of the recipient and the tributary – the design water level is an envelope of the maximum water levels obtained by calculating the water level lines for the following combinations of discharges:
  - theoretical value of the highest annual discharge of the recipient upstream from the confluence for the selected probability of occurrence p and corresponding

discharge of the tributary, for the same coincidence probability $(QOUT_{max};QTR_{cor2})_p$,

- corresponding discharge of the recipient downstream from the confluence and theoretical value of the highest annual discharge of the tributary for the selected probability of occurrence p and the same coincidence probability $(QTR_{max};QOUT_{cor2})_p$,

- for the recipient upstream from the zone of mutual influence of the recipient and the tributary – the design water levels are obtained by hydraulic calculations of the water level lines for the theoretical value of the highest annual discharge of the recipient (at the upstream hydrologic station), for the selected probability of occurrence $QIN_{max,p}$,
- for the tributary upstream from the zone of mutual influence of the recipient and the tributary – the design water levels are obtained from hydraulic calculations of the water levels lines for the theoretical value of the highest annual discharge of the tributary (at the upstream hydrologic station), $QTR_{max,p}$, for the selected probability of occurrence p.

The design water level lines for the zone of mutual influence of the recipient and the tributary are determined as schematically represented in Fig. 1. The adopted level of protection corresponds to the selected probability of occurrence p [6].

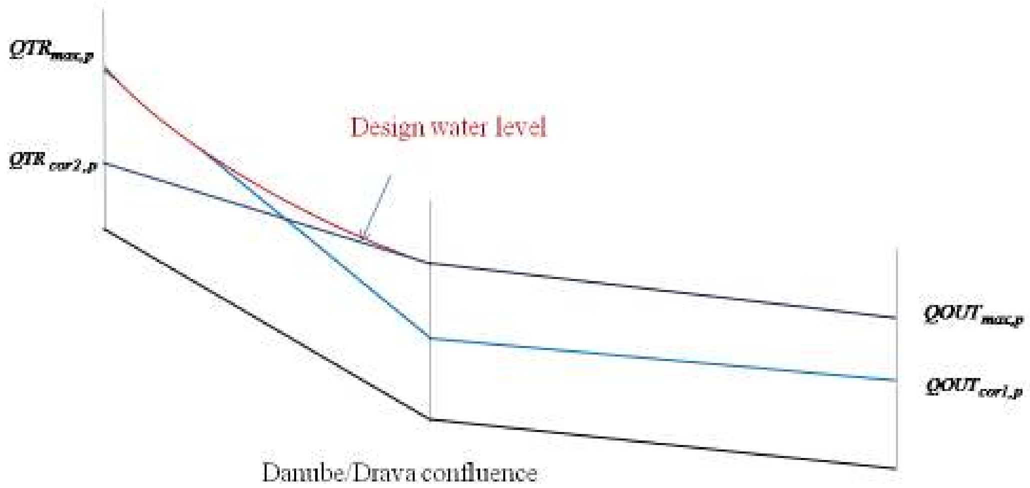

Figure 1. **Schematic representation of the selection of the design water level in the zone of the confluence**

Estimation of coinciding flood discharges to define design water levels at partially gauged confluences

A partially gauged confluence refers to the extended sector of the confluence, where data are not available at one station. All the necessary probabilities and coincidences of the variables described in Eqs. (22), (23) and (24) are defined on the basis of available data.

To simplify the procedure, shown below is how calculations are made in the absence of data on the output cross-section of the recipient [7]. This means that time-series of daily discharges are available for: the input cross-section of the recipient $\left(QIN_{max}\right)$ and the input cross-section of the tributary $\left(QTR_{max}\right)$.

In this case it is necessary to define the coincidences (lines of the same probability of occurrence $f(x,y)$ and cumulative lines of exceedance probabilities $\Phi(x,y)$) for the following combinations of variables, only for synchronous occurrences:

- the highest annual discharge of the recipient upstream from the mouth of the tributary – corresponding discharge of the tributary $\left(QIN_{max};QTR_{cor1}\right)$, and
- the highest annual discharge of the tributary – corresponding discharge of the recipient upstream from the mouth of the tributary $\left(QTR_{max};QIN_{cor2}\right)$.

In this case study, the maximum discharge of a certain probability of occurrence – $QOUT_{max,p}$ was determined based on two points of intersection (1 and 2) of the previously-mentioned coincidence lines in two cases:

$$P\left[\left(QIN_{max} > qIN_{max}\right)\cap\left(QTR_{cor1} > qTR_{cor1x}\right)\right] = p,\quad f\left(QIN_{max},QTR_{cor1}\right) = p, \tag{25}$$

$$P\left[\left(QTR_{max} > qTR_{max}\right)\cap\left(QIN_{cor2} > qIN_{cor2x}\right)\right] = p,\quad f\left(QTR_{max},QIN_{cor2}\right) = p, \tag{26}$$

where: p is the probability of occurrence.

The coordinates of the intersected points were:

- Case 1:
  - Point 1 $\left(QIN^{1}_{max} : QTR^{1}_{cor1}\right)_p$
  - Point 2 $\left(QIN^{2}_{max} : QTR^{2}_{cor1}\right)_p$
- Case 2:
  - Point 1 $\left(QTR^{1}_{max} : QIN^{1}_{cor2}\right)_p$
  - Point 2 $\left(QTR^{2}_{max} : QIN^{2}_{cor2}\right)_p$

The design value of the highest discharge along the reach of the recipient downstream from the mouth of the tributary, for the probability of occurrence p – $QOUT_{max,p}$ Eq. (27), is equal to the mean value of the sum of coordinates of the two points in the two graphs, i.e.:

$$QOUT_{max,p} = \left(\begin{array}{l} \sum_{1}^{2}\left(QIN^{1}_{max,p} + QTR^{1}_{cor1,p}\right) + \sum_{1}^{2}\left(QIN^{2}_{max,p} + QTR^{2}_{cor1,p}\right) + \\ + \sum_{1}^{2}\left(QTR^{1}_{max,p} + QIN^{1}_{cor2,p}\right) + \sum_{1}^{2}\left(QTR^{2}_{max,p} + QIN^{2}_{cor2,p}\right) \end{array}\right) / 4. \tag{27}$$

It should be kept in mind that the basic assumption of this approach is that the intermediate catchment in the considered sector between the input cross-sections and the output cross-section has no significant effect on flood wave formation at the output cross-section of the recipient.

Results.

***Flood discharges of characteristic probabilities of occurrence at a gauged confluence*.** The primary criterion related to the construction of flood defenses in the zone of the confluence of the Drava and the Danube is cost-effective sizing of all structural flood protection measures [3]. In the specific case, the main structures are levees. The sector of the Danube River from the hydrologic stations at Bezdan on the Danube and Donji Miholjac on the Drava to the hydrologic station at Bogojevo on the Danube is shown in Fig. 2.

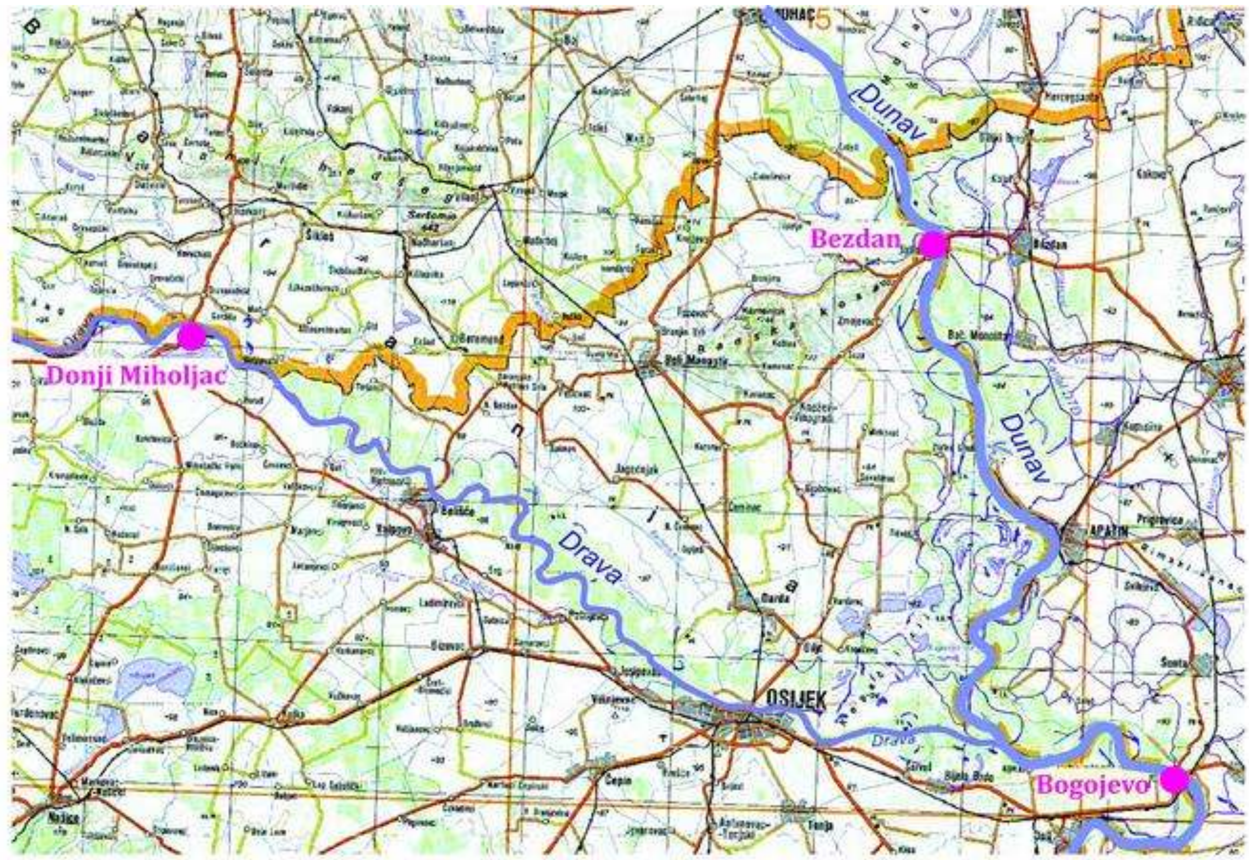

Figure 2. **Reach of the Danube River from the hydrologic station at Bezdan to the hydrologic station at Bogojevo**

The theoretical discharges of different return periods at the considered river cross-sections, derived by the conventional approach for statistically significant coincidences, based on time-series of annual discharges from 1931 to 2014, served as a basis for sizing of flood defenses. Goodness-of-fit tests (χ^2, Kolmogorov-Smirnov and $n\omega^2$) revealed that Gumbel's theoretical probability distribution function best fitted the empirical values of the data recorded by the three hydrologic stations. The theoretical discharges of characteristic probabilities of occurrence are shown in Table 2.

Table 2. **Theoretical values of annual maximum discharges of the Danube and the Drava for different probabilities of occurrence – $Q_{max,p}$ (m³/s)**

Probability of occurrence (%)	Danube		Drava at Donji Miholjac
	at Bezdan	at Bogojevo	
0.1	11020	12350	3384
1	8810	9910	2652
5	7249	8186	2136

However, values that represent derived quantities, which depend on the strength of coincidence of flood discharges of the Danube and the Drava according to criteria (19), (20) and (21), need to be considered in relation to the recipient upstream from the confluence, within the zone of mutual influence of the two rivers, in order to define design discharges for sizing of flood protection. The optimal approach is to adopt the most likely combination of discharge coincidence variables of the Danube and the Drava from the exceedance probability curve, taking into consideration the place of origin of the flood wave for the selected level of protection (i.e. return period).

In the present case study, the coincidence of flood discharges of the Danube and the Drava was estimated for the following combinations of variables:

- highest annual discharge at Bezdan – corresponding discharge at Bogojevo $\left(Q_{max}^{Bez};Q_{cor1}^{Bog}\right)\equiv\left(\text{IMOC}\right)$,
- corresponding discharge at Bezdan – highest annual discharge at Bogojevo $\left(Q_{cor1}^{Bez};Q_{max}^{Bog}\right)\equiv\left(\text{ICOM}\right)$,
- highest annual discharge at Bezdan – corresponding discharge at Donji Miholjac $\left(Q_{max}^{Bez};Q_{cor1}^{DM}\right)\equiv\left(\text{IMTC}\right)$,
- corresponding discharge at Bezdan – highest annual discharge at Donji Miholjac $\left(Q_{cor2}^{Bez};Q_{max}^{DM}\right)\equiv\left(\text{ICTM}\right)$,
- highest annual discharge at Bogojevo – corresponding discharge at Donji Miholjac $\left(Q_{max}^{Bog};Q_{cor2}^{DM}\right)\equiv\left(\text{TCOM}\right)$,
- corresponding discharge at Bogojevo – highest annual discharge at Donji Miholjac $\left(Q_{cor2}^{Bog};Q_{max}^{DM}\right)\equiv\left(\text{TMOC}\right)$.

The results are graphically represented in Figs. 3 through 8, including lines of the same probabilities of occurrence (density functions), lines of exceedance probabilities (distribution functions) and empirical points.

To assess the statistical significance of the calculated coincidences of flood discharges of the Danube and the Drava, Table 3 shows the main indicators of the strengths of the established coincidence correlations – the coefficient of linear correlation and standard error.

The above results lead to the conclusion that there is statistically significant coincidence between the combinations of the highest annual discharges of the tributary at Donji Miholjac and the corresponding discharges of the recipient at Bezdan and Bogojevo, as well as between all combinations of discharges at the upstream station (Bezdan) and the downstream station (Bogojevo) on the recipient. None of the combinations of the highest annual discharges of the recipient (Bezdan or Bogojevo) and the corresponding discharges of the tributary is statistically significant.

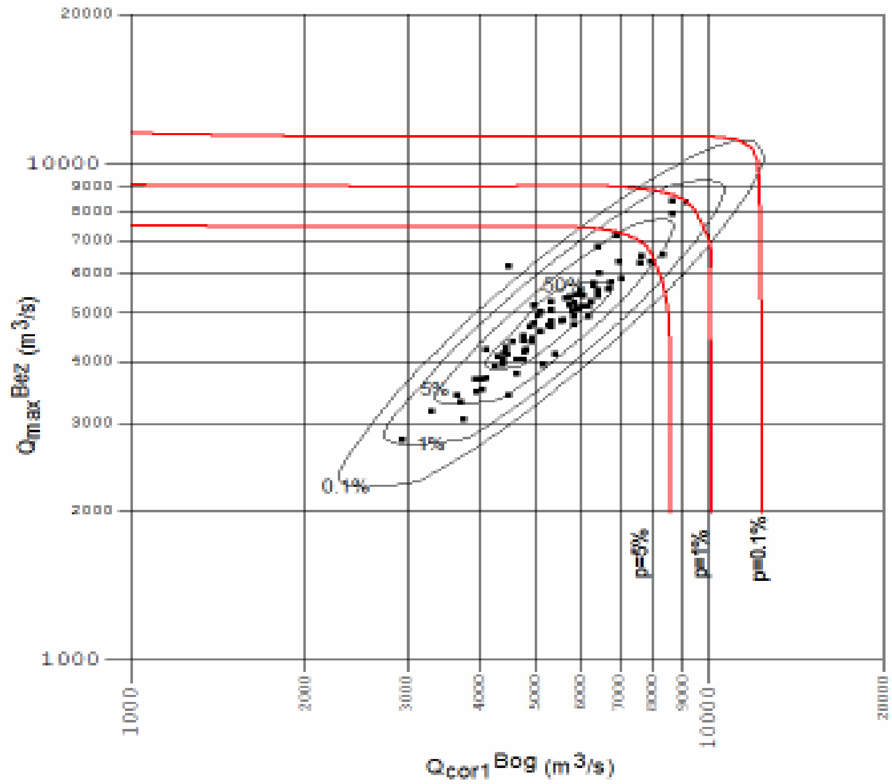

Figure 3. **Coincidence of the maximum annual discharge of the Danube at Bezdan and the corresponding discharge of the Danube at Bogojevo (*IMOC*)**

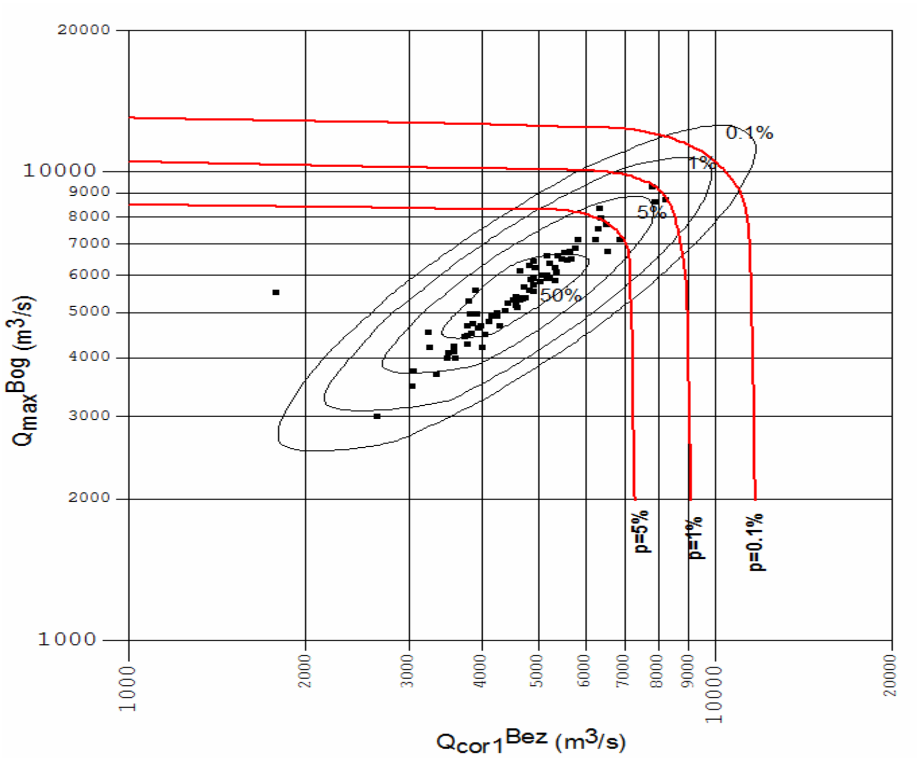

Figure 4. **Coincidence of the maximum annual discharge of the Danube at Bogojevo and the corresponding discharge of the Danube at Bezdan (*ICOM*)**

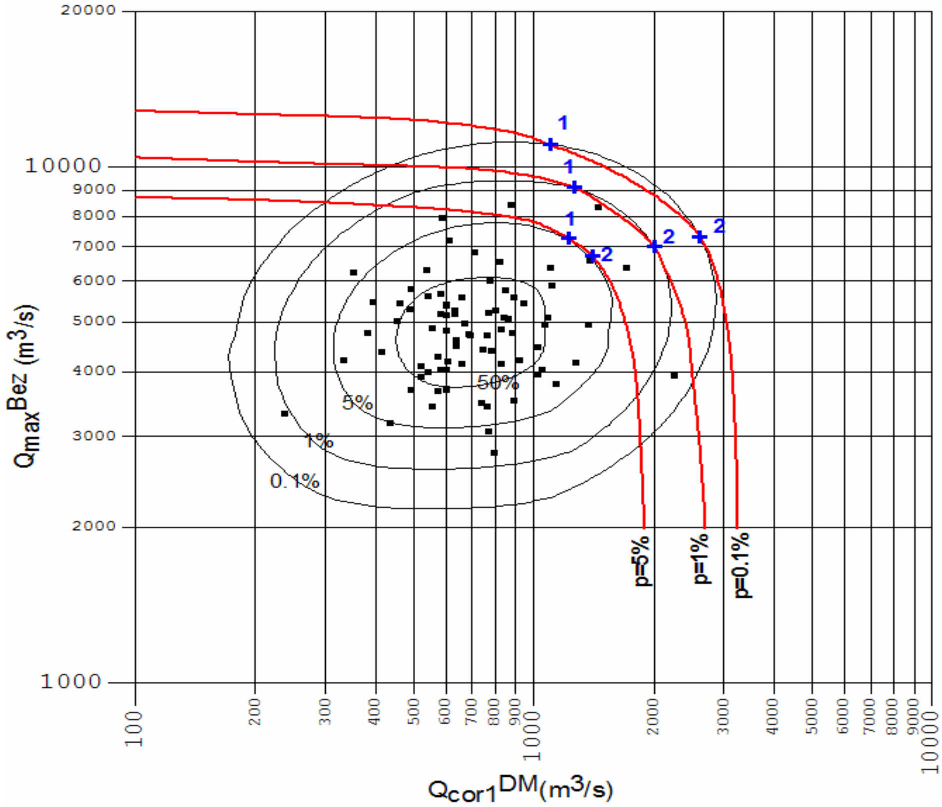

Figure 5. **Coincidence of the maximum annual discharge of the Danube at Bezdan and the corresponding discharge of the Drava at Donji Miholjac (*IMTC*)**

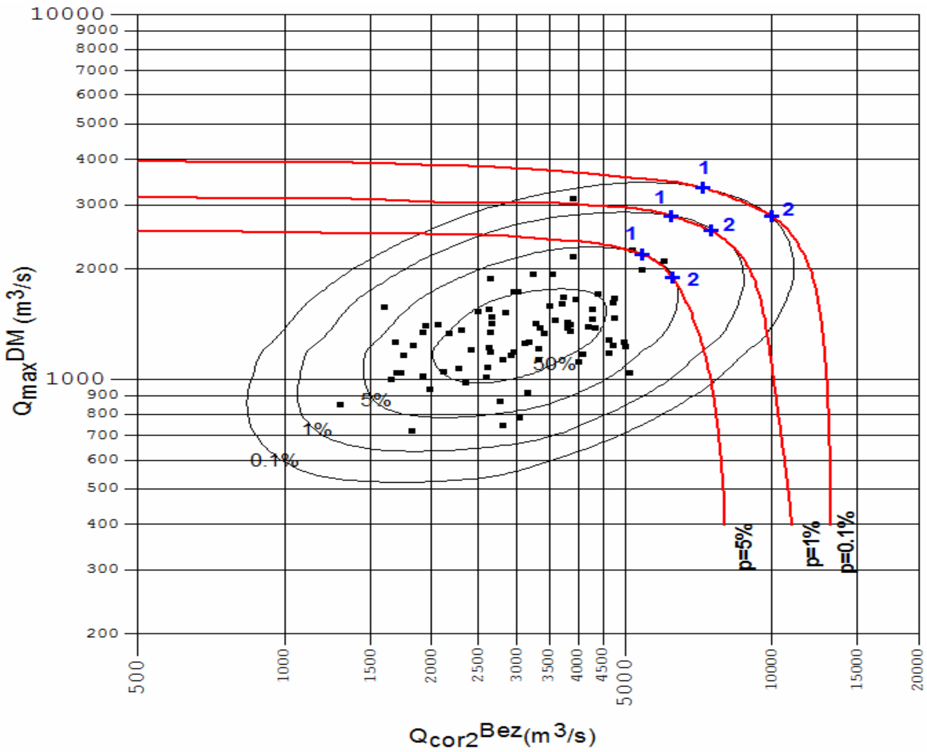

Figure 6. **Coincidence of the maximum annual discharge of the Drava at Donji Miholjac and the corresponding discharge of the Danube at Bezdan (*ICTM*)**

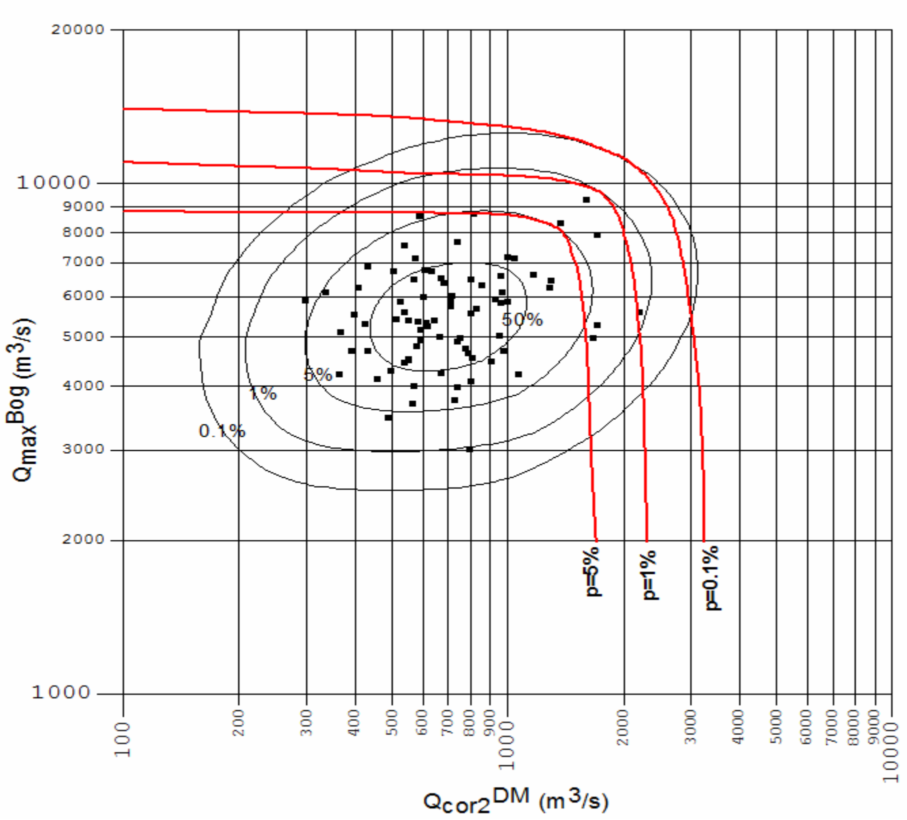

Figure 7. **Coincidence of the maximum annual discharge of the Danube at Bogojevo and the corresponding discharge of the Drava at Donji Miholjac (*TCOM*)**

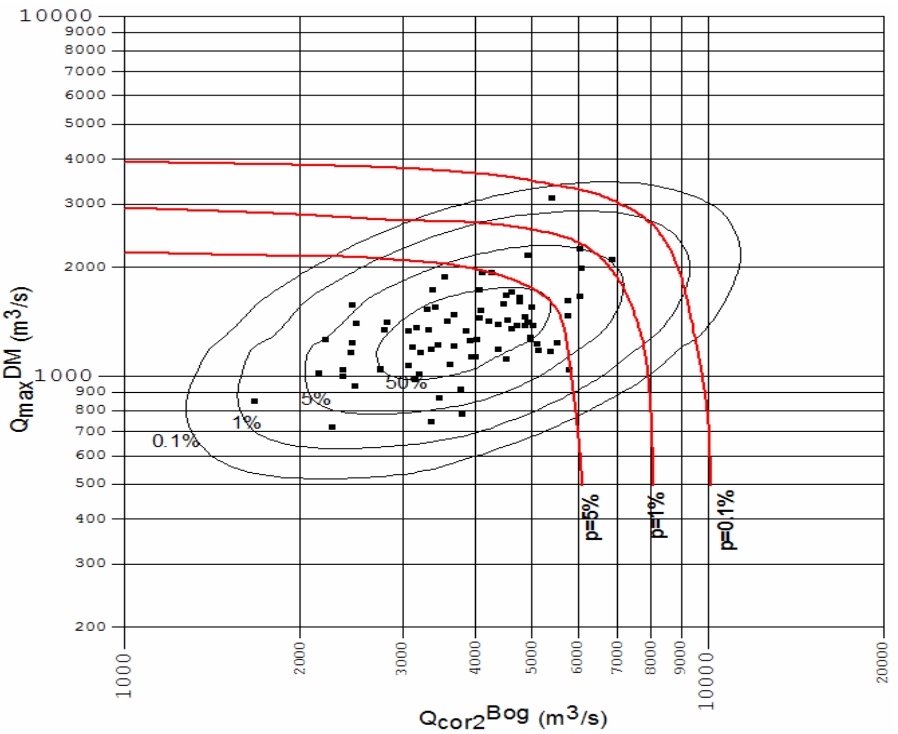

Figure 8. **Coincidence of the maximum annual discharge of the Drava at Donji Miholjac and the corresponding discharge of the Danube at Bogojevo (*TMOC*)**

Table 3. **Statistical significance of the considered combinations of variables**

Hydrologic stations	Combination of variables	R	N	σ	3σ	Statistical significance
Bezdan – Bogojevo	max – cor	0.91809	79	0.017676	0.053029	YES
	cor – max	0.8561	79	0.030050	0.090151	YES
Bezdan – Donji Miholjac	max – cor	0.15869	79	0.109676	0.329027	NO
	cor – max	0.45362	79	0.089358	0.268073	YES
Donji Miholjac – Bogojevo	cor – max	0.24087	79	0.105981	0.317944	NO
	max – cor	0.51906	79	0.082196	0.246589	YES

The values shown in Table 3 corroborate the validity of the proposed approach for the estimation of the coincidence of flood discharges in the extended zone of the confluence of the Drava and the Danube.

The analytical discharges for the different combinations of variables are shown in Table 4.

Table 4. **Analytical flood discharges of the Danube and the Drava for different coincidence probabilities**

p%	Danube upstream from confluence			Danube downstream from confluence			Drava upstream from confluence		
	Q_{max}^{Bez}	Q_{cor1}^{Bog}	Q_{cor1}^{DM}	Q_{max}^{Bog}	Q_{cor2}^{Bez}	Q_{cor1}^{DM}	Q_{max}^{DM}	Q_{cor2}^{Bog}	Q_{cor1}^{Bez}
0.1	11020	11750	1700	12350	10000	2000	3384	8000	9000
1.0	8810	9100	1650	9910	8000	1770	2652	6800	7000
5.0	7249	7800	1300	8186	6500	1180	2136	5400	5800

The analytical discharges for determining design water levels for sizing flood defenses along the Danube from its point of entry into Serbia to the hydrologic station at Bogojevo, and along the Drava from the hydrologic station at Donji Miholjac to the confluence with the Danube, are schematically represented in Figs. 9 and 10, respectively.

For the reach of the Danube upstream from the mouth of the Drava, the water level envelopes of a 100-year return period would be obtained on the basis of the following combinations of discharges: $Q_{max,1\%}^{Bog} = 9910\ \mathrm{m^3/s}$ and the corresponding discharge from Fig. 4, $Q_{cor1,1\%}^{Bez} = 8000\ \mathrm{m^3/s}$, and $Q_{max,1\%}^{Bez} = 8810\ \mathrm{m^3/s}$ and the corresponding discharge from Fig. 3, $Q_{cor1,1\%}^{Bog} = 9100\ \mathrm{m^3/s}$.

For the reach of the Drava upstream from its mouth, the water level envelope of a 100-year return period would be obtained from the combinations: $Q_{max,1\%}^{DM} = 2652\ \mathrm{m^3/s}$ and the corresponding discharge from Fig. 8, $Q_{cor2,1\%}^{Bog} = 6800\ \mathrm{m^3/s}$, and $Q_{max,1\%}^{Bog} = 9910\ \mathrm{m^3/s}$ and the corresponding discharge from Fig. 7, $Q_{cor2,1\%}^{DM} = 1770\ \mathrm{m^3/s}$.

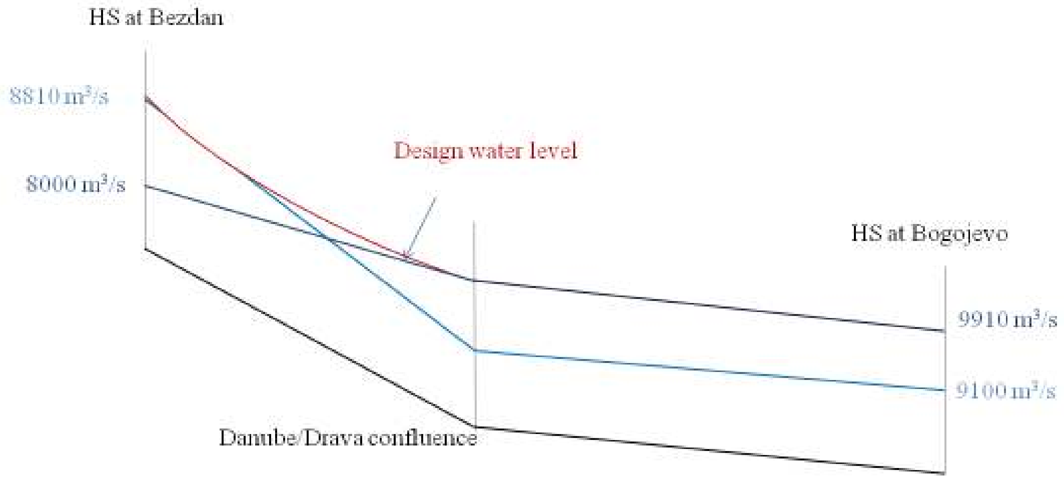

Figure 9. **Maximum design discharges for estimating the 100-year water level along the considered reach of the Danube**

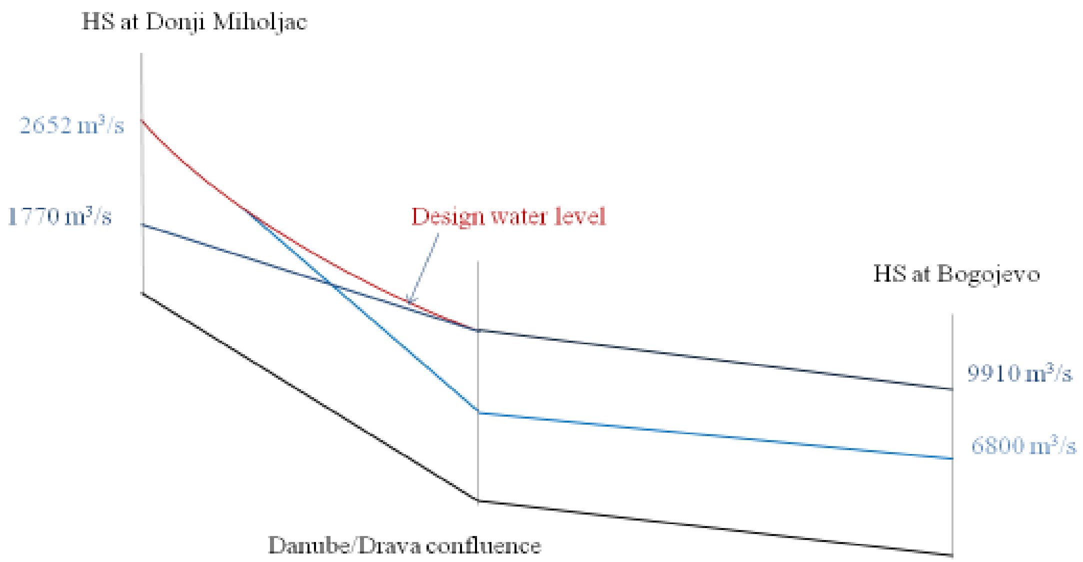

Figure 10. **Maximum design discharges for estimating the 100-year water level along the Danube to the mouth of the Drava and along the Drava to the hydrologic station at Donji Miholjac**

Design flood discharges at an insufficiently gauged cross-section of the recipient. In order to apply the proposed approach to the estimation of flood discharge coincidence for defining design water levels at a partially gauged cross-section, it was assumed that there are only two upstream gauging stations, at Bezdan and Donji Miholjac, in the considered sector of the Danube and the Drava, and that there are no data on the reach downstream from the mouth of the Drava. Data from the "non-existent" station at Bogojevo were used only to verify the results.

The results of coincidence calculations for the following combinations of variables at the hydrologic stations at Bezdan and Donji Miholjac were used:

- maximum annual discharge of the Danube at Bezdan – corresponding discharge of the Drava at Donji Miholjac $\left(Q_{max}^{Bez}; Q_{cor1}^{DM}\right)$, and

- annual maximum discharge of the Drava at Donji Miholjac – corresponding discharge of the Danube at $\left(Q_{\max}^{DM}; Q_{cor2}^{Bez}\right)$.

Table 5 shows the analytical results for maximum design discharges of the Danube at the 'non-existent' station at Bogojevo, for the probabilities of occurrence p = 0.1, 1.0 and 5.0 %.

It follows from the results that the proposed method for estimating the flood discharge coincidence of a recipient and a tributary is also suitable for defining theoretical values of maximum discharges of certain probabilities of occurrence along the recipient downstream from the mouth of the tributary, if time-series of daily and annual maximum discharges at the two input cross-sections in the upstream sector are available. It was assumed in the above example concerning the estimation of annual maximum discharges of the Danube at Bogojevo (Table 5, Figs. 5 and 6) that data were available only on the upstream sector, at Bezdan on the Danube and Donji Miholjac on the Drava. The resulting analytical values of annual maximum discharges of the Danube at Bogojevo, based on defined coincidence functions, matched very well the results of conventional probabilistic analysis (Table 5). The differences between the analytical values based on coincidence and the statistical analysis were minimal – the errors were in the interval from -7.0% (for a 1000-year return period) to -1.3% (50-year return period).

Table 5. **Analytical discharges of the Danube at the 'non-existent' Bogojevo station for different probabilities of occurrence**

Combi-nation	Variable	5%			1%			0.1%		
		Point		∑∑	Point		∑∑	Point		∑∑
		1	2		1	2		1	2	
$Q_{\max}^{Bez}; Q_{cor1}^{DM}$	$Q_{\max}^{Bez}$	7250	6700	13950	9100	7000	16400	11020	7300	18320
	Q_{cor1}^{DM}	1220	1400	2620	1260	2000	3200	1100	2610	3710
	∑			16570			19600			22030
$Q_{\max}^{DM}; Q_{cor2}^{Bez}$	$Q_{\max}^{DM}$	2200	1900	4100	2800	2550	5150	3350	2800	6150
	Q_{cor2}^{Bez}	5400	6250	11650	6200	7500	13200	7200	10000	17000
∑				15750			18350			23150
∑∑				32320			37950			45180
$Q_{\max,p}^{Ra\check{c}} = \sum\sum/4$				**8080**			**9487**			**11295**
Bogojevo	$Q_{\max,p}^{racGumbel}$			**8186**			**9910**			**12150**
	$\Delta Q_{\max,p}(\%)$			**-1.3**			**-4.3**			**-7.0**

Conclusion. The practical significance of the results of coincidence estimation is that flood protection could be sized on the basis of design discharges that provide a lower level of protection in the zone of mutual influence of a recipient and a tributary, from a conventional one-dimensional design approach perspective, while ensuring the same level of protection from a flood risk standpoint. The proposed coincidence estimation approach yields representative quantitative indicators of optimal combinations of the considered random variables, from the standpoint of cost-effectiveness and safety. The results can be used to define design water levels at river mouths, where the required (appropriate) data are not available at one gauging station.

The developed method for estimating the coincidence of flood discharges of the recipient and a tributary is also suitable for defining maximum design discharges if no data are available on a river cross-section. The calculations in the case of no data at the hydrologic station of Bogojevo showed that the errors were minimal, up to 10%.

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Расчетные расходы воды для зашиты от наводнений в месте слияния Дуная и Дравы

Стеван Прохаска, Александра Илич, Владислава Бартош Дивац

В статье представлены теоретические основы для определения совпадений (одновременных) паводковых расходов воды в зоне сильного взаимного влияния между основной рекою (реципиентом) и притоком, в том числе математические основы и порядок определения линий той же самой вероятности появления и линий вероятности превышений двух случайных величин. В случае сложной речной системы, ограниченной двумя входными сечениями (реципиента и притоки) и одним выходом поперечного сечения (реципиента), соответствующие комбинации ежегодных максимальных расходов воды рек и соответствующие (синхронные) расходы воды определены в других входных / выходных сечениях, при условии, если не существует значительного влияния притока из промежуточного водосборного бассейна.

Результаты моделирования совпадений расходов воды паводков в секторе Дуная представлены от точки въезда в Сербию (гидрологический пост в Бездане) до контрольно-измерительной станции в Богоево, а также в нижнем течении реки Дравы - от контрольно-измерительной станции в Дони-Михоляц до ее устья. В работе численно и графически проиллюстрированы результатами расчетов.

Ключевые слова: *совпадение паводковых расходов воды, линии равных вероятностей появления, вероятностей превышения, расчетная схема паводковых расходов воды.*

Розрахункові витрати води для захисту від повеней в місці злиття Дунаю і Драви

Стеван Прохаска, Олександра Іліч, Владислава Бартош Дівац

У статті представлені теоретичні основи для визначення збігів (одночасних) паводкових витрат води в зоні сильного взаємовпливу між основною рікою (реципієнтом) і припливом, в тому числі математичні основи та порядок визначення ліній тієї ж самої ймовірності появи і ліній ймовірності перевищень двох випадкових величин. У разі складної річкової системи, обмеженої двома вхідними перетинами (реципієнта і притоки) і одним виходом поперечного перерізу (реципієнта), відповідні комбінації щорічних максимальних витрат води річок і відповідні (синхронні) витрати води визначені в інших вхідних / вихідних перетинах, за умови, якщо не існує значного впливу припливу з проміжного водозбірного басейну.

Результати моделювання збігів витрат води паводків в секторі Дунаю представлені від точки в'їзду до Сербії (гідрологічний пост в Бездане) до контрольно-вимірювальної станції в Богоево, а також в нижній течії річки Драви - від контрольно-вимірювальної станції в Доні-Міхоляц до її гирла. У роботі чисельно і графічно проілюстровані результатами розрахунків.

Ключові слова: *збіг паводкових витрат води, лінії рівних ймовірностей появи, ймовірностей перевищення, розрахункова схема паводкових витрат води.*

Flood protection design discharge at the confluence of the Danube and the Drava

Stevan Prohaska, Aleksandra Ilić, Vladislava Bartoš Divac

The theoretical background for defining coinciding (concurrent) flood discharges in the zone of strong mutual influence between the recipient and a tributary is presented in the paper, including the mathematical basis and a procedure for defining lines of the same probability of occurrence and lines of probability exceedance of two random variables. In the case of a complex river system, bounded by two input cross-sections (of the recipient and the tributary) and one output cross-section (of the recipient), relevant combinations of annual maximum river discharges and corresponding (synchronous) discharges are defined at other input/output cross-sections, if there is no significant influence of inflow from the intermediate catchment.

The results of the simulation of coinciding flood discharges for the sector of the Danube from the point of entry into Serbia (gauging station at Bezdan) to the gauging station at Bogojevo are presented, including the lower course of the Drava – from the gauging station at Donji Miholjac to its mouth. The paper is numerically and graphically illustrated with calculation results.

Keywords: *coinciding flood discharges, lines of equal probabilities of occurrence, exceedance probabilities, design flood discharges.*

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